6th GRADE MATH

Curriculum Map 2016-2017

CANYONS
School District

MATH CURRICULUM MAP
CANYONS SCHOOL
DISTRICT 2016-2017

Curriculum Mapping Purpose

Canyons School District's curriculum math maps are standards-based maps driven by the Utah Core State Standards for Mathematics and implemented using Big Ideas. Student achievement is increased when both teachers and students know where they are going, why they are going there, and what is required of them to get there. Additional instructional days were intentionally built into the map to allow teachers to go into more depth on concepts. Supporting resources for these additional days can be found in the General Information section.

Curriculum Maps are a tool for:

- **ALIGNMENT:** Provides support and coordination between concepts, skills, standards, curriculum, and assessments
- **COMMUNICATION:** Articulates expectations and learning goals for students
- **PLANNING:** Focuses instruction and targets critical information
- **COLLABORATION:** Promotes professionalism and fosters dialogue between colleagues about best practices in both instruction and assessment.
- **SCAFFOLDED INSTRUCTION AND GROUPING STRUCTURES:** The organization of a scaffolded classroom includes whole group, small group (e.g., teacher-led skill-based, cooperative learning), partner, and independent work where students are provided support towards mastery. As students assume more responsibility for the learning, gradual support is decreased in order to shift the responsibility for learning from the teacher to the students.

Canyons School District secondary math maps are created by CSD secondary
teachers and published by the CSD Office of Instructional Supports.

General Information

Pacing

This curriculum map provides guidance for intertwining the Utah Core Math Standards and the Big Ideas curriculum. Following the map, allows students to access all core standards by the end of the year.

Homework

The struggle to develop new concepts should occur while the teacher is available to support and scaffold the learning and correct students' errors in thinking. Work that is sent home for students to complete should consist of concepts that have already been taught in class, been practiced, and the student can already do independently. Math homework should be used to build automaticity of skills already acquired and not for development of new skills without instruction. Practicing concepts incorrectly at home can reinforce errors in thinking and cause frustration for students and families. Practicing the skill to automaticity with homework assignments is appropriate after students have acquired the skill. *Reflex Math* is available for students in grades 2-5 and can be accessed at home as well as at school. *Reflex* Math *helps* students develop fluency with their basic facts in addition, subtraction multiplication and division and could be assigned as homework to support students' automaticity.

Online Supports for Unpacking the Core

For additional information about teaching math standards, please visit the following websites:

USOE Curriculum Guides http://www.schools.utah.gov/CURR/mathelem/Core.aspx

EngageNY—Mathematics Modules--http://www.engageny.org/mathematics

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Canyons School District secondary math maps are created by CSD secondary teachers and published by the CSD Office of Instructional Supports.

Canyons School District Academic Framework to Support Effective Instruction

Multi-Tiered System of Supports (MTSS) for Academics and Behavior

RTI Multi-Tiered System of Support	**(1) Providing high quality core instruction (and intervention) matched to students' needs**	**(2) using data over time (i.e. rate of learning, level of performance, fidelity of implementation)**	**(3) to make important educational decisions.**
CANYONS School District **Student Achievement Principles**	• All CSD students and educators are part of ONE proactive educational system. • Evidence-based instruction and interventions are aligned with rigorous content standards.	• Data are used to guide instructional decisions, and allocate resources. • CSD educators use assessments that are reliable, valid, and connected to standards	• CSD educators problem solve collaboratively to meet student needs.
	• Culture centers around building positive relationships, setting high expectations, and committing to every student's success. • Ongoing, targeted, quality professional development and coaching supports effective instruction for ALL students. • Leadership at all levels is vital.		

Core Expectations for ALL Teachers in the Classrooms and Common Areas

Standards for Instruction	**Evidence-based Instructional Priorities**	**Time Allocation for Instruction**	**Teacher Learning Data**	**Student Performance Data**	**Collaborative Problem Solving for Improvement**
Standards clarify what we want students to learn and do.	Planning, instruction, and assessment techniques to increase student engagement and achievement.	School culture ensures that instructional time is maximized to increase student growth.	Teacher learning and professional growth fostered through public practice and ongoing feedback.	Student academic and behavioral performance is assessed using a variety of reliable and valid methods.	Use data to problem solve and make decisions
Curriculum maps with common pacing guides Instructional content aligned with the Utah Core Standards Scientifically research-based programs Standards-based grades and report cards Cognitive Rigor (Depth of Knowledge – DOK) International Society for Technology in Education Standards (ISTE) School-wide Positive Behavioral Interventions and Supports (PBIS) World-class Instructional Design and Assessment (WIDA) Federal and state requirements (IEP, 504, ELs)	Classroom Positive Behavioral Interventions and Supports (PBIS) Explicit Instruction (I, We, Y'all, You) Instructional Hierarchy: Acquisition, Automaticity, Application (AAA) Systematic Vocabulary Development Maximizing Opportunities to Respond (OTR) Feedback Cycle Scaffolded Instruction & Grouping (SIG) Structures	Master schedule takes into consideration the learning needs of the student population. Scheduling is ensured for: • Intervention and skill-based instruction • Special Education services • English Language Development (ELD) Classroom instructional time is prioritized for instruction of standards Individual and team planning time is used to intentionally increase the application of evidence-based instructional priorities and standards for instruction	Annual setting of goals and documentation of progress (e.g. CSIP, LANDTrust, CTESS) Supporting teacher growth Formalized protocols and checklists to monitor and support implementation Public practice applications: • Coaching cycles with peer coaches, teacher specialists, achievement coach, and/or new teacher coach • Instructional Professional Learning Communities (IPLCs) • Learning walkthroughs and targeted observations • Lesson Study • Video Analysis	Assessment practices: • Inform instruction • Provide feedback about learning to students, parents, and teachers • Build student efficacy • Monitor student achievement and behavioral growth • Celebrate teaching and learning successes Assessment Types: • Classroom Assessing • Teams and Schoolwide Assessment • Districtwide Standards-based Benchmarks • Comprehensive Assessments • Screening Assessments (DIBELS, SRI, SMI) • Specialized Assessments (WIDA, IDEA, eligibility assessment, Phonics surveys)	Problem solving process: identify, analyze, plan, and evaluate Early warning system for identification of risk (academic, behavior, and attendance) Timely and consistent review of relevant data by teams (e.g. BLT, IPLC, CST): • Evaluate effectiveness of academic and behavior instruction for all groups of students using valid and reliable data (student and teacher data) • Determine needs for academic and behavior intervention

Public Practice and Coaching Supports

INSTRUCTIONAL PRIORITIES

Techniques to Increase Student Achievement and Engagement

Classroom Positive Interventions & Supports (PBIS)

Effect Size: .52

Explicit Instruction (I do, We do, Y'all Do, You do)

Effect Size: .57

Instructional Hierarchy (Acquisition, Automaticity, Application)

Effect Size: .57

Systematic Vocabulary Development

Effect Size: .67

Maximizing Opportunities to Respond (OTR)

Effect Size: .60

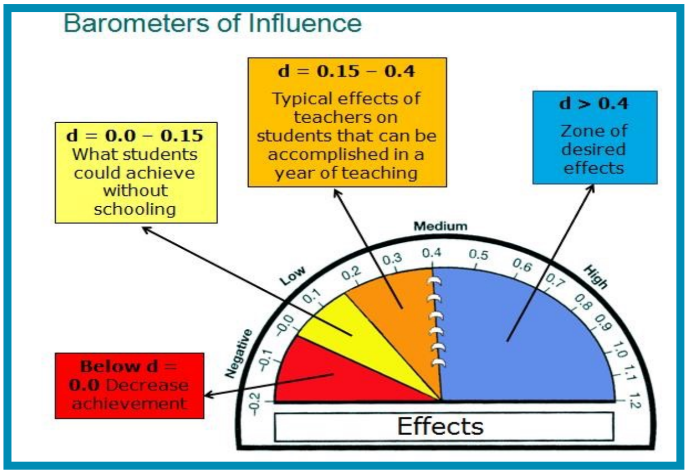
Feedback Cycle

Effect Size: .75

Scaffolded Instruction & Grouping

Effect Size: .49

Our time with students is limited and valuable. Every minute we spend with them should be spent using the practices that are most likely to be successful. This requires us to shift our perspective from looking at instructional practices that work to looking at what instructional practices work BEST.



Works Best?

Meta-analysis offer the strongest evidence base for determining what works best. "A Meta-analysis is a summary, or synthesis of relevant research findings. It looks at all of the individual studies done on a particular topic and summarizes them." (Marzano, 2000). A meta-analysis is simply, a study of studies. Meta-analysis explain the results across studies examined using effect size (ES). Average effects for instruction is 0.20 to 0.40 growth per year (Hattie, 2009). Thus the hinge point for determining what works best is 0.40. Instructional practices above the 0.40 have a high likelihood of increasing learning than those practices below the hinge-point (Hattie, 2009).

INSTRUCTIONAL PRIORITIES

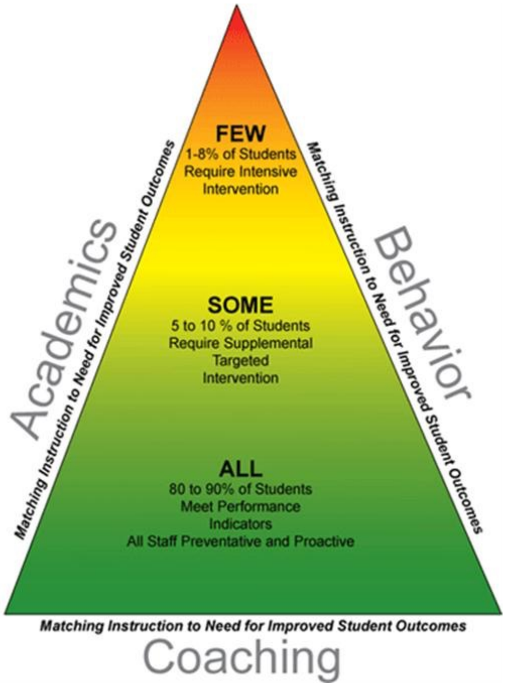
Techniques to increase Student Achievement and Engagement.

Overview

Priority	Critical Actions for Educators
Classroom Positive Behavioral Interventions and Supports (PBIS)	*Clearly identify behavior expectations and explicitly teach them to your students. *Implement reinforcement system for appropriate behavior and routinely evaluate the system for effectiveness. *Recognize students for positive behavior. *Systematically correct problem behaviors.
Explicit Instruction (I do, We do, Y'all do, You do)	*Give clear, straightforward, and unequivocal directions. *Explain, demonstrate and model. Introduce skills in a specific and logical order. Support this sequence of instruction in your lesson plans. *Break skills down into manageable steps. Review frequently. *Demonstrate the skills for students and give opportunity to practice skills independently.
Instructional Hierarchy: Acquisition, Automaticity, then Application (AAA)	*Explicitly teach a skill to students by explaining, demonstrating, and modeling. *Build the skill through practice and use, to gain automaticity. *Provide students with multiple opportunities to apply the skill.
Systematic Vocabulary Development	*Explicitly teach critical vocabulary before students are expected to use it in context. *Teach students to say, define, and use critical vocabulary in discreet steps. *Explicitly teach common academic vocabulary across all content areas.
Maximizing Opportunities to Respond (OTR)	*Actively engage ALL students in learning; students are active when they are saying, writing, or doing. *Pace instruction to allow for frequent student responses. *Call on a wide variety of students throughout each period.
Feedback Cycle	*Provide timely prompts that indicate when students have done something correctly or incorrectly. *Give students the opportunity to use the feedback to continue their learning process. *End feedback with the student performing the skill correctly and receiving positive acknowledgement.
Scaffolded Instruction and Grouping Structures	*Present information at various levels of difficulty. *Use data to identify needs and create small groups to target specific skills. *Frequently analyze current data and move students within groups depending on their changing needs.

CLASSROOM PBIS

Effect Size: 0.52



Critical Actions for Educators

*Clearly identify behavior expectations and explicitly teach them to students.

*Implement reinforcement system for appropriate behavior and routinely evaluate the system for effectiveness.

*Recognize students for positive behavior.

*Systematically correct problem behaviors.

The heart of classroom management is developing routines and environments that promote student success through the active teaching of positive social behaviors.

A well-implemented positive classroom management system will:

- Increase positive behavior in students.
- Help students feel more positive towards their teacher, administrator and school.
- Help students feel safer in school.
- Increase time for academic instruction and decrease teacher time spent correcting problem behaviors.

PBIS, or Positive Behavioral Interventions and Supports, is an evidence-based system that helps define the key components of a well-managed classroom. The key components include:

- Clearly establishing student rules
- Explicitly teaching rules
- Reinforcing positive behaviors and correcting negative behaviors
- Creating a supportive classroom

CLASSROOM PBIS

Effect Size: 0.52

Key Component	Definition
Clearly Establishing Student Rules	• Select 3-5 positively stated & easily remembered rules that align with the school wide rules in your building. ◦ The school's rules might be: Be Safe, Be Kind, Be Responsible. It is appropriate to adopt these same rules for your classroom, and add one or two additional rules that fit the needs of your setting if necessary. It is important to explicitly describe what these rules look like in your classroom. • Publicly post rules in the classroom in a prominent location. • Determine which routines are needed for your classroom (a routine is a set of skills explicitly taught to students to help them be successful with following the rules). Examples may include: ◦ Walking in the hallway ◦ Classroom exit ◦ Starting and ending class ◦ Sharpening pencils ◦ Going to the restroom ◦ Transitioning from one activity to the next ◦ Technology use in the classroom
Explicitly Teaching Rules	• Explicitly teach classroom rules and routines to students. ◦ Define and model positive examples and non-examples of what the rules look like in your classroom. ◦ Have students model and practice performing the desired behaviors. ◦ Provide positive feedback and corrective feedback as needed during practice of the desired behaviors. • Review and practice the rules with students throughout the school year. ◦ Rules should be reviewed more comprehensively at the beginning of each year, after significant breaks in the school schedule (e.g. Thanksgiving, Christmas, Spring), and as needed. • Example Routine ◦ Classroom exit: Describe and model the routine to your students, have students practice lining up, and going back to their seats. Make sure that 100% of students demonstrate the behavior correctly. This may require you to practice several times while providing positive and corrective feedback.

CLASSROOM PBIS

Effect Size: 0.52

Key Component	Definition

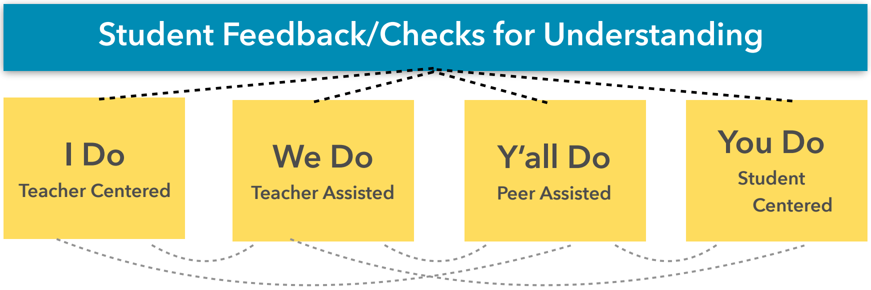
Reinforcing Positive Behaviors and Correcting Negative Behaviors	• It is important to publicly recognize positive behavior, while individually providing corrective feedback when needed. Students should be monitored closely while in the classroom and feedback should be given often. Public positive statements often prompt other students to exhibit the desired behavior.
	  • Example: "I really like the way Sarah is waiting for instructions. She has her materials ready, and she's sitting quietly at her desk."
	• When correcting negative behavior, provide a precision request to students (whole group) to describe desired behavior. Based on student response, provide positive feedback to the group. If undesired behaviors continue follow-up with a statement of the desired behavior directed to the target student in a private manner as needed. Give the student an opportunity to comply and perform the behavior correctly, and then reward the student with positive feedback.
	  • Example: "I need everyone to be in their seats, have materials ready, and wait quietly for instructions." Teacher observes Sarah talking during the transition, so he/she approaches Sarah quietly. "Sarah, the rule in our class is to wait quietly for instructions. I need you to show me how you sit quietly for instructions." While Sarah is performing the desired behavior, you might say, "Sarah, I appreciate how you are waiting quietly. Great job."

Creating a Supportive Classroom	Creating a safe and respectful learning environment allows students to feel supported while learning. It is necessary for teachers to find opportunities to establish positive connections with all students. A teacher's daily interactions influence the students' perception of safety and sense of trust. Considerations for creating a supportive classroom include: • Make personal connections with students • Help students feel like they belong • Establish clear classroom norms to demonstrate respect for others • Create consistent rules, routines, and arrangements (fosters predictability) • Weave positive feedback into daily interactions with students and parents • Be available for students (e.g. to ask questions, seek guidance) • Actively listen • Set a positive tone for learning and problem solving • Be aware of your personal emotions, assumptions, and biases and how they may impact your interactions with students

EXPLICIT INSTRUCTION

Effect Size: 0.57

Explicit instruction is a systematic method of teaching with emphasis on; proceeding in small steps, checking for student understanding, and achieving active and successful participation by all students.



The model is generally characterized with the following components: I Do, We Do, Y'all Do, and You Do. Teachers use student feedback to determine how to progress through the model. For instance, if students are in the "We Do" phase, and the teacher has determined that students aren't understanding, they should move back to the "I Do" phase to provide more examples.

Explicit Instruction	
I Do (Modeling)	Demonstrate & Describe Use Think-Alouds Involve Students
We Do (Guided Practice)	Heavily Scaffolded with Prompts • Tell them what to do. • Ask them what to do. • Remind them what to do. Continual Checks for Understanding
Y'all Do (Group Practice)	Practice Skill in Small Groups/Partners Continual Checks for Understanding Use Precision Partnering
You Do (Individual Practice)	Monitored Individual Practice Show Mastery of Skill

Critical Actions for Educators

*Give clear, straightforward, and unequivocal directions.

*Explain, demonstrate and model. Introduce skills in a specific and logical order. Support this sequence of instruction in your lesson plans.

*Break skills down into manageable steps. Review frequently.

*Demonstrate the skills for students and then give the opportunity to practice skills independently.

* I do, We Do, Y'all Do, You Do.

INSTRUCTIONAL HIERARCHY

Effect Size: 0.57

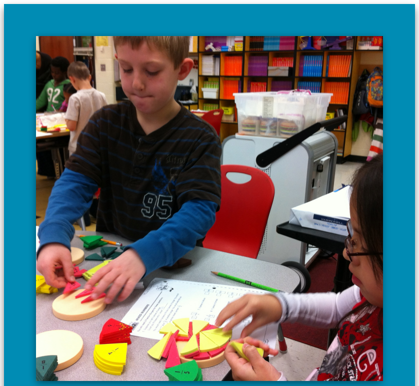
Critical Actions for Educators

*Explicitly teach a skill to students by explaining, demonstrating, and modeling.
*Build the skill through practice and use, to gain automaticity.
*Provide students with multiple opportunities to apply the skill.

Learners follow predictable stages. To begin, the learner is usually halting and uncertain as she tries to use a new skill. With feedback and a lot of practice, the learner becomes increasingly accurate, then automatic (fluent), and confident in using the skill.

Acquisition, automaticity, and application are progressive stages of the instructional hierarchy. Each stage requires its own set of pedagogical approaches and assessment strategies.

The learning stages, along with the goal of each phase and the teacher and student actions present in each stage are listed in the table below.



Accurate at Skill

- If no, teach skill.
- If yes, move to automaticity.

Automatic at Skill

- If no, teach automaticity.
- If yes, move to application.

Able to Apply Skill

- If no, teach application.
- If yes, move to higher level/concept or repeat cycle with new knowledge.

INSTRUCTIONAL HIERARCHY

Effect Size: 0.57

Learning Stage	Goal	Teacher and Student Actions
Acquisition • First learning stage • Teacher feedback to increase accuracy • Typically associated with DOK 1	The student can perform the skill accurately with little adult support. If goal met proceed to automaticity stage; if not teach skill.	• Teacher actively demonstrates target skill • Teacher uses 'think-aloud' strategy-- especially for thinking skills that are otherwise covert • Student has models of correct performance to consult as needed (e.g., correctly completed math problems on board) • Student gets feedback about correct performance • Student receives praise, encouragement for effort • Students take notes, outlines, points
Automaticity • Builds habits and fluent skills through repetition and deliberate practice with timely and descriptive feedback • Typically associated with DOK 2	The student has learned skill well enough to retain, to combine with other skills, and is as fluent as peers. If observed proceed to application; if not continue or move back to acquisition.	• Teacher structures learning activities to give student opportunity for active (observable) responding • Student has frequent opportunities to drill (direct repetition of target skill) and practice (blending target skill with other skills to solve problems) • Student gets feedback on fluency and accuracy of performance • Student receives praise, encouragement for increased fluency
Application • Applying knowledge or skills to relevant application • Typically associated with DOK 3 & 4	The student uses the skill across situations and settings solving real life problems. If observed, move to new skills and knowledge or move to a higher level concept; if no observed try again or go back to building automaticity	• Teacher structures academic tasks to require that the student use the target skill regularly in assignments. • Student receives encouragement, praise for using skill in new settings, situations • Teacher works with parents to identify tasks that the student can do outside of school to practice target skill • Teacher helps student to articulate the 'big ideas' or core element(s) of target skill that the student can modify to face novel tasks, situations • Encourage student to set own goals for adapting skill to new and challenging situations.

EXPLICIT VOCABULARY

Effect Size: 0.57

Explicit vocabulary instruction is clear, concise vocabulary instruction presenting the meaning and contextual examples of a word through multiple exposures. It is not the traditional procedure of having students copy a list of words, looking up words, copying definitions, or memorizing definitions.

Systematic vocabulary instruction increases reading comprehension, allows for greater access to content material, increases growth in vocabulary knowledge, and supports struggling readers.

Effective vocabulary/academic language instruction comes down to:

- Connection: Connect the new word to what the student knows, which helps to build the "semantic network" in the brain.
- Use: Academic speaking and writing is constructed as we apply it, not by simply memorizing.

Teacher should explicitly teach worlds that are:

- Based on essential concepts
- Unknown
- Critical to the future
- Difficult to obtain independently (or through context

Critical Actions for Educators

*Explicitly teach critical vocabulary before students are expected to use it in context.

*Teach students to say, define, and use critical vocabulary in discreet steps.

*Explicitly teach common academic vocabulary across all content areas.

Basic Instructional Protocol

1. Introduce the Word
2. Provide Student Friendly Definition of the Word
3. Identify Word Parts, Families, and Origin
4. Illustrate word with Examples
5. Check students' understanding
6. Deepen students' understanding
7. Check students' understanding
8. Review & Coach Use (possible extensions)

OPPORTUNITIES TO RESPOND

Effect Size: 0.57

Critical Actions for Educators

*Actively engage ALL students in learning; students are active of they are saying, writing, or doing.

*Pace instruction to allow for frequent student responses.

*Call on a wide variety of students throughout each period.

Maximizing the opportunities to respond in a classroom increases students engagements. Engagement allows for positive interactions between teacher and student, creates opportunities for teachers to provide authentic feedback on learning, and decreases inappropriate student behavior.

Students are engaged through opportunities to respond when they are saying, writing, or doing (Feldman). When tied to learning objectives, these opportunities give the teacher and students feedback on their learning and understanding.

Engagement opportunities can be focused on an individual student or a group of students. Each of these approaches has different purposes. The teacher may choose to use a group OTR to minimize the risk the student feels in responding and to increase engagement for all students. Through group OTRs, students not only receive feedback from the teacher, but their peers as well as they hear and see other student responses. When seeking individual student understanding, teacher may choose to use individual OTRs.

Opportunities to respond can be verbal or non-verbal. Verbal responses help students to summarize and share their thoughts with others while non-verbal responses can increase writing skills or give students the opportunity to move around the room.

Structured Non-Verbal	Structured Verbal	Structured Writing	Structured Reading
• Cold Calling (Teacher Chosen) • Cold Calling (Random) • Choral Response • Think Pair Share • Precision Partner • Small Group Discussion	• Hand Signals • Point at Something • 4 Corners • Response Cards • White Boards • Student Response System	• Note-Taking: Cloze, Cornell • Graphic Organizer • Sentence Starter/ Quick Write • White Boards • Summarizing • Technology	• Partner Reading w/ Comprehension Strategy • Choral Reading • Cloze Reading Guide • Model Reading Strategies • Task for each Reading Segment

FEEDBACK

BETWEEN TEACHERS & STUDENTS

Effect Size: 0.75

Feedback lets the learner know whether or not a task was performed correctly, and how it might be improved. Feedback is most effective when it is clear, purposeful, compatible with prior knowledge, immediate, and non-threatening.

Feedback from Students:

Educational research indicates that feedback is one of the most powerful drivers of student achievement. John Hattie's synthesis of the overall effect size of feedback is very high (ES = .75). He states that feedback from students as to what they understand, when they are not engaged, where they make errors, and when they have misconceptions helps make student learning visible to the teacher.

Feedback to Students:

Positive academic and behavioral feedback, or teacher praise has been statistically correlated with student on-task behavior (Apter, Arnold & Stinson, 2010) and has strong empirical support for both increasing academic and behavioral performance and decreasing problem behaviors (Gable, Hester, Rock & Hughes, 2009). With regard to reprimands and corrective feedback, there is a continued assertion that teachers maintain a ratio of praise to correction at 3:1 or 4:1 (Gable, Hester, Rock, & Hughes, 2009; Stichter, Lewis, & Wittaker, 2009).

Critical Actions for Educators

*Provide timely prompts that indicate when students have done something correctly or incorrectly.

*Give students the opportunity to use the feedback to continue their learning process.

*End feedback with the student performing the skill correctly and receiving positive acknowledgement.

Feedback Types:

Type	Description	Example	Non-Example
Positive	Teacher indicates that a target academic or social behavior is correct.	"Correct! 7 X 4 is 28"	"Johnny, pick up your pencil off the floor please
Corrective	Teacher indicates that a behavior is incorrect.	"That's not quite right, let me give you another clue . . . "	"Try harder on your math worksheet; I know you can do better."
Harsh	Teacher shows frustration or is critical of the student.	I can't believe you **still** can't figure this out!	"Let me give you another clue . . . "
Neutral	Teacher redirects the student or describes what she would like the student to do.	"Johnny, turn to page 4 and start reading."	"Nice work! You really showed justification for your reasons."

FEEDBACK CYCLE

Effect Size: 0.75

	Example	Non-Example
Corrective Sequence	Teacher provides an opportunity to respond Student responds incorrectly Teacher indicates that the response was not correct and provides an opportunity for correction Student gives correct response Teacher affirms that response was correct	Teacher provides an opportunity to respond Student responds incorrectly Teacher indicates that the response was not correct but does not provide an opportunity for the student to answer correctly
Expansive Sequence	Teacher provides an opportunity to respond Student response is a partial response or could be expanded into a higher quality response Teacher affirms response and provides guidance for expansion/refinement Student revises or elaborates upon previous response Teacher acknowledges response as an improvement.	Teacher provides an opportunity to respond Student response is a partial response or could be expanded into a higher quality response Teacher affirms response but does not provide guidance for expansion/refinement
Challenge Sequence	Teacher provides and opportunity to respond Student response is fully correct Teacher affirms student response and asks a more difficult question on the same topic as a follow up Student answers Teacher responds with positive or corrective feedback	Teacher provides and opportunity to respond Student response is fully correct Teacher affirms student response but does not ask a more difficult question on the same topic as a follow up

SCAFFOLDING & GROUPING

Effect Size: 0.57

Scaffolding is a process in which students are given support until they can apply new skills and strategies independently (Rosenshine & Meister, 1992). When students are learning new or challenging task, they are given more assistance. As they begin to demonstrate task mastery, the assistance or support is decreased gradually in order to shift the responsibility for learning from the teacher to the students. Thus, as the students assume more responsibility for learning, the teacher provides less support.

Structure of the Scaffolded Classroom:

The organization of the scaffolded classroom includes whole group, small group (skill-based or station teaching), partners, and independent work. The scaffolding supports that will be put in place for diverse learners should include interventions for striving and accelerated learners. When using small groups, identify the groups as skill-based or station teaching. Skill-based groups are organized homogeneously based upon the needs of students. Station teaching groups are organized heterogeneously to create diverse groups.

Critical Actions for Educators

*Present information at various levels of difficulty.

*Use data to identify needs and create small groups to target specific skills.

*Frequently analyze current data and move students within groups depending on their changing needs.

Types of Scaffolds

Scaffold	Ways to use Scaffolds in an Instructional Setting
Advance Organizers	Tools used to introduce new content and tasks to help student learn about the topic: Venn diagrams to compare and contrast information; flow charts to illustrate processes; organizational charts to illustrate hierarchies; outlines that represent content; mnemonics to assist recall; statements to situate the task or content; rubrics that provide task expectations.
Checklists	Prepare a list of items required, things to be done, or points to be considered, used as a reminder as the student proceeds through the learning task.
Collaborative Grouping	Having students work in partners or small groups with students who can support/model students who may struggle with content.
Concept and Mind Maps	Maps that show relationships: Partially or completed maps for students to complete; students create their own maps based on their current knowledge of the task or concept.
Cue Cards	Prepared cards given to individual groups of students to assist in their discussion about a particular topic or content area: Vocabulary words to prepare for exams; content-specific stem sentences to complete; formula to associate with a problem; concepts to define.
Examples	Samples, specimens, illustrations, problems, modeling: Real objects; illustrative problems used to represent something. Demonstrate and model how to do something, giving an example of what it should look like.
Explanations	More detailed information to move students along on a task or in their thinking of a concept: Written instructions for a task; verbal explanation of how a process works.

Scaffold	Ways to use Scaffolds in an Instructional Setting
Handouts	Prepared handouts that contain task and content-related information, but with less detail and room for student note taking.
Images and Multimedia	Providing an image or other graphic representation, such as a video, that represents the word(s)/concept(s) being taught in conjunction with the explicit vocabulary routine can help to support students in learning new vocabulary and concepts. Images help provide a non-linguistic representation and allow students to recall the term more readily. This technique can be used with any Reading Street Vocabulary (Amazing Words, Story/Lesson Vocabulary), Math Vocabulary, or Content Vocabulary or concepts.
Manipulatives	Manipulatives, such as markers, toothpicks, blocks, or coins, are used to support hands-on learning and provide concrete models to help students solve problems and develop concepts. The students can manipulate the items to increase their understanding and come to accurate conclusions. May also include virtual manipulatives.
Pair-Share	Pose a problem, students have time to think about it individually, and then they work in pairs to solve the problem and share their ideas with the class. Providing think time increase the quality of the response.
Precision Partnering	Strategically appointed partners with assigned roles.
Previewing Text	Before reading a text, preview the text by providing students with an overview/synopsis of the text. This will allow students to know what to expect when they are reading and give them background knowledge to help them understand the text.
Prompts	A physical or verbal cue to remind—to aid in recall of prior or assumed knowledge. Physical: Body movements such as pointing, nodding the head, eye blinking, foot tapping. Verbal: Words, statements and questions such as "Go," "Stop," "It's right there," "Tell me now," "What toolbar menu item would you press to insert an image?" "Tell me why the character acted that way."
Question Cards	*Prepared cards with content and task-specific questions* given to individuals or groups of students to ask each other pertinent questions about a particular topic or content area.
Question Stems	*Incomplete sentences which students complete:* Encourages deep thinking by using higher order "What if" questions.
Realia	Anytime the real object, concept, or phenomena can be presented with the actual object helps to support learners in acquiring new ideas and concepts. For example, when teaching about the three types of rocks, having examples of those types for students to see and touch can help them to make deeper connections.
Rubrics	A rubric is an easily applicable form of authentic assessment. A rubric simply lists a set of criteria, which defines and describes the important components of the work being planned or evaluated.
Sentence Frames	*Sentence frames provide an opportunity for students to use key vocabulary while providing a structure* that may be higher than what they could produce on their own. For example, if students are to compare two ocean creatures, they might say something like "Whales have lungs, but fish have gills." In the preceding sentence, the simple frame is "_______ have __________, but ________ have ________. Note the sentence can be filled in with any content; this differs from closed sentences that often have only a few possibilities.
Setting & Reviewing Objectives	*Providing students with a purpose and intended outcome* will help students to know what to focus their attention on and what they should be learning. Having student self-assess their progress towards the objectives at the end of the lesson will provide the teacher with information on their current levels of understanding.
Socratic Seminar	The purpose of a Socratic Seminar is to achieve a deeper understanding about the ideas and values in a text. In the Seminar, participants systematically question and examine issues and principles related to a particular content, and articulate different points-of-view. The group conversation assists participants in constructing meaning through disciplined analysis, interpretation, listening, and participation. Prepare several questions in advance in addition to questions that students may bring to class. Questions should lead participants into the core ideas and values and to the use of the text in their answers. Questions must be open-ended, reflect genuine curiosity, and have no "one-right answer."
Stories	*Stories relate complex and abstract material to situations more familiar with students:* Recite stories to inspire and motivate learners.
Student Work Exemplars	*Providing students with example student work samples can provide models for students to use to support their development of the skill.* For example, an anchor paper for a writing assignment of how a sample student responded to the assignment previously will provide an example of what the assignment looks like.
Visual Scaffolds	Pointing to call attention to an object; representational gestures (holding cured hands apart to illustrate roundness; moving rigid hands diagonally upward to illustrate steps or process), diagrams such as charts and graphs; methods of highlighting visual information.

Hess' Cognitive Rigor Matrix & Curricular Examples: Applying Webb's Depth-of-Knowledge Levels to Bloom's Cognitive Process Dimensions – *M-Sci*

Revised Bloom's Taxonomy	**Webb's DOK Level 1 Recall & Reproduction**	**Webb's DOK Level 2 Skills & Concepts**	**Webb's DOK Level 3 Strategic Thinking/ Reasoning**	**Webb's DOK Level 4 Extended Thinking**
Remember Retrieve knowledge from long-term memory, recognize, recall, locate, identify	Recall, observe, & recognize facts, principles, properties Recall/ identify conversions among representations or numbers (e.g., customary and metric measures)			
Understand Construct meaning, clarify, paraphrase, represent, translate, illustrate, give examples, classify, categorize, summarize, generalize, infer a logical conclusion (such as from examples given), predict, compare/contrast, match like ideas, explain, construct models	Evaluate an expression Locate points on a grid or number on number line Solve a one-step problem Represent math relationships in words, pictures, or symbols Read, write, compare decimals in scientific notation	Specify and explain relationships (e.g., non-examples/examples; cause-effect) Make and record observations Explain steps followed Summarize results or concepts Make basic inferences or logical predictions from data/observations Use models /diagrams to represent or explain mathematical concepts Make and explain estimates	Use concepts to solve non-routine problems Explain, generalize, or connect ideas using supporting evidence Make and justify conjectures Explain thinking when more than one response is possible Explain phenomena in terms of concepts	Relate mathematical or scientific concepts to other content areas, other domains, or other concepts Develop generalizations of the results obtained and the strategies used (from investigation or readings) and apply them to new problem situations
Apply Carry out or use a procedure in a given situation; carry out (apply to a familiar task), or use (apply) to an unfamiliar task	Follow simple procedures (recipe-type directions) Calculate, measure, apply a rule (e.g., rounding) Apply algorithm or formula (e.g., area, perimeter) Solve linear equations Make conversions among representations or numbers, or within and between customary and metric measures	Select a procedure according to criteria and perform it Solve routine problem applying multiple concepts or decision points Retrieve information from a table, graph, or figure and use it solve a problem requiring multiple steps Translate between tables, graphs, words, and symbolic notations (e.g., graph data from a table) Construct models given criteria	Design investigation for a specific purpose or research question Conduct a designed investigation Use concepts to solve non-routine problems Use & show reasoning, planning, and evidence Translate between problem & symbolic notation when not a direct translation	Select or devise approach among many alternatives to solve a problem Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze Break into constituent parts, determine how parts relate, differentiate between relevant-irrelevant, distinguish, focus, select, organize, outline, find coherence, deconstruct	Retrieve information from a table or graph to answer a question Identify whether specific information is contained in graphic representations (e.g., table, graph, T-chart, diagram) Identify a pattern/trend	Categorize, classify materials, data, figures based on characteristics Organize or order data Compare/ contrast figures or data Select appropriate graph and organize & display data Interpret data from a simple graph Extend a pattern	Compare information within or across data sets or texts Analyze and draw conclusions from data, citing evidence Generalize a pattern Interpret data from complex graph Analyze similarities/differences between procedures or solutions	Analyze multiple sources of evidence analyze complex/abstract themes Gather, analyze, and evaluate information
Evaluate Make judgments based on criteria, check, detect inconsistencies or fallacies, judge, critique			Cite evidence and develop a logical argument for concepts or solutions Describe, compare, and contrast solution methods Verify reasonableness of results	Gather, analyze, & evaluate information to draw conclusions Apply understanding in a novel way, provide argument or justification for the application
Create Reorganize elements into new patterns/structures, generate, hypothesize, design, plan, construct, produce	Brainstorm ideas, concepts, or perspectives related to a topic	Generate conjectures or hypotheses based on observations or prior knowledge and experience	Synthesize information within one data set, source, or text Formulate an original problem given a situation Develop a scientific/mathematical model for a complex situation	Synthesize information across multiple sources or texts Design a mathematical model to inform and solve a practical or abstract situation

	DOK Level Descriptions	Teacher's Role	Student's Role	Sample Tasks
Level 1	**Recall & Reproduction** requires recognition of information, such as a fact, definition, term, principle, or performance of a simple process or procedure. Responding to a Level 1 task or question involves following a well-known rule, procedure, or formula. You either know it, or you don't know it.	• Questions to direct or focus attention *(Who? What? Where? How? When?)* • Directs, leads, demonstrates, defines • Examines, breaks down • Uses concrete objects, nonverbal and visual cues to teach concepts, procedures, and vocabulary • Builds background knowledge to build upon later • Provides resources and procedures • Uses mentor texts as unambiguous models	• Learns rules (spells, decodes, edits for grammar, usage, mechanics, principles of design) • Learns processes (order of operations, evaluates expression, measures, key word searches) • Acquires vocabulary, facts • Memorizes, recites, quotes • Practices, restates • Locates/retrieves information • Identifies/names parts • Reports/shares solutions /findings	- Reads orally, reads fluently - Draws/labels/acts to illustrate an event, parts of the whole, phases in a cycle - Writes a variety of sentences - Represents math/fine arts relationships with words, symbols, objects, visuals - Recalls math facts, terms, dates, formulas, rules - Calculates, measures, follows steps - Uses tools, records data - Reads or reproduces maps, diagrams - Highlights key words
Level 2	**Basic Application of Skills/Concepts** requires engagement of some mental processing beyond recall or reproduction - basic comprehension and subsequent processing of content. Students apply more than one concept and make some decisions about how to approach the question or problem, what tools to use, and how ideas relate.	• Questions to differentiate/classify, draw out inferences, check conceptual understanding (*Why? What conditions? Give example?)* • Provides examples and non-examples to build conceptual understanding • Provides graphic organizers to show relationships or organizational schemas • Matches readers with texts • "Thinks aloud" to explore possible options or connections	• Explains relationships, sorts, classifies, compares • Makes predictions based on observations, estimates, proposes • Compiles and organizes information • Distinguishes relevant-irrelevant, fact-opinion, example-non-example • Selects appropriate strategy and applies it • Explains steps taken to complete a task	- Solves routine, multi-step math word problems - Makes science observations, organizes data (graph, table, spreadsheet, etc.) - Writes a caption, paragraph, summary - Creates a timeline of events - Makes and uses models - Interprets simple graphics, tables, etc. - Retrieves information and uses it to answer a question or solve a problem - Creates survey to research a topic
Level 3	**Strategic Thinking/Reasoning** gets at deeper understanding of concepts within novel or new contexts. Students develop their reasoning underlying an interpretation, generalization, or connection, and provide supporting evidence for judgments made. Cognitive demands are more complex and abstract, often with more than one possible answer or approach.	• Questions to probe reasoning and underlying thinking *(How do you know? What is the evidence? But what if? Is this supported by the facts?)* • Asks open-ended questions • Encourages varied approaches • Acts as a resource, coach, mentor • Provides criteria for making judgments • Guides how and what materials encourage in-depth explorations • Models and scaffolds complex thinking	• Uncovers relevant, accurate, and credible information • Uncovers flaws in a design • Develops supporting evidence for conclusions or claims • Tests ideas, predictions, hypotheses • Transfers knowledge to solve non-routine problems • Revises work to establish a progression of ideas or chain of reasoning	- Interprets complex graphics, tables, etc. - Sets up a data base - Conducts a designed investigation - Develops both sides of a fact-based argument for debate or speech - Creates a website, podcast, multi-media presentation matched to purpose - Critiques an essay, performance , or novel, using discipline-based criteria - Analyzes theme, perspective, author's craft in a piece of work
Level 4	**Extended Thinking** requires complex reasoning, planning, and designing own research focus, probably over an extended time. Tasks require significant conceptual understanding and application of skills across disciplines, using multiple sources or resources.	• Questions to extend thinking, explore sources, broaden perspectives *(What are the potential biases? Can you propose an alternative? Can you design a model? What is the importance/value?)* • Facilitates teaming, collaboration, self-monitoring • Models and scaffolds integrating sources	• Initiates learning focus and structures tasks needed to complete complex projects • Locates relevant and credible mentors and resources • Transfers and constructs knowledge • Modifies, creates, elaborates • Investigates real-world problems and issues • Revises work to establish a progression of ideas or chain of reasoning	- Produces a short film, play, or short story based on a theme or issue - Designs own research or investigation as an extension of concepts or issues studied - Critiques importance of policies or events from different perspectives (e.g., historical, social, economic, cultural) - Analyzes theme, perspectives, authors' craft across multiple pieces of work

Table 1: *Math Descriptors* – Applying Depth of Knowledge Levels for Mathematics (Webb, 2002) & NAEP 2002 Mathematics Levels of Complexity (M. Petit, Center for Assessment 2003, K. Hess, Center for Assessment, updated 2006)

Level 1 **Recall**	**Level 2** **Skills/Concepts**	**Level 3** **Strategic Thinking**	**Level 4** **Extended Thinking**
a. Recall, observe, or recognize a fact, definition, term, or property b. Apply/compute a well-known algorithm (e.g., sum, quotient) c. Apply a formula d. Determine the area or perimeter of rectangles or triangles given a drawing and labels e. Identify a plane or three dimensional figure f. Measure g. Perform a specified or routine procedure (e.g., apply rules for rounding) h. Evaluate an expression i. Solve a one-step word problem j. Retrieve information from a table or graph k. Recall, identify, or make conversions between and among representations or numbers (fractions, decimals, and percents), or within and between customary and metric measures l. Locate numbers on a number line, or points on a coordinate grid m. Solve linear equations n. Represent math relationships in words, pictures, or symbols o. Read, write, and compare decimals in scientific notation	a. Classify plane and three dimensional figures b. Interpret information from a simple graph c. Use models to represent mathematical concepts d. **Solve a routine problem** requiring multiple steps/decision points, or the application of multiple concepts e. Compare and/or contrast figures or statements f. Construct 2-dimensional patterns for 3-dimensional models, such as cylinders and cones g. Provide justifications for steps in a solution process h. Extend a pattern i. Retrieve information from a table, graph, or figure and use it solve a problem requiring multiple steps j. Translate between tables, graphs, words and symbolic notation k. Make direct translations between problem situations and symbolic notation l. Select a procedure according to criteria and perform it m. Specify and explain relationships between facts, terms, properties, or operations n. Compare, classify, organize, estimate, or order data	a) Interpret information from a complex graph b) Explain thinking when more than one response is possible c) Make and/or justify conjectures d) Use evidence to develop logical arguments for a concept e) Use concepts to solve non-routine problems f) Perform procedure with multiple steps and multiple decision points g) Generalize a pattern h) Describe, compare, and contrast solution methods i) Formulate a mathematical model for a complex situation j) Provide mathematical justifications k) Solve a multiple- step problem and provide support with a mathematical explanation that justifies the answer l) Solve 2-step linear equations/inequalities in one variable over the rational numbers, interpret solution(s) in the original context, and verify reasonableness of results m) Translate between a problem situation and symbolic notation that is not a direct translation n) Formulate an original problem, given a situation o) Analyze the similarities and differences between procedures p) Draw conclusion from observations or data, citing evidence	a) Relate mathematical concepts to other content areas b) Relate mathematical concepts to real-world applications in new situations c) Apply a mathematical model to illuminate a problem, situation d) Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results e) Design a mathematical model to inform and solve a practical or abstract situation f) Develop generalizations of the results obtained and the strategies used and apply them to new problem situations g) Apply one approach among many to solve problems h) Apply understanding in a novel way, providing an argument/justification for the application ***NOTE**: Level 4 involves such things as complex restructuring of data or establishing and evaluating criteria to solve problems.*

Elements of Rigor

Conceptual Understanding:

Conceptual understanding refers to the kind of thinking that demonstrates understanding of the underlying mathematics in a situation and is an important part of learning mathematics deeply. Conceptual understanding provides flexibility in thinking about mathematics. It can be assessed at a variety of DOK levels. For example, a DOK 1 conceptual question might have a student provide a basic definition of a word. A higher level DOK question would require students to engage in mathematical practices such as making conjectures based on their understanding of mathematics or constructing an argument. Conceptual understanding can be seen when students connect different representations of mathematical ideas, such as graphs, tables, and equations or when they answer questions such as “why?” or “under what conditions?” Standards that begin with verbs such as understand, explain, or identify most likely fall into the conceptual category.

Procedural Skill and Fluency:

Procedural skill and fluency refer to demonstrations of completing mathematical processes such as solving arithmetic problems or manipulating algebraic expressions. Procedures are often rooted in algorithms, but may also be evident where students use repeated reasoning to solve problems. Procedural skill and fluency include efficiency and accuracy. Standards that begin with verbs such as solve, calculate, or construct most likely fall into the procedural skill and fluency category.

Applications and Modeling:

Applications and modeling are student demonstrations of applying mathematics to contextual problems. These problems engage students in problem solving and often include multiple steps and complex thinking. Applications come from a variety of academic subject areas such as science or social studies and can also be rooted in real life scenarios. Standards that begin with verbs such as use, apply, or model most likely fall into the applications and modeling category.

Utah SAGE Elementary Mathematics Blueprints

Grade 6

50 Operational Items

Reporting Category	Min.	Max.
Ratios and Proportional Relationships (RP)	28%	32%
The Number System (NS)	18%	22%
Expressions and Equations (EE)	28%	34%
Geometry/Statistics and Probability (G/SP)	16%	20%
DOK 1	18%	32%
DOK 2	46%	62%
DOK 3	8%	20%

Note: The percentages shown represent target aggregate values; individual student experiences will vary based on the adaptive algorithm.

Disclosure: Depth of Knowledge (DOK) and Elements of Rigor are essential components of the Utah Mathematics Core Standards. As such, DOK and Elements of Rigor are integrated into the Student Assessment of Growth and Excellence (SAGE) assessment items. All students will see a variety of DOK and Elements of Rigor on the SAGE summative assessment. For more information about DOK and Elements of Rigor please seehttp://www.schools.utah.gov/assessment/Adaptive- Assessement-System/Math.aspx

Acronym and Key Term Glossary for Secondary Teachers

- **ACT:** The ACT was designed to measure academic skills required for success in college and university settings. College and universities commonly use results to help determine which students to admit. There are four college readiness benchmark areas: 1) English, 2) Mathematics, 3) Reading, and 4) Science. Student's reaching ACT benchmarks have a 75% or better chance of getting a "C" or higher and a 50% chance or better of getting a "B" or higher in a college course in that subject. The ACT is administered to all 11th graders within the Canyons School District in the spring.

- **BLT:** Building leadership teams are comprised of key members of the school staff and an external coach. Each school's BLT is charged with the following tasks:
    - To identify, plan, and develop the instruction, intervention, and supports for all students to be successful
    - To sustain improvement over time
    - To develop collective capacity for quality instruction (e.g. support all teachers in professional learning and growth)

- **Canvas:** Canvas is a LMS, Learning Management System, (i.e. a software application for the administration, documentation, tracking, reporting and delivery of online learning). Canvas was selected as the LMS for Canyons schools because of its extensive use in Utah institutes of higher learning, along with its ability to increase collaboration among students, teachers, and parents.

- **CBM:** Curriculum-Based Measurement – a brief standardized measurement procedure designed to ascertain a student's overall academic performance in a basic subject area: e.g. reading, spelling, or writing. CBMs were designed to help teachers monitor academic growth over time, so that instruction could be modified and learning rates accelerated.

- **CFA:** Common Formative Assessment – An assessment typically created collaboratively by a team of teachers responsible for the same grade level or course, in order to improve instruction with a current group of students. Common formative assessments are frequently administered throughout the year to identify:
    - Individual students who need additional time and support for learning
    - The teaching strategies most effective in helping students acquire the intended knowledge and skills
    - Program concerns – areas in which students generally are having difficulty achieving the intended standard, and
    - Improvement goals for individual teachers and the team
    - *Dufour (2004). *Learning by Doing*, p. 214

- **CSA:** Common Summative Assessment – An assessment typically created collaboratively by a team of teachers responsible for the same grade level or course in order to evaluate whether or not students reached common standards at the completion of an instruction cycle.

- **CTESS:** Canyons Teacher Effectiveness Support System- In compliance with Senate Bill 64, this is Canyons School District teacher evaluation system that includes documentation of student growth, evidence of instructional quality, and response to stakeholder input.

- **District-Wide Standards-Based Assessment**: These assessments are given in all content areas at key times during the school year. Data from these benchmarks will be used for student growth in compliance with House Bill 201.

- **DLT:** The District Leadership Team supports implementation of the CSD Academic Framework and is comprised of representatives from school and district administration. The DLT is charged with the following:
    - Develop tools necessary for successful scaling-up of CSD Framework (i.e. evidence-based practices)
    - Provide a consistent feedback loop between school leaders and district leaders
    - Provide cascading levels of support to building leaders
    - Implement the district academic plan

- **HMH Math Inventory:** Houghton Mifflin Harcourt math inventory is a research-based, adaptive assessment that measures math abilities and longitudinal progress from Kindergarten through Algebra II

- **IPLC:** Instructional Professional Learning Communities meet regularly to focus on data and instruction to improve student achievement.

- **IPOP**: Instructional Priorities Observation Protocol – The classroom observation tool used for evidence of instructional quality.

- **ISD:** The Instructional Supports Department (commonly known as the curriculum department). This is where you will find the content leads and support for the curriculum.

- **LMS:** **Learning Management System -** A software application for the administration, documentation, tracking, reporting and delivery of online learning. **Canvas** was selected as the LMS for Canyons schools because of its extensive use in Utah institutes of higher learning, along with its ability to increase collaboration among students, teachers, and parents.

- **MTSS:** Multi-tiered Systems of Support (see RtI) is practice of providing high quality instruction, using data to make decisions about instruction and intervention for students that is based upon the students' performance, and providing multiple levels of support for both academic and behavioral standards.

- **PBIS:** Positive Behavioral Intervention and Supports is an evidence-based system that helps define the key components of a well-managed classroom.

- **Progress Monitoring:** A procedure that involves frequent measurement of student performance for the purpose of evaluating a student's growth toward a targeted objective. For example, the trajectory of reading growth can be measured with weekly administration of R-CBM.

- **Lexile Scores:** Lexiles can be a measure of text difficulty or of reading proficiency. They range from 0 to 1700. Below is a list of descriptors of Lexile scores by grade level. Students reading in the Proficient and Advanced levels are on track to graduate college and career ready.

- **SEM:** Standard error of measurement is one standard deviation of error around a student's true score.

- **SRI:** Scholastic Reading Inventory is a computer administered reading test that measures inferential and literal reading comprehension skills. Scores are reported in a numeric Lexile scores. Percentile ranks are also available. SRI was designed primarily to match students with books of an appropriate level of difficulty. It measures both literal and inferential comprehension. It is a particularly good assessment for identifying advanced readers. It has a disadvantage of not being as sensitive to growth as are CBM measures, of being subject to student sloughing, and having limited reliability if administered a few number of times.

- **R-CBM:** Reading Curriculum-Based Measurement (R-CBM) also known as Oral Reading Fluency (ORF) and CBM-Read Aloud, this is a one-minute measure which results in two primary numerical scores: number of words read correctly per minute (or correct words per minute, CWPM), and percentage of correctly read words (accuracy rate). This measure is highly correlated with reading comprehension in elementary school but outlives its usefulness once students read at the same rate at which they speak. Maze has been identified as a more appropriate CBM once students are reading grade-level texts at rates above 130 words read correctly per minute, with greater than 97% accuracy.

- **Reliability:** The degree to which a measure is free of error. All tests contain error and it results from characteristics of the test (such as poorly designed questions), characteristics of the test taker (bad day, lack of sleep, misreading questions, anxiety, and lack of effort), and characteristics of the environment (distracting noises, room temperature, and distracting odors).

- **RtI:** "Response to Intervention" (see MTSS) is the practice of (1) providing high-quality instruction/intervention matched to student needs and (2) using learning rate over time and level of performance to (3) make important educational decisions". (Batsche et al, 2007).

- **Turnitin Revision Assistant:** A core-aligned formative writing tool that gives students immediate feedback on their writing.

- **Universal Screening:** A procedure in which all students are evaluated for the purpose of identifying those students who need more intensive interventions. For example, reading is a critical and foundational academic skill, for which CSD screens in middle school with the SRI.

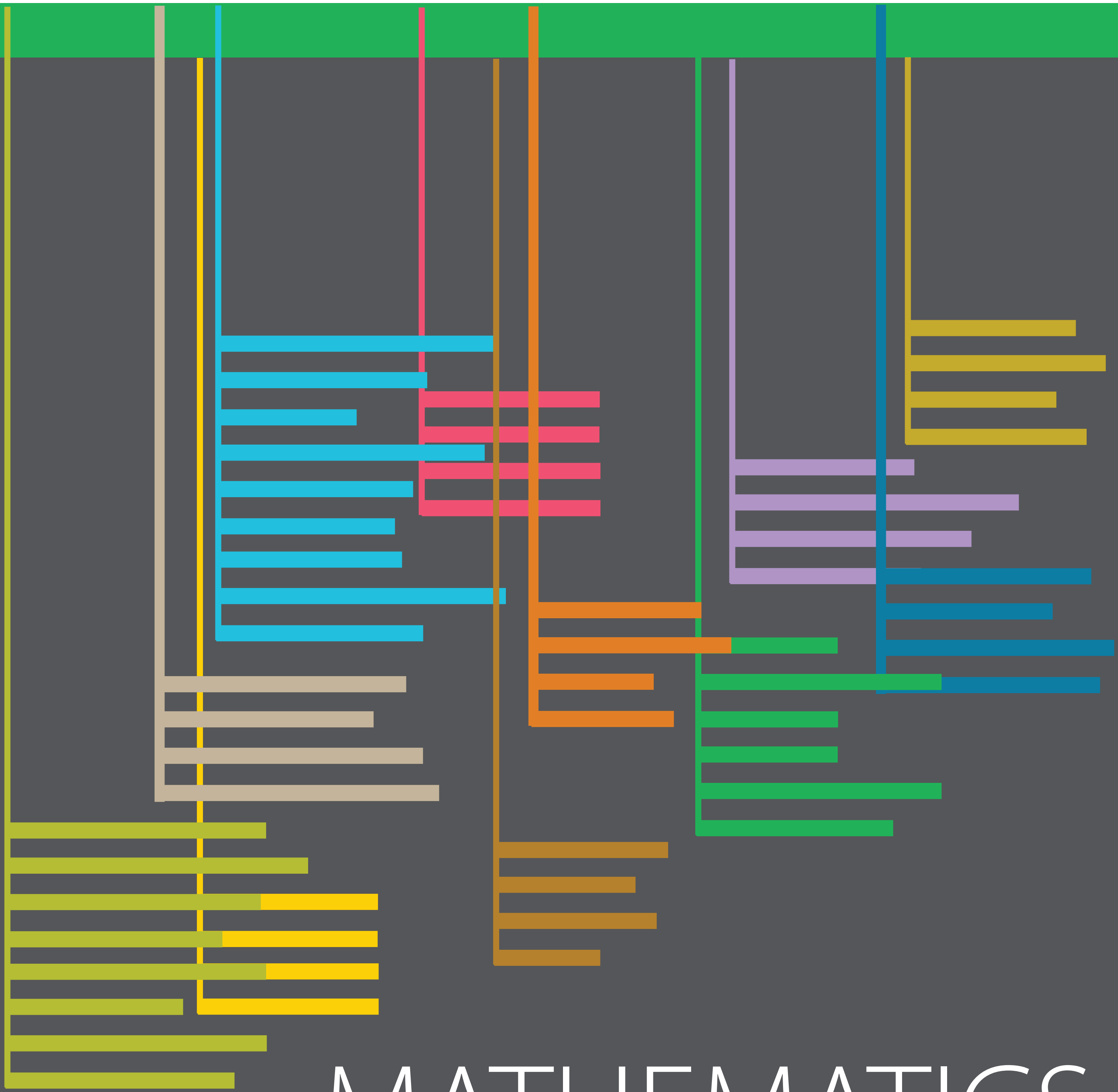
- **Utah Core Standards:** The standards for teaching and learning adopted by the Utah State Board of Education and implemented by local school districts and charter schools with guidance and support from the Utah State Office of Education.

- **Validity:** The degree to which a test measures what it is intended to measure. Establishing the validity of a measurement procedure involves empirical study of item content, accurate prediction, and alignment with theories about what is being measured.

Common Core State Standards
Standards for Mathematical Practice
Questions for Teachers to Ask

Make sense of problems and persevere in solving them	**Reason abstractly and quantitatively**	**Construct viable arguments and critique the reasoning of others**	**Model with mathematics**
Teachers ask: • What is this problem asking? • How could you start this problem? • How could you make this problem easier to solve? • How is ___'s way of solving the problem like/different from yours? • Does your plan make sense? Why or why not? • What tools/manipulatives might help you? • What are you having trouble with? • How can you check this?	*Teachers ask:* • What does the number ____ represent in the problem? • How can you represent the problem with symbols and numbers? • Create a representation of the problem.	*Teachers ask:* • How is your answer different than _____'s? • How can you prove that your answer is correct? • What math language will help you prove your answer? • What examples could prove or disprove your argument? • What do you think about _____'s argument • What is wrong with ____'s thinking? • What questions do you have for ____? **it is important that the teacher poses tasks that involve arguments or critiques*	*Teachers ask:* • Write a number sentence to describe this situation • What do you already know about solving this problem? • What connections do you see? • Why do the results make sense? • Is this working or do you need to change your model? **It is important that the teacher poses tasks that involve real world situations*
Use appropriate tools strategically	**Attend to precision**	**Look for and make use of structure**	**Look for and express regularity in repeated reasoning**
Teachers ask: • How could you use manipulatives or a drawing to show your thinking? • Which tool/manipulative would be best for this problem? • What other resources could help you solve this problem?	*Teachers ask:* • What does the word ____ mean? • Explain what you did to solve the problem. • Compare your answer to _____'s answer • What labels could you use? • How do you know your answer is accurate? • Did you use the most efficient way to solve the problem?	*Teachers ask:* • Why does this happen? • How is ____ related to ____? • Why is this important to the problem? • What do you know about ____ that you can apply to this situation? • How can you use what you know to explain why this works? • What patterns do you see? **deductive reasoning (moving from general to specific)*	*Teachers ask:* • What generalizations can you make? • Can you find a shortcut to solve the problem? How would your shortcut make the problem easier? • How could this problem help you solve another problem? **inductive reasoning (moving from specific to general)*

UTAH CORE STANDARDS



MATHEMATICS

Middle/Junior High (6–8)

UTAH STATE OFFICE OF EDUCATION   250 EAST 500 SOUTH   P.O. BOX 144200   SALT LAKE CITY, UTAH 84114-4200   SYDNEE DICKSON, INTERIM STATE SUPERINTENDENT OF PUBLIC INSTRUCTION

Utah State Office of Education

UTAH CORE STATE STANDARDS

for

MATHEMATICS

MIDDLE/JUNIOR HIGH SCHOOL

GRADES (6–8)

Adopted August 2010

by the

Utah State Board of Education

1896

Revised September 2015–April 2016

1896

The Utah State Board of Education, in January of 1984, established policy requiring the identification of specific core standards to be met by all K–12 students in order to graduate from Utah's secondary schools. The Utah State Board of Education regularly updates the Utah Core Standards, while parents, teachers, and local school boards continue to control the curriculum choices that reflect local values.

The Utah Core Standards are aligned to scientifically based content standards. They drive high quality instruction through statewide comprehensive expectations for all students. The standards outline essential knowledge, concepts, and skills to be mastered at each grade level or within a critical content area. The standards provide a foundation for ensuring learning within the classroom.

1896

UTAH STATE BOARD OF EDUCATION

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2/2016

INTRODUCTION

Organization of the Standards

The Utah Core Standards are organized into **strands**, which represent significant areas of learning within content areas. Depending on the core area, these strands may be designated by time periods, thematic principles, modes of practice, or other organizing principles.

Within each strand are **standards**. A standard is an articulation of the demonstrated proficiency to be obtained. A standard represents an essential element of the learning that is expected. While some standards within a strand may be more comprehensive than others, all standards are essential for mastery.

Understanding Mathematics

These standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The standards set grade-specific standards but do not dictate curriculum or teaching methods, nor do they define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the standards do provide clear signposts along the way to the goal of college and career readiness for all students.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, "Students who already know… should next come to learn ..." Grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

Utah State Office of Education

UTAH CORE STATE STANDARDS
for
MATHEMATICS

6–8

Mathematics | Grade 6

In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

(1) Students will use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students will connect their understanding of multiplication and division with ratios and rates. Thus students will expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students will solve a wide variety of problems involving ratios and rates.

(2) Students will use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students will use these operations to solve problems. Students will extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They will reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students will understand the use of variables in mathematical expressions. They will write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students will understand that expressions in different forms can be equivalent, and they will use the properties of operations to rewrite expressions in equivalent forms. Students will know that the solutions of an equation are the values of the variables that make the equation true. Students will use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students will construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they will use equations (such as $3x = y$) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students will begin to develop their ability to think statistically. Students will recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students will recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students will learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

(5) Students in Grade 6 will also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They will find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students will discuss, develop, and justify formulas for areas of triangles and parallelograms. Students will find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They will reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They will prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

Strand: MATHEMATICAL PRACTICES (6.MP)

The Standards for Mathematical Practice in Sixth Grade describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards 6.MP.1–8).**

- **Standard 6.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, "Does this make sense?" Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method.

- **Standard 6.MP.2 Reason abstractly and quantitatively.** Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem.

- **Standard 6.MP.3 Construct viable arguments and critique the reasoning of others.** Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others.

- **Standard 6.MP.4 Model with mathematics.** Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

- **Standard 6.MP.5 Use appropriate tools strategically.** Consider the available tools and be sufficiently familiar with them to make sound decisions about when each tool might be helpful, recognizing both the insight to be gained as well as the limitations. Identify relevant external mathematical resources and use them to pose or solve problems. Use tools to explore and deepen their understanding of concepts.

- **Standard 6.MP.6 Attend to precision.** Communicate precisely to others. Use explicit definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Specify units of measure and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context.

- **Standard 6.MP.7 Look for and make use of structure.** Look closely at mathematical relationships to identify the underlying structure by recognizing a simple structure within a more complicated structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, see 5 – 3(x – y)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

- **Standard 6.MP.8 Look for and express regularity in repeated reasoning.** Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details.

Strand: RATIOS AND PROPORTIONAL RELATIONSHIPS (6.RP)

Understand ratio concepts and use ratio reasoning to solve problems **(Standards 6.RP.1–3).**

- **Standard 6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. The following are examples of ratio language: *"The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every two wings there was one beak." "For every vote candidate A received, candidate C received nearly three votes."*

- **Standard 6.RP.2** Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with $b \neq 0$, and use rate language in the context of a ratio relationship. The following are examples of rate language: *"This recipe has a ratio of four cups of flour to two cups of sugar, so the rate is two cups of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* (In sixth grade, unit rates are limited to non-complex fractions.)

- **Standard 6.RP.3** Use ratio and rate reasoning to solve real-world (with a context) and mathematical (void of context) problems, using strategies such as reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations involving unit rate problems.

  a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

  b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took four hours to mow eight lawns, how many lawns could be mowed in 32 hours? What is the hourly rate at which lawns were being mowed?*

  c. Find a percent of a quantity as a rate per 100. Solve problems involving finding the whole, given a part and the percent. *(For example, 30% of a quantity means 30/100 times the quantity.)*

  d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Strand: THE NUMBER SYSTEM (6.NS)

Apply and extend previous understandings of multiplication and division of whole numbers to divide fractions by fractions **(Standard 6.NS.1).** Compute (add, subtract, multiply and divide) fluently with multi-digit numbers and decimals and find common factors and multiples **(Standards 6.NS.2–4).** Apply and extend previous understandings of numbers to the system of rational numbers **(Standards 6.NS.5–8).**

■ **Standard 6.NS.1** Interpret and compute quotients of fractions.

- **a.** Compute quotients of fractions by fractions, *for example, by applying strategies such as visual fraction models, equations, and the relationship between multiplication and division, to represent problems.*
- **b.** Solve real-world problems involving division of fractions by fractions. *For example, how much chocolate will each person get if three people share 1/2 pound of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mile and area 1/2 square mile?*
- **c.** Explain the meaning of quotients in fraction division problems. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.)*

■ **Standard 6.NS.2** Fluently divide multi-digit numbers using the standard algorithm.

■ **Standard 6.NS.3** Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

- **a.** Fluently divide multi-digit decimals using the standard algorithm, limited to a whole number dividend with a decimal divisor or a decimal dividend with a whole number divisor.
- **b.** Solve division problems in which both the dividend and the divisor are multi-digit decimals; develop the standard algorithm by using models, the meaning of division, and place value understanding.

■ **Standard 6.NS.4** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4(9 + 2).*

■ **Standard 6.NS.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values *(for example, temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge)*; use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of zero in each situation.

- **Standard 6.NS.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
  - **a.** Recognize opposite signs of numbers as indicating locations on opposite sides of zero on the number line; recognize that the opposite of the opposite of a number is the number itself. *For example, -(-3) = 3, and zero is its own opposite.*
  - **b.** Understand that the signs of numbers in ordered pairs indicate their location in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
  - **c.** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
- **Standard 6.NS.7** Understand ordering and absolute value of rational numbers.
  - **a.** Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.*
  - **b.** Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write $-3°C > -7°C$ to express the fact that $-3°C$ is warmer than $-7°C$.*
  - **c.** Understand the absolute value of a rational number as its distance from zero on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world context. *For example, for an account balance of –30 dollars, write $|-30| = 30$ to describe the size of the debt in dollars.*
  - **d.** Distinguish comparisons of absolute value from statements about order. *For example: Recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*
- **Standard 6.NS.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same *x*-coordinate or the same *y*-coordinate.

Strand: EXPRESSIONS AND EQUATIONS (6.EE)

Apply and extend previous understandings of arithmetic to algebraic expressions involving exponents and variables **(Standards 6.EE.1–4).** They reason about and solve one-variable equations and inequalities **(Standards 6.EE.5–8).** Represent and analyze quantitative relationships between dependent and independent variables in a real-world context **(Standard 6.EE.9).**

- **Standard 6.EE.1** Write and evaluate numerical expressions involving whole-number exponents.

- **Standard 6.EE.2** Write, read, and evaluate expressions in which letters represent numbers.
  - **a.** Write expressions that record operations with numbers and with letters representing numbers. *For example, express the calculation "Subtract* y *from 5" as* $5 - y$ *and express "Jane had \$105.00 in her bank account. One year later, she had* x *dollars more. Write an expression that shows her new balance" as* \$105.00 + x.
  - **b.** Identify parts of an expression using mathematical terms (for example, sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity and a sum of two terms. *For example, describe the expression* $2(8 + 7)$ *as a product of two factors; view* $(8 + 7)$ *as both a single entity and a sum of two terms.*
  - **c.** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, applying the Order of Operations when there are no parentheses to specify a particular order. *For example, use the formulas* $V = s^3$ *and* $A = 6s^2$ *to find the volume and surface area of a cube with sides of length* $s = 1/2$.
- **Standard 6.EE.3** Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression* $3(2 + x)$ *to produce the equivalent expression* $6 + 3x$; *apply the distributive property to the expression* $24x + 18y$ *to produce the equivalent expression* $6(4x + 3y)$; *apply properties of operations to* $y + y + y$ *to produce the equivalent expression* $3y$.
- **Standard 6.EE.4** Identify when two expressions are equivalent. *For example, the expressions* $y + y + y$ *and* $3y$ *are equivalent because they name the same number, regardless of which number* y *represents.*
- **Standard 6.EE.5** Understand solving an equation or inequality as a process of answering the question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- **Standard 6.EE.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- **Standard 6.EE.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + a = b$ and $ax = b$ for cases in which a, b and x are all non-negative rational numbers.
- **Standard 6.EE.8** Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
- **Standard 6.EE.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity,

thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation* $d = 65t$ *to represent the relationship between distance and time.*

Strand: GEOMETRY (6.G)

Solve real-world and mathematical problems involving area, surface area, and volume **(Standards 6.G.1–4).**

- **Standard 6.G.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing and decomposing into rectangles, triangles and/or other shapes; apply these techniques in the context of solving real-world and mathematical problems.
- **Standard 6.G.2** Find the volume of a right rectangular prism with appropriate unit fraction edge lengths by packing it with cubes of the appropriate unit fraction edge lengths *(for example, 3½ x 2 x 6)*, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. (Note: Model the packing using drawings and diagrams.)
- **Standard 6.G.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same x coordinate or the same y coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
- **Standard 6.G.4** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Strand: STATISTICS AND PROBABILITY (6.SP)

Develop understanding of statistical variability of data **(Standards 6.SP.1–3).** Summarize and describe distributions **(Standards 6.SP.4–5).**

- **Standard 6.SP.1** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*
- **Standard 6.SP.2** Understand that a set of data collected to answer a statistical question has a distribution that can be described by its center, spread/range and overall shape.

- **Standard 6.SP.3** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

- **Standard 6.SP.4** Display numerical data in plots on a number line, including dot plots, histograms and box plots. Choose the most appropriate graph/plot for the data collected.

- **Standard 6.SP.5** Summarize numerical data sets in relation to their context, such as by:

  a. Reporting the number of observations.

  b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

  c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations (for example, outliers) from the overall pattern with reference to the context in which the data were gathered.

  d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Math Olympiads (6th Grade Honor's)

Ideally, you will practice with your honor's classes about once every chapter. The following practice dates are only suggestions, as you do what is best for your class. The practice dates are weeks, so that you can plan on one day during that week to practice.

The actual contest dates are when the data is due, so you can administer the contest before or on that date, once the contest is open.

Type	Date
Practice 1	Sept. 6 – 9
Practice 2	Oct. 10 – 14
Practice 3	Oct. 31 – Nov. 3
Contest 1 due date	**November 15**
Practice 4	Nov. 28 – Dec. 2
Contest 2 due date	**December 13**
Practice 5	Jan. 3 – 7
Contest 3 due date	**January 10**
Practice 6	Jan. 31 – Feb. 3
Contest 4 due date	**February 14**
Practice 7	Feb. 27 – Mar. 3
Contest 5 due date	**March 7**
Practice 8	Mar. 27 – Mar. 31
Practice 9	Apr. 17 – Apr. 21
Practice 10	May 8 – May 12

6th Grade Year at a Glance

	1st Trimester	**1st Trimester**	**2nd Trimester**	**2nd Trimester**	**3rd Trimester**	**3rd Trimester**
Big Idea	**Historical Narratives**	**Embracing Heritage**	**Discovery**	**Figuring it Out**	**Characters**	**Dreaming Big**
Standards	Numerical Expressions and Factors 6.NS.2 6.NS.4 6.EE.1 Dividing Fractions 6.NS.1 **Practice Standards**	Operations with Decimals 6.NS.3 Numerical Expressions and Factors 6.NS.4 Algebraic Expressions and Properties 6.NS.4 6.EE.2 6.EE.3 6.EE.4 6.EE.6 **Practice Standards**	Area of Polygons 6.G.1 6.G.3 Ratios, Rates, Percentages 6.RP.1 6.RP.2 6.RP.3 **Practice Standards**	Integers & Coordinate Plane 6.NS.5 6.NS.6 6.NS.7 6.NS.8 Ratios, Rates, Percentages 6.RP.3 Equations 6.EE.7 **Practice Standards**	Equations and Inequalities 6.EE.5 6.EE.8 6.EE.9 Surface Area and Volume 6.G.2 6.G.4 Statistical Measures 6.SP.1 **Practice Standards**	Statistical Measures 6.SP.2 6.SP.3 Data Displays 6.SP.4 6.SP.5 Summative Assessments **Practice Standards**
Essential Question	How can I fluently divide rational numbers?	How can I use mental math to multiply two numbers?	What is the relationship between rates, ratios, and percents?	What is the relationship between a number and its opposite?	How do I use equations to solve Geometry problems?	How I use Statistics to describe numerical information?
District Standards-Based Assessment	Benchmark 1: October 19 – November 2 Standards: 6.NS.1, 6.NS.2, 6.NS.3, 6.NS.4, 6.EE.1		Benchmark 2: January 4 – January 18 Standards: 6.NS.4, 6.EE.2, 6.EE.3, 6.EE.4, 6.EE.6, 6.G.1, 6.G.3, 6.RP.1, 6.RP.2, 6.RP.3		Benchmark 3: March 15 – March 29 Standards: 6.NS.5, 6.NS.7, 6.NS.8, 6.EE.5, 6.EE.7, 6.EE.8, 6.EE.9, 6.G.2, 6.G.4	
Summary Math Concepts	Numerical Expressions & Factors Dividing Fractions	Operations with Decimals Algebraic Expressions and Properties	Areas of Polygons Ratios, Rates, & Percentages	Percentages Converting Measures Integers and the Coordinate Plane Equations in One Variable	Equations Inequalities Surface Area and Volume Introduction to Statistics	Statistical Measures Data Displays

Writing Focus	Narrative	Informational/ Expository	Argument	Informational/ Expository	Argument	Argument
Prioritized Vocabulary	• Analyze • Conclude • Sequence	• Compare • Contrast • Summarize	• Discover • Evidence • Calculate	• Scale • Evaluate • Infer	• Claim • Source • Characteristics	• Bibliography • Credibility • Paraphrase
Math Concepts	• Whole Number operations • Powers and Exponents • Order of Operations • Prime Factorization • Greatest Common Factor • Least Common Multiple • Dividing Fractions • Dividing Mixed Numbers	• Adding and Subtracting Decimals • Multiplying Decimals • Dividing Decimals • Multiplying Decimals • Dividing Decimals • Algebraic Expressions • Writing Expressions • Properties of Addition and Multiplication • The Distributive Property	• Area of Parallelograms • Area of Triangles • Area of Trapezoids • Polygons in the Coordinate Plane • Ratios • Ratio Tables • Rates • Comparing and Graphing Ratios • Percentages	• Solving Percent Problems • Converting Measures • Integers • Comparing and Ordering Integers • Fractions and Decimals on the Number Line • Absolute Value • The Coordinate Plane • Reflecting Points on the Coordinate Plane • Writing Equations with 1 and 2 variables • Addition and Subtraction equations • Multiplication and Division Equations	• Writing Equations in 2 variables • Writing and Graphing inequalities • Three Dimensional Figures • Surface Area of Prisms • Surface Area of pyramids • Volume of Rectangular Prisms • Intro to Statistics • Mean • Measures of Center	• Measures of Variation • Mean Absolute Deviation • Stem-and-Leaf Plots • Histograms • Shapes of Distributions • Box-and-Whisker Plots
Science Concepts	• Heat • Light • Sound	• Moon Phases	• Seasons	• Solar System	• Universe	• Microorganisms

English Language Arts Concepts	• Narrative Writing • Citing Textual Evidence • Determining a Theme • Pronouns	• Point of View • Discussion	• Compare & Contrast • Use Evidence	• Word Meaning • Gathering & Interpreting Information • Figurative Language	• Master Discussions • Presenting Claims	• Determining a Central Idea • Analyzing Text Structure • Supporting Arguments with Evidence • Conduct Research
Social Studies Concepts	• Historical Skills & Geography • Introduction to Civilizations: Mesopotamia	• Characteristics of Civilizations • Comparison of Civilizations	• Transformation of Cultures • World Religions • Middle Ages & Renaissance	• Process & Impact of Revolutions • 1750-1914	• Building Today's World • WWI & WWII	• Our World Today • 1945-Today • (Cold War, Human Rights, Middle East)
PE & Health Concepts	• Sportsmanship • Physical Fitness • Game Play	• Understanding Emotions • Resiliency & Stress	• Substance Abuse • Nutrition • Consumer Health	• Sportsmanship • Physical Fitness • Game Play	• Body Image • Hygiene • HIV/AIDS	• Sportsmanship • Physical Fitness • Game Play

Scope and Sequence Broken Down by Chapters

Chapter 1: Numerical Expressions & Factors	Chapter 2: Fractions & Decimals	Chapter 3: Algebraic Expressions and Properties	Chapter 4: Areas of Polygons	Chapter 5: Ratios and Rates	Chapter 6: Integers and the Coordinate Plane	Chapter 7: Equations and Inequalities	Chapter 8: Surface Area and Volume	Chapter 9: Statistical Measures	Chapter 10: Data Displays
1.1	2.2	3.1	4.1	5.1	6.1	7.1	8.1	9.1	10.2
1.2	2.3	3.2	4.2	5.2	6.2	7.2	8.2	9.2	10.3
1.3	2.4	3.3	4.3	5.3	6.3	7.3	8.3	9.3	10.3e
1.4	2.5	3.4	4.3e	5.4	6.4	7.4	8.4	9.4	10.4
1.5	2.6	3.4e	4.4	5.5	6.5	7.5		9.5	
1.6				5.6					
				5.7					

*An "e" after the section number denotes the extension part of that section, as outlined in Big Ideas.

The following sections from Big Ideas are not in the core:
1.6e
2.1
6.5e
7.6
7.7
10.1

Mathematical Practices
6.MP.1 Make sense of problems and persevere in solving them. Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, “Does this make sense?” Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method.
6.MP.2 Reason abstractly and quantitatively. Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem.
6.MP.3 Construct viable arguments and critique the reasoning of others. Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others.
6.MP.4 Model with mathematics. Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
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6.MP.8 Look for and express regularity in repeated reasoning. Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details.

Content Standard	**Big Ideas (2014)**	**Performance Task(s)**
6.NS.2 – Fluently divide multi-digit numbers using the standard algorithm.	1.1 Whole Number Operations	Big Ideas (2014): 6.NS.2 Festival Trees I spent $504 on 28 tickets for a concert. How much did I spend on each ticket? Justify your reasoning.
6.EE.1 – Write and evaluate numerical expressions involving whole-number exponents.	1.2 Powers and Exponents 1.3 Order of Operations	Big Ideas (2014) 6.EE.1 Band Competition Certain biological cells quadruple each hour. Start with one cell at 2:00 and find out how many cells there will be by 5:00. Create a diagram to represent the cell growth. Include an equation using exponential notation.
6.NS.4 – Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1 – 100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4(9 + 2).*	1.4 Prime Factorization 1.5 Greatest Common Factor 1.6 Least Common Multiple 3.4 The Distributive Property	Big Ideas (2014): 6.NS.4 Fruits Hot dogs come in packs of 8. Buns come in packs of 12. How many packs of hot dogs and bags of buns would you have to buy to have an equal number of hot dogs and buns? You need to make gift bags for a party with the same number of balloons and candy in each bag. One package of candy has 24 pieces. One package of balloons has 20 balloons. You need to use all the candy and all the balloons. What is the greatest number of gift bags that you can make containing an equal number of items?
6.NS.1 – Compute quotients of fractions by fractions. a. Solve real-world problems involving division of fractions by fractions, and explain the meaning of quotients in fraction division problems. b. Apply strategies such as using visual fraction models, applying the relationship between multiplication and division, and using equations to represent such problems	2.2 Dividing Fractions 2.3 Dividing Mixed Numbers	Big Ideas (2014): 6.NS.1 Amusement Park You have 5/8 pound of Skittles. You want to give your friends 1/4 lb. each. How many friends can you give Skittles to? Explain your answer. You have a 3/4-acre lot. You want to divide it into 3/8-acre lots. How many lots will you have? Draw a diagram to justify your solution. You have a 3/4-acre lot. You want to divide it into 2

as: *How much chocolate will each person get if 3 people share ½ lb. of chocolate equally?* c. Create a story context for (2/3) + (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) + (3/4) = 8/9 because ¾ of 8/9 is 2/3. (In general, (a/b) + (c/d) = ad/bc)		sections. How many acres in each section will you have? Draw a diagram to justify your solution. How wide is a rectangular strip of land with length 3/4 mi. and area 1/2 square mi.?
6.NS.3 – Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. a. Fluently divide multi-digit decimals using the standard algorithm, limited to a whole number dividend with a decimal divisor or a decimal dividend with a whole number divisor. b. Solve division problems in which both the dividend and the divisor are multi-digit decimals; develop the standard algorithm by using models, the meaning of division, and place value understanding.	2.4 Adding and Subtracting Decimals 2.5 Multiplying Decimals 2.6 Dividing Decimals	Big Ideas (2014): 6.NS.3 Gold The school had a bake sale and raised $75.55. If each cookie cost $0.05, how many cookies were sold? Explain how you got your answer.
6.EE.2 – Write, read, and evaluate expressions in which letters represent numbers. a. Write expressions that record operations with numbers and with letters representing numbers. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity and a sum of two terms. c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including	3.1 Algebraic Expressions 3.2 Writing Expressions 3.4 The Distributive Property	Big Ideas (2014): 6.EE.2 Geometry Hannah is 3 years younger than Katie. Joey is twice as old as Hannah. Let k stand for Katie's age. Write an expression to represent Hannah's age. Using k, write an expression for Joey's age. You know that you can find the area of a triangle using the formula A =1/2 bh. If a triangle has an area of 48 cm^2, what can its base and height be? Draw diagrams to justify your thinking.

those involving whole-number exponents, applying the Order of Operations when there are no parentheses to specify a particular order.		
6.EE.3 – Apply the properties of operations to generate equivalent expressions.	3.3 Properties of Addition and Multiplication 3.4 The Distributive Property 3.4 Extension Factoring Expressions	Big Ideas (2014): 6.EE.3 Lacrosse In one packet of nuts, there are two different types of nuts. There are 5 peanuts (p) and 7 cashews (c) in each container. I have 6 packets of nuts; write two expressions that show how many nuts I have all together. Possible answers: 6 (5 + 7) or (6 x 5) + (6 x 7) Sarah says that the two expressions 3(2 + x) and 6 + x are equivalent. Is she right? If not, explain.
6.EE.4 – Identify when two expressions are equivalent.	3.3 Properties of Addition and Multiplication 3.4 The Distributive Property 3.4e Factoring Expressions	Big Ideas (2014): 6.EE.4 Perimeter of Geometric Figures Have students generate equivalent expressions for the number 48, using at least two operations and verifying that their notation is correct.
6.EE.6 – Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, and number in a specified set.	3.2 Writing Expressions 3.4 The Distributive Property	Big Ideas (2014): 6.EE.6 Earnings An appliance repairman charges $50 for coming to a home for a service call and $40 an hour for the service. Write an expression to represent her earnings for h hours. Sally delivered 7 newspapers and John delivered x number of newspapers. Write an expression showing how many total newspapers were delivered. Write an expression to represent how many John delivered if Sally delivered seven more newspapers than John.
6.G.1 – Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing and decomposing into rectangles, triangles and/or other shapes;	4.1 Areas of Parallelograms 4.2 Areas of Triangles 4.3 Areas of Trapezoids	Big Ideas (2014): 6.G.1 Concert Stages

apply these techniques in the context of solving real-world and mathematical problems.	4.3e Areas of Composite Figures	
6.G.3 – Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same x coordinate or the same y coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	4.4 Polygons in the Coordinate Plane	Big Ideas (2014): 6.G.3 Facial Recognition Given the coordinates A (2,5), B (-4,5), C (-4,1), and D (2,1) Jose says that the distance between A and subtracting 2 from 5 can find D. Prove or disprove. Explain your answer with words, pictures, and equations.
6.RP.1 – Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.	5.1 Ratios 5.2 Ratio Tables 5.4 Comparing and Graphing Ratios	Big Ideas (2014): 6.RP.1 Ships The newspaper reported, "For every vote candidate A received, candidate B received three votes." Describe possible election results using at least three different ratios. Explain your answer.
6.RP.2 – Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0 and use rate language in the context of a ratio relationship.	5.3 Rates 5.4 Comparing and Graphing Ratios	Big Ideas (2014): 6.RP.2 Factory Production Is the following example a ratio or rate? (60 heartbeats per minute) Explain your answer. Give a real-life example of a unit rate and justify your answer.
6.RP.3 – Use ratio and rate reasoning to solve real-world (with a context) and mathematical (void of context) problems, using strategies such as reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations involving unit rate problems. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.	5.2 Ratio Tables 5.3 Rates 5.4 Comparing and Graphing Ratios	Big Ideas (2014): 6.RP.3 Windmills Given a table, graph the information on a coordinate plane and explain the relationship. Joe's Gas and Go has drinks for the following prices: *12 fl. oz. for $0.89. *16 fl. oz. for $0.99. *20 fl. oz. for $1.09. *32 fl. oz. for $1.19.

b. Solve unit rate problems including those involving unit pricing and constant speed.		Which drink costs the least per ounce? You may round to the nearest cent and use a calculator if you desire.
6.RP.3 – Use ratio and rate reasoning to solve real-world (with a context) and mathematical (void of context) problems, using strategies such as reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations involving unit rate problems. c. Find a percent of a quantity as a rate per 100. Solve problems involving finding he whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	5.5 Percents 5.6 Solving Percent Problems 5.7 Converting Measures	Big Ideas (2014): 6.RP.3 Windmills Stop and Shop has pants for $30 with a 10% discount, while Stay and Shop has pants for $45 with a 20% discount. Which store has the pants for a better price? Use a table of equivalent values, number lines, or diagrams to solve and explain your reasoning. In the store a package of candy that weighs 150 grams costs $1.00. A package of 200 candies that each weigh 200 milligrams also costs $1.00. Which package is the better deal? Explain why.
6.NS.5 – Understand that positive and negative numbers are used together to describe quantities having opposite directions or values; use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	6.1 Integers	Big Ideas (2014): 6.NS.5 Lakes of North America Create situations in which integers have opposite values and explain what zero means in this situation.
6.NS.6 – Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself.	6.2 Comparing and Ordering Integers 6.3 Fractions and Decimals on the Number Line 6.4 Absolute Value 6.5 The Coordinate Plane	Big Ideas (2014): 6.NS.6 Temperature

b. Understand that the signs of numbers in ordered pairs indicate their location in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.		
6.NS.7 – Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. b. Write, interpret, and explain statements of order for rational numbers in a real-world context. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in real-world context. d. Distinguish comparisons of absolute value from statements about order.	6.4 Absolute Value	Big Ideas (2014): 6.NS.7 Temperature on Planets On Tuesday the temperature was -7°F and on Wednesday the temperature was -5°F. Which day was colder? Write the inequality and show it on a number line. Explain how you know your answer is correct. A scuba diver is 30 ft. below sea level and a submarine is 75 ft. below sea level. Jim thinks the inequality for this situation should be -30 ft. below sea level > -75 ft. below sea level. Sally thinks the inequality should be -30 ft. below sea level < -75 ft. below sea level. Who is correct? Why? A whale swims 40 ft. below sea level. Express the whale's location as an integer and tell how many feet below the surface the whale is swimming. Explain your answers for both parts of the problem. A mother dolphin is 150.25 meters below sea level. Her calf is 45 meters below sea level. Which dolphin is farthest from the surface? A mother whale is at 35 meters below the surface and her calf is at the surface. How far does the calf have to swim to get to its mother? Which

		statement deals with absolute value? Which statement deals with ordering? Justify your answer.
6.NS.8 – Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same x-coordinate or the same y-coordinate.	6.5 The Coordinate Plane	Big Ideas (2014): 6.NS.8 Pain Ball Bill's house is at (-4, 6), the library is at (-4, -2) and the Bakery is at (3, -2). What is the distance between Bill's house and the library? The library and the bakery? Show two different methods to find the difference.
6.EE.7 – Solve real-world and mathematical problems by writing and solving equations of the form x + a = b and ax = b for cases in which a, b and x are all non-negative rational numbers.	7.1 Writing Equations in One Variable 7.2 Solving Equations Using Addition or Subtraction 7.3 Solving Equations Using Multiplication or Division	Big Ideas (2014): 6.EE.7 Dinosaurs There were some grapes on the table. Logan ate 1/6 of them. He ate 5 grapes. Write an equation to represent the situation and solve. Angela bought 5 shirts that each cost the same amount. She spent $34.65. How much did she spend on each shirt? Write and solve an equation to solve the problem. Ronnie earned $.50, giving her a total of $3.17. Write an equation that allows you to find her beginning amount.
6.EE.5 – Understand solving an equation or inequality as a process of answering the question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	7.2 Solving Equations Using Addition or Subtraction 7.3 Solving Equations Using Multiplication or Division 7.5 Writing and Graphing Inequalities	Big Ideas (2014): 6.EE.5 Bees
6.EE.9 – Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationships between	7.4 Writing Equations in Two Variables	Big Ideas (2014): 6.EE.9 Hiking You are training for a 13-mile race. On the first day you run 1.5 miles. Each day you run 0.5 mile longer than you ran on the previous day. How many days will it take you to work up to 13 miles? Create a table, graph, and

the dependent and independent variables using graphs and tables, and relate these to the equation.		equation and explain the relationship between the dependent and independent variables.
6.EE.8 – Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	7.5 Writing and Graphing Inequalities	Big Ideas (2014): 6.EE.8 Country Fair Water boils at 100ºC. Write an inequality that represents all the temperatures at which water does not boil. Represent the solution on a number line.
6.G.4 – Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	8.1 Three-Dimensional Figures 8.2 Surface Areas of Prisms 8.3 Surface Areas of Pyramids	Big Ideas (2014): 6.G.4 Tents Belinda had two boxes to wrap for a birthday party. Box A has a length of 12 in, width of 8 in, and height of 6 in. Box B has a length of 11 in, width of 9 in, and height of 7 in. Which box will require the least amount of wrapping paper?
6.G.2 – Find the volume of a right rectangular prism with appropriate unit fraction edge lengths by packing it with cubes of the appropriate unit fraction edge lengths (for example 3½ x 2 x 6) and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = lwh and V = Bh to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. (Note: Model the packing using drawings and diagrams)	8.4 Volumes of Rectangular Prisms	Big Ideas (2014): 6.G.2 Money Build 3 rectangular prisms with the volume of 36 cubic units. At least one of the side lengths of each prism is a fractional unit. What are the dimensions of each of the rectangular prisms you built?
6.SP.1 – Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.	9.1 Introduction to Statistics	Big Ideas (2014): 6.SP.1 Softball

6.SP.2 – Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread/range and overall shape.	9.1 Introduction to Statistics 10.3 Shapes of Distributions	Big Ideas (2014): 6.SP.2 Movies Provide a box score from a college or professional basketball game, have the students pick out the points scored by each player. The students will find the center of the data (in this case, let's use the median), and the spread of the data. Have the students graph the data and describe the overall shape. Then have the students answer the following questions: All players who don't score at or above the median points scored have to ride a stationary bicycle for 20 minutes. List the players who have to ride the bicycle. The coach is trying to get the team to play more as a team. He is using the spread of the data as a way to determine if they are playing as a team. How might the coach use the spread to accomplish his goal?
6.SP.3 – Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	9.2 Mean 9.3 Measures of Center 9.4 Measures of Variation 9.5 Mean Absolute Deviation	Big Ideas (2014): 6.SP.3 Olympic Medals Have students create statistical questions that have meaning to them (e.g. how much allowance they get, how far they walk or ride to school) in groups. Students survey students in other grade levels and/or classes to gather data, and they then graph the data. Have them then analyze and summarize the data using the vocabulary in this lesson.
6.SP.5 – Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall	9.1 Introduction to Statistics 9.2 Mean 9.3 Measures of Center 9.4 Measures of Variation 9.5 Mean Absolute Deviation 10.2 Histograms	Big Ideas (2014): Speed 6.SP.5 Have students use data found in newspaper or other media to interpret total number of observations in that data set. Have them explain why the number of observations is important for that set of data.

pattern and any striking deviations (for example, outliers) from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	10.3 Shapes of Distributions 10.3e Choosing Appropriate Measures	
6.SP.4 – Display numerical data in plots on a number line, including dot plots, histograms and box plots. Choose the most appropriate graph/plot for the data collected.	9.1 Introduction to Statistics 10.2 Histograms 10.4 Box-and-Whisker Plots	Big Ideas (2014): 6.SP.4 Game Show Have students count the number of steps they take to get to school. If they ride in a car or bus have them count the steps they take to get to the vehicle and then into school. Have the students graph the data using a dot plot, histogram, and box plot.

6th Grade Numerical Expressions & Factors/Dividing Fractions Unit 1 Theme: Historical Narratives				
Suggested Pacing: 6 Weeks				
Thematic Question	Supporting Questions	Key Terms	Practice Standards Task(s)	Cross Curricular Connections
How can I fluently divide rational numbers?	• How do you know which operation to choose when solving a real-life problem? • How can you use repeated factors in real-life situations? • What is the effect of inserting parentheses into a numerical expression? • Without dividing, how can you tell when a number is divisible by another number? • How can you find the greatest common factor of two numbers? • How can you find the least common multiple of two numbers? • How can you divide by a fraction? • How can you model division by a mixed number?	• Fluently • Standard Algorithm • Greatest Common Factor • Least Common Multiple • Distributive Property • Dividend • Division Notation Quotient • Factorization	• Big Ideas (2014): 6.NS.2 Festival Trees • Big Ideas (2014): 6.EE.1 Band Competition • Big Ideas (2014): 6.NS.4 Fruits • Big Ideas (2014): 6.NS.1 Amusement Park	• Analyze • Conclude • Sequence

Utah Core Standards for Mathematics	Student Learning Targets	Curriculum Resources
Standard 6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.	• I can divide multi-digit numbers.	Big Ideas (2014): Section 1.1 Task: 6.NS.2 Festival Trees
Standard 6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.	• I can write and understand numerical expressions involving whole number exponents. • I can use my knowledge of the order of operations to evaluate and create equivalent expressions.	Big Ideas (2014): Section 1.2 Section 1.3 Task: 6.EE.1 Band Competition
Standard 6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 {9 + 2}*.	• I can find the greatest common factor of two whole numbers. • I can find the least common multiple of two whole numbers.	Big Ideas (2014): Section 1.4 Section 1.5 Section 1.6 Task: 6.NS.4 Fruits
Standard 6.NS.1 Compute quotients of fractions by fractions. a. Solve real-world problems involving division of fractions by fractions, and explain the meaning of quotients in fraction division problems. b. Apply strategies such as using visual fraction models, applying the relationship between multiplication and division, and using equations to represent such problems as: *How much chocolate will each person get if 3 people share ½ lb. of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*	• I can divide two fractions. • I can solve word problems involving the division of fractions by fractions. • I can model division fractions concretely (manipulatives),	Big Ideas (2014): Section 2.2 Section 2.3 Task: 6.NS.1 Amusement Park

c. Create a story context for (2/3) + (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that *(2/3) + (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) + (c/d) = ad/bc.)*	visually (diagrams), and abstractly (equations).	
Additional Resources		

Science and Technical Subject Literacy Standards		Literacy Implementation Ideas
Reading	**6.RST.3** - Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks. **6.RST.4** - Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to 6th *grades topics*. **6.RST.7 -** Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).	
Writing	**6.WHST.2** - Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. **6.WHST.3** - Provide a concluding statement or section that follows from and supports the information or explanation presented. **6.WHST.7** - Conduct short research projects to answer a question (including a self-generated question), drawing on several sources and generating additional related, focused questions that allow for multiple avenues of exploration.	

6th Grade
Operations with Decimals/Algebraic Expressions and Properties
Unit 2 Theme: Embracing Heritage

Suggested Pacing: 6 weeks – 3 weeks for operations with decimals and 3 weeks for expressions and properties

Thematic Question	Supporting Questions	Key Terms	Practice Standards Task(s)	Cross Curricular Connections
How can I use mental math to multiply two numbers?	• How can you add and subtract decimals? • How can you multiply and divide decimals? • How can you write and evaluate an expression that represents a real-life problem? • How can you write an expression that represents an unknown quantity? • Does the order in which you perform and operation matter? • How do you use the distributive property to multiply two numbers?	• Addend • Sum • Difference • Factor • Product • Divisor • Dividend • Quotient • Term • Equal • Equation • Equivalent expression • Expression • Variable • Associative property • Commutative property • Distributive property • Identity property • Variable	• Big Ideas (2014): 6.NS.3 Gold • Big Ideas (2014): 6.EE.2 Geometry • Big Ideas (2014): 6.EE.3 Lacrosse • Big Ideas (2014): 6.EE.4 Perimeter of Geometric Figures • Big Ideas (2014): 6.EE.6 Earnings	• Compare • Contrast • Summarize
Utah Core Standards For Mathematics			Student Learning Targets	Curriculum Resources

Standard 6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. a. Fluently divide multi-digit decimals using the standard algorithm, limited to a whole number dividend with a decimal divisor or a decimal dividend with a whole number divisor. b. Solve division problems in which both the dividend and the divisor are multi-digit decimals; develop the standard algorithm by using models, the meaning of division, and place value understanding.	• I can add multi-digit numbers involving decimals. • I can subtract multi-digit numbers involving decimals. • I can multiply multi-digit numbers involving decimals. • I can divide multi-digit numbers involving decimals.	Big Ideas (2014): Section 2.4 Section 2.5 Section 2.6 Task: 6.NS.3 Gold
Standard 6.EE.2 Write, read, and evaluate expressions in which letters represent numbers. a. Write expressions that record operations with numbers and with letters representing numbers. For example: Express the calculation *"Subtract y from 5" as 5 - y. "Jane had $105.00 in her bank account. One year later she had x dollars more. Write an expression that shows her new balance" as $105.00 + x.* b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity and a sum of two terms. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, applying the Order of Operations when there are no parentheses to specify a particular order. *For example, use the formulas V = s3 and A = 6 s2 to find the volume and surface area of a cube with sides of length s = 1/2.*	• I can write an expression with variables. • I can identify the parts of an expression (sum, term, product, factor, quotient, & coefficient). • I can explain that a quantity in parenthesis is a both a number by itself and two numbers with an operation. • I can evaluate expressions using order of operations when given values of the variables.	Big Ideas (2014): Section 3.1 Section 3.2 Section 3.4 Task: 6.EE.2 Geometry

Standard 6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 {9 + 2}.*	• I can use the distributive property to show the sum of two whole numbers 1-100 in different ways.	Big Ideas (2014): Section 3.4 Extension 3.4 Task: 6.NS.4 Fruits
Standard 6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*	• I can create an equivalent expression using the distributive property. • I can create an equivalent expression through the use of properties of operations such as the commutative, associative, and factoring	Big Ideas (2014): Section 3.3 Section 3.4 Task: 6.EE.3 Lacrosse
Standard 6.EE.4 Identify when two expressions are equivalent. *For example: the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y represents.*	• I can identify when two expressions are equivalent.	Big Ideas (2014): Section 3.3 Section 3.4 Task: 6.EE.4 Perimeter of Geometric Figures
Standard 6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	• I can write an expression or equation using a variable that helps me solve a real-world problem.	Big Ideas (2014): Section 3.2 Section 3.4 Task: 6.EE.6 Earnings

Additional Resources

To assist in lining up decimals, use lined paper turn on it’s side.
Color coding numbers in the distributive property
Use puzzles, games, and challenges to make practice more fun and increase automaticity.

Reference pages for properties and vocabulary

Science and Technical Subject Literacy Standards		Literacy Implementation Ideas
Reading	**6.RST.3** - Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks. **6.RST.4** - Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to 6th *grades topics*. **6.RST.7 -** Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).	
Writing	**6.WHST.2** - Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. **6.WHST.3** - Provide a concluding statement or section that follows from and supports the information or explanation presented. **6.WHST.7** - Conduct short research projects to answer a question (including a self-generated question), drawing on several sources and generating additional related, focused questions that allow for multiple avenues of exploration.	

<table>
<tr><th colspan="5">6th Grade
Areas of Polygons/Ratios, Rates, & Percentages
Unit 3 Theme: Discovery</th></tr>
<tr><td colspan="5">Suggested Pacing: Areas of Polygons: 2 weeks; Ratios: 6 weeks</td></tr>
<tr><th>Thematic Question</th><th>Supporting Questions</th><th>Key Terms</th><th>Practice Standards Task(s)</th><th>Cross Curricular Connections</th></tr>
<tr><td>What is the relationship between rates, ratios, and percents?</td><td>• How can you derive a formula for the area of a parallelogram?
• How can you derive a formula for the area of a triangle?
How can you derive a formula for the area of a trapezoid?
• How can you find the lengths of line segments in a coordinate plane?
• How can you represent a relationship between two quantities?
• How can you find two ratios that describe the same relationship?
• How can you use rates to describe changes in real-life problems?
• How can you compare two ratios?</td><td>• Area
• Compose
• Base
• Height
• Polygon
• Decompose
• Right triangles
• Quadrilaterals
• Perpendicular
• Coordinate Plane
• Coordinates
• Origin
• x-axis
• y-axis
• Vertex
• Vertices
• Ratio
• Terms
• Rate
• Unit Rate
• Equivalent Ratio</td><td>• Big Ideas (2014): 6.G.1 Concert Stages
• Big Ideas (2014): 6.G.3 Facial Recognition
• Big Ideas (2014): 6.RP.3 Windmills</td><td>Discover
Evidence
Calculate</td></tr>
<tr><th colspan="3">Utah Core Standards For Mathematics</th><th>Student Learning Targets</th><th>Curriculum Resources</th></tr>
<tr><td colspan="3">Standard 6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing and decomposing into</td><td>• I can put together and take apart shapes to help me</td><td>Big Ideas (2014):
Section 4.1
Section 4.2</td></tr>
</table>

rectangles, triangles and/or other shapes; apply these techniques in the context of solving real-world and mathematical problems.	find the area of right triangles, other triangles, special quadrilaterals and polygons. • I can make a line plot to display data sets of measurements in fractions. • I can apply what I know about taking apart and putting together shapes to find the area in real world situations.	Section 4.3 Extension 4.3 Task: 6.G.1 Concert Stages
Standard 6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same x coordinate or the same y coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	• I can draw polygons in the coordinate plane when I am given the coordinates for the vertices. • I can use the coordinates of the vertices of a polygon on the coordinate plane to find the length of a side, joining points with the same first coordinate or the same second coordinate. • I can apply what I have learned about polygons on coordinate planes to real world and mathematical situations.	Big Ideas (2014): Section 4.4 Task: 8.G.3 Facial Recognition
Standard 6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. The following are examples of ratio language: *"The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received three votes."*	• I can use different notations to represent a ratio (1:3, 1 to 3, 1/3) • I can understand ratio relationships between two quantities.	Big Ideas (2014): Section 5.1 Section 5.2 Section 5.4 Task: 6.RP.1 Ships

Standard 6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b≠0, and use rate language in the context of a ratio relationship. The following are examples of rate language: *"This recipe has a ratio of 4 cups of flour to 2 cups of sugar, so the rate is 2 cups of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." (In sixth grade, unit rates are limited to non-complex fractions.)*	• I can understand that a rate is a special ratio that compares two quantities with different units of measure. • I can find the unit rate. • I can use rate language to describe rates (per, each, or the @ symbol)	Big Ideas (2014): Section 5.3 Section 5.4 Task: 6.RP.2: Factory Production
Standard 6.RP.3 Use ratio and rate reasoning to solve real-world (with a context) and mathematical (void of context) problems, using strategies such as reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations involving unit rate problems. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example: if it took 4 hours to mow 8 lawns, how many lawns could be mowed in 32 hours? What is the hourly rate at which lawns were being mowed?*	• I can create a tables of equivalent ratios and use it to: o find missing values in the tables, o plot those values on a coordinate plane o use the tables to compare ratios • I can solve real world problems using unit rates.	Big Ideas (2014): Section 5.2 Section 5.3 Section 5.4 Task: 6.RP.3 Windmills

Additional Resources

Science and Technical Subject Literacy Standards		Literacy Implementation Ideas
Reading	**6.RST.3** - Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks. **6.RST.4** - Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to 6th *grades topics*. **6.RST.7 -** Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).	
Writing	**6.WHST.2** - Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical	

	processes. **6.WHST.3** - Provide a concluding statement or section that follows from and supports the information or explanation presented. **6.WHST.7** - Conduct short research projects to answer a question (including a self-generated question), drawing on several sources and generating additional related, focused questions that allow for multiple avenues of exploration.	

6th Grade
Percentages, Converting Measures, Integers and the Coordinate Plane, Equations in One Variable
Unit 4 Theme: Figuring it Out

Suggested Pacing: 7 weeks

Thematic Question	Supporting Questions	Key Terms	Practice Standards Task(s)	Cross Curricular Connections
What is the relationship between a number and its opposite?	• What is the connection between ratios, fractions, and percents? • How can you use mental math to find the percent of a number? • How can you compare lengths between customary and metric systems? • How can you represent numbers that are less than 0? • How can you use a number line to order real-life events? • How can you use a number line to compare positive and negative fractions and decimals? • How can you describe how far an object is from sea level? • How can you graph and locate points that contain negative numbers in a coordinate plane? • How does rewriting a word problem help you solve the word problem? • How can you use multiplication or division to solve an equation?	• Percent • Convert • Integer • Negative • Positive • Rational • Opposite • Rational Number • Point • Absolute Value • Magnitude • Coordinate • Ordered Pair • Point • Quadrant • x-axis • y-axis • Balance • Equations	• Big Ideas (2014): 6.RP.3 Windmills • Big Ideas (2014): 6.NS.5 Lakes of North America • Big Ideas (2014): 6.NS.6 Temperature • Big Ideas (2014): 6.NS.8 Pain Ball • Big Ideas (2014): 6.EE.7 Dinosaurs	• Scale • Evaluate • Infer

Utah Core Standards For Mathematics	Student Learning Targets	Curriculum Resources
Standard 6.RP.3 Use ratio and rate reasoning to solve real-world (with a context) and mathematical (void of context) problems, using strategies such as reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations involving unit rate problems. c. Find a percent of a quantity as a rate per 100. Solve problems involving finding the whole, given a part and the percent. *(For example: 30% of a quantity means 30/100 times the quantity)* d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	• I can find a percent of a quantity as a rate per hundred. • I can find the percent of a number using rate methods. • I can solve problems involving finding the whole if I am given a part and the percent. • I can convert customary units using ratios. • I can convert metric units by multiplying and dividing.	Big Ideas (2014): Section 5.5 Section 5.6 Section 5.7 Task: 6.RP.3 Windmills
Standard 6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values *(for example, temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.*	• I can understand that positive and negative numbers are used to describe amounts having opposite values. • I can use positive and negative numbers to show amounts in real-world situations and explain what the number 0 means in those situations.	Big Ideas (2014): Section 6.1 Task: 6.NS.5 Lakes of North America
Standard 6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of	• I can understand that a rational number is a point on a number line. • I can extend number line diagrams to show positive and negative numbers on the line and in the plane	Big Ideas (2014): Section 6.2 Section 6.3 Section 6.4 Section 6.5 Task: 6.NS.6 Temperature

a number is the number itself, for example: -(-3) = 3, and that 0 is its own opposite. b. Understand that the signs of numbers in ordered pairs indicate their location in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	• I can recognize opposite signs of numbers as indicating places on opposite sides of 0 on the number line. • I can understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane. (Ex: when two ordered pairs differ only by signs, the locations appear to be reflections of each other on the coordinate plane.) • I can place integers and other numbers on a number line diagram. • I can place ordered pairs on a coordinate plane.	
Standard 6.NS.7 Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example: Interpret –3 > –7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right.* b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 °C > –7 °C to express the fact that –3 °C is warmer than –7 °C.* c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world context. *For example: for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*	• I can understand the absolute value as the number's distance from 0 on the number line. • I can understand absolute value of rational numbers. • I can write, understand and explain what rational numbers means in real-world situations. • I can understand absolute values as they apply to real-world situations. (Ex: for an account balance of	Big Ideas (2014): Section 6.4 Task: 6.NS.7 Temperature on Planets

d. Distinguish comparisons of absolute value from statements about order. For example: Recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.	-30 dollars, write (-30) =30 to describe the size of the debt in dollars.) • I can tell the difference between comparing absolute values and ordering positive and negative numbers.	
Standard 6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same x-coordinate or the same y-coordinate.	• I can graph in all four quadrants of the coordinate plane to help me solve real world and mathematical problems. • I can determine the distance between points in the same first coordinate or the same second coordinate.	Big Ideas (2014): Section 6.5 Task: 6.NS.8 Pain Ball
Standard 6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + a = b$ and $ax= b$ for cases in which a, b and x are all non-negative rational numbers.	• I can solve one-step equations with all four operations. • I can write and solve equations that represent real-world problems.	Big Ideas (2014): Section 7.1 Section 7.2 Section 7.3 Task: 6.EE.7 Dinosaurs
Additional Resources		
Activities: Hands-on Equations 1-2 days; graphing projects in all four quadrants (good formative assessment); reflecting ordered pairs projects; percent survey project; trouble: unit rates, percents		

Science and Technical Subject Literacy Standards		Literacy Implementation Ideas
Reading	**6.RST.3** - Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks. **6.RST.4** - Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to 6th *grades topics*. **6.RST.7 -** Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).	
Writing	**6.WHST.2** - Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. **6.WHST.3** - Provide a concluding statement or section that follows from and supports the information or explanation presented. **6.WHST.7** - Conduct short research projects to answer a question (including a self-generated question), drawing on several sources and generating additional related, focused questions that allow for multiple avenues of exploration.	

<table>
<tr><th colspan="5">6th Grade
Inequalities and Equations/Surface Area and Volume/Statistics
Unit 5 Theme: Characters</th></tr>
<tr><td colspan="5">Suggested Pacing: 6 Weeks</td></tr>
<tr><th>Thematic Question</th><th>Supporting Questions</th><th>Key Terms</th><th>Practice Standards Task(s)</th><th>Cross Curricular Connections</th></tr>
<tr><td>How do I use equations to solve Geometry problems?</td><td>• How can you write an equation in two variables?
• How can you use a number line to represent solutions of an inequality?
• How can you draw three-dimensional figures?
• How can you find the area of the entire surface of a prism?
• How can you use a net to find the surface area of a pyramid?
• How can you find the volume of a rectangular prism with fractional edge lengths?
• How can you tell whether a question is a statistical question?</td><td>• Equation
• Inequality
• Solution
• Substitution
• Set
• Graph
• Table
• Variable
• Independent Variable
• Dependent Variable
• Equivalent
• Volume
• Rectangular Prism
• Length
• Width
• Height
• Base
• Cubic units
• Edge
• Unit Fraction
• Data
• Statistics
• Statistical Question/Variability
• Bias
• Fair Questions</td><td>• Big Ideas (2014): 6.EE.5 Bees
• Big Ideas (2014): 6.EE.9 Hiking
• Big Ideas (2014): 6.EE.8 Country Fair
• Big Ideas (2014): 6.G.2 Money
• Big Ideas (2014): 6.G.4 Tents
• Big Ideas (2014): 6.SP.1 Softball</td><td>• Claim
• Source Characteristics</td></tr>
</table>

Utah Core Standards For Mathematics	Student Learning Targets	Curriculum Resources
Standard 6.EE.5 Understand solving an equation or inequality as a process of answering the question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	• I can explain if a value from a set makes an inequality or equation true/false.	Big Ideas (2014): Section 7.5 Task: 6.EE.5 Bees
Standard 6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*	• I can write an equation involving dependent and independent variables and evaluate that equation. • I can make a table, graph or equation to represent a problem. • I can organize and display data using tables and graphs.	Big Ideas (2014): Section 7.4 Task: 6.EE.9 Hiking
Standard 6.EE.8 Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	• I can write an inequality about a real-world situation and recognize that it has infinite solutions. • I can graph an inequality on a number line.	Big Ideas (2014): Section 7.5 Task: 6.EE.8 Country Fair
Standard 6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	• I can show how three-dimensional figures can be made using two -dimensional nets. • I can figure out the surface area of a three dimensional shape by using a net.	Big Ideas (2014): Section 8.1 Section 8.2 Section 8.3 Task: 6.G.4 Tents
Standard 6.G.2 Find the volume of a right rectangular prism with appropriate unit fraction edge lengths by packing it with cubes of the appropriate unit fraction edge lengths (for example, 3½ x 2 x 6) and show that the volume is	• I can use unit cubes to find the volume of a right rectangular prism and I	Big Ideas (2014): Section 8.4 Task: 6.G.2 Money

the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = Bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. (Note: Model the packing using drawings and diagrams.)	understand that the mathematical formula ($V=lwh$ or $V=bh$) will give me the same result. • I can use the mathematical formulas $V=lwh$ or $V=bh$ to determine the volume of real world objects.	
Standard 6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, “How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students’ ages.*	• I can identify and write statistical questions.	Big Ideas (2014): Section 9.1 Task: 6.SP.1: Softball
Additional Resources		

Science and Technical Subject Literacy Standards		Literacy Implementation Ideas
Reading	**6.RST.3** - Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks. **6.RST.4** - Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to 6th *grades topics*. **6.RST.7 -** Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).	
Writing	**6.WHST.2** - Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. **6.WHST.3** - Provide a concluding statement or section that follows from and supports the information or explanation presented. **6.WHST.7** - Conduct short research projects to answer a question (including a self-generated question), drawing on several sources and generating additional related, focused questions that allow for multiple avenues of exploration.	

6th Grade Statistical Measures/Data Displays Unit 6 Theme: Dreaming Big Suggested Pacing: 5 Weeks				
Thematic Question	Supporting Questions	Key Terms	Practice Standards Task(s)	Cross Curricular Connections
How I use Statistics to describe numerical information?	• How can you find an average value of a data set? • In what other ways can you describe an average of a data set? • How can you describe the spread of a data set? • How can you use the distances between each data value and the mean of a data set to measure the spread of a data set? • How can you use intervals, tables, and graphs to organize data? • How can you describe the shape of the distribution of a data set? • How can you use quartiles to represent data graphically?	• Center • Distribution • Spread • Shape • Center • Mean • Median • Mode • Range • Variability • Interquartile Range • Mean Absolute Deviation • Data set • Observation • Sample Size • Upper Quartile • Lower Quartile • Box Plot • Histogram • Median	• Big Ideas (2014): 6.SP.2 Movies • Big Ideas (2014): 6.SP.3 Olympic Medals • Big Ideas (2014): 6.SP.4 Game Show • Big Ideas (2014): 6.SP.5 Speed	• Bibliography • Credibility • Paraphrase

Utah Core Standards For Mathematics	Student Learning Targets	Curriculum Resources
Standard 6.SP.5 Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations (for example, outliers) from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	• I can summarize sets of numerical data that are different. • I can summarize data by stating the number of observations. • I can summarize data by describing the characteristics of what is being investigated, including how it was measured. • I can summarize data by giving numerical measures of center and variability • I can summarize data by describing the overall pattern of the data and noticing unusual deviations from the overall pattern. • I can summarize data by explaining how the distribution of the data on a graph determines its measure of center (median and/or mean).	Big Ideas (2014): Chapter 9 Chapter 10 Task: 6.SP.5 Speed
Standard 6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread/range and overall shape.	• I can describe a set of data using its center (mode, median, or mean), spread (range) and its shape.	Big Ideas (2014): Section 9.1 Section 10.3 Task: 6.SP.2 Movies

Standard 6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	• I can understand that a set of numerical data has a measure of center (median and/or mean) that summarizes all of its values with a single number. • I can understand that in a set of numerical data, the measure of variation describes how its values vary with a single number.	Big Ideas (2014): Section 9.2 Section 9.3 Section 9.4 Section 9.5 Task: 6.SP.3 Olympic Medals
Standard 6.SP.4 Display numerical data in plots on a number line, including dot plots, histograms and box plots. Choose the most appropriate graph/plot for the data collected.	• I can show numerical data on a number line. • I can display data with a dot plot. • I can display data with a box plot. • I can display data with a histogram.	Big Ideas (2014): Section 9.1 Section 10.2 Section 10.4 Section 10.5 Task: 6.SP.4 Game Show

Additional Resources
● Geometer's Sketchpad - Use it to create data tables and turn them into the different types of graphs. Student's can add or remove outliers and otherwise manipulate data. ● Most difficult concepts appear to be mean, median, mode, and measures of central tendencies. ● Utah State Virtual Manipulatives - Use it to create and manipulate graphs.

Science and Technical Subject Literacy Standards		Literacy Implementation Ideas
Reading	**6.RST.3** - Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks. **6.RST.4** - Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to 6th *grades topics*. **6.RST.7 -** Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).	

Writing	**6.WHST.2** - Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. **6.WHST.3** - Provide a concluding statement or section that follows from and supports the information or explanation presented. **6.WHST.7** - Conduct short research projects to answer a question (including a self-generated question), drawing on several sources and generating additional related, focused questions that allow for multiple avenues of exploration.	

What are Mathematical Tasks? Why are they important to the implementation of the new Utah Core Standards for Mathematics (UCSM)?

With the implementation of the new UCSM comes not only a shift of what mathematical concepts are taught, but also a shift in how students show their work. Previously, it was enough for a student to "show their work" to demonstrate they mastered a concept. With standards that are asking students to "Understand" mathematics, computational steps will not be enough to show mastery.

"Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness." (www.corestandards.org/math)

By integrating mathematical tasks into their instruction and assessment, teachers will be able to fully assess if students truly understand mathematical concepts.

A mathematical task is a problem or set of problems that focuses students' attention on a particular mathematical idea and provides an opportunity for students to develop or use particular mathematical skills. A task has a well-‐defined purpose and is accessible for all students (www.commoncoretools.me). Tasks also require students to defend and justify their solutions.

The following documents outline tasks that are aligned to Canyons School District Scope and Sequence and are supported through our text resource: Big Ideas Mathematics. The rubric can help guide teachers when planning for integration of mathematical tasks with the Standards for Mathematic Practices.

6th Grade Performance Tasks from Big Ideas Learning

Standard	Title	For Use After
The Number System:		
6.NS.1	Amusement Park	2.3
6.NS.2	Festival Trees	1.1
6.NS.3	Gold	2.6
6.NS.4	Fruits	3.4
6.NS.5	Lakes of North America	6.1
6.NS.6	Temperature	6.5
6.NS.7	Temperature on Planets	6.4
6.NS.8	Pain Ball	6.5
Expressions and Equations:		
6.EE.1	Band Competition	1.3
6.EE.2	Geometry	3.4
6.EE.3	Lacrosse	3.4e
6.EE.4	Perimeter of Geometric Figures	3.4e
6.EE.5	Bees	7.5
6.EE.6	Earnings	3.4
6.EE.7	Dinosaurs	7.3
6.EE.8	Country Fair	7.5
6.EE.9	Hiking	7.4
Ratios & Proportions:		
6.RP.1	Ships	5.4
6.RP.2	Factory Production	5.4
6.RP.3	Windmills	5.7
Geometry		
6.G.1	Concert Stages	4.3e
6.G.2	Money	8.4
6.G.3	Facial Recognition	4.4
6.G.4	Tents	8.3
Statistics & Probability		
7.SP.1	Softball	9.1
7.SP.2	Movies	10.3
7.SP.3	Olympic Medals	9.5
7.SP.4	Game Show	10.4
7.SP.5	Speed	10.2

The Cornerstone of WIDA's Standards: Guiding Principles of Language Development

1. **Students' languages and cultures are valuable resources to be tapped and incorporated into schooling.**
   Escamilla & Hopewell (2010); Goldenberg & Coleman (2010); Garcia (2005); Freeman, Freeman, & Mercuri (2002); González, Moll, & Amanti (2005); Scarcella (1990)

2. **Students' home, school, and community experiences influence their language development.**
   Nieto (2008); Payne (2003); Collier (1995); California State Department of Education (1986)

3. **Students draw on their metacognitive, metalinguistic, and metacultural awareness to develop proficiency in additional languages.**
   Cloud, Genesee, & Hamayan (2009); Bialystok (2007); Chamot & O'Malley (1994); Bialystok (1991); Cummins (1978)

4. **Students' academic language development in their native language facilitates their academic language development in English. Conversely, students' academic language development in English informs their academic language development in their native language.**
   Escamilla & Hopewell (2010); Gottlieb, Katz, & Ernst-Slavit (2009); Tabors (2008); Espinosa (2009); August & Shanahan (2006); Genesee, Lindholm-Leary, Saunders, & Christian (2006); Snow (2005); Genesee, Paradis, & Crago (2004); August & Shanahan (2006); Riches & Genesee (2006); Gottlieb (2003); Schleppegrell & Colombi (2002); Lindholm & Molina (2000); Pardo & Tinajero (1993)

5. **Students learn language and culture through meaningful use and interaction.**
   Brown (2007); Garcia & Hamayan, (2006); Garcia (2005); Kramsch (2003); Díaz-Rico & Weed (1995); Halliday & Hasan (1989); Damen (1987)

6. **Students use language in functional and communicative ways that vary according to context.**
   Schleppegrell (2004); Halliday (1976); Finocchiaro & Brumfit (1983)

7. **Students develop language proficiency in listening, speaking, reading, and writing interdependently, but at different rates and in different ways.**
   Gottlieb & Hamayan (2007); Spolsky (1989); Vygotsky (1962)

8. **Students' development of academic language and academic content knowledge are inter-related processes.**
   Gibbons (2009); Collier & Thomas (2009); Gottlieb, Katz, & Ernst-Slavit (2009); Echevarria, Vogt, & Short (2008); Zwiers (2008); Gee (2007); Bailey (2007); Mohan (1986)

9. **Students' development of social, instructional, and academic language, a complex and long-term process, is the foundation for their success in school.**
   Anstrom, et.al. (2010); Francis, Lesaux, Kieffer, & Rivera (2006); Bailey & Butler (2002); Cummins (1979)

10. **Students' access to instructional tasks requiring complex thinking is enhanced when linguistic complexity and instructional support match their levels of language proficiency.**
    Gottlieb, Katz, & Ernst-Slavit (2009); Gibbons (2009, 2002); Vygotsky (1962)

Performance Definitions for the Levels of English Language Proficiency in Grades K-12

At the given level of English language proficiency, English language learners will process, understand, produce, or use:

6 **Reaching**	• specialized or technical language reflective of the content areas at grade level • a variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level • oral or written communication in English comparable to English-proficient peers
5 **Bridging**	• specialized or technical language of the content areas • a variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays, or reports • oral or written language approaching comparability to that of English-proficient peers when presented with grade-level material
4 **Expanding**	• specific and some technical language of the content areas • a variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences, or paragraphs • oral or written language with minimal phonological, syntactic, or semantic errors that do not impede the overall meaning of the communication when presented with oral or written connected discourse with sensory, graphic, or interactive support
3 **Developing**	• general and some specific language of the content areas • expanded sentences in oral interaction or written paragraphs • oral or written language with phonological, syntactic, or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative, or expository descriptions with sensory, graphic, or interactive support
2 **Beginning**	• general language related to the content areas • phrases or short sentences • oral or written language with phonological, syntactic, or semantic errors that often impede the meaning of the communication when presented with one- to multiple-step commands, directions, questions, or a series of statements with sensory, graphic, or interactive support
1 **Entering**	• pictorial or graphic representation of the language of the content areas • words, phrases, or chunks of language when presented with one-step commands, directions, WH-, choice, or yes/no questions, or statements with sensory, graphic, or interactive support • oral language with phonological, syntactic, or semantic errors that often impede meaning when presented with basic oral commands, direct questions, or simple statements with sensory, graphic, or interactive support

North Carolina Unpacked Standards

*Note: Since Utah has modified and adopted their own core, some standards may be different. However, this is a good resource for tasks for your class.

North Carolina Department of Public Instruction

INSTRUCTIONAL SUPPORT TOOLS

FOR ACHIEVING NEW STANDARDS

6th Grade Mathematics ● Unpacked Contents

For the new Common Core standards that will be effective in all North Carolina schools in the 2012-13 School Year.

This document is designed to help North Carolina educators teach the Common Core (Standard Course of Study). NCDPI staff are continually updating and improving these tools to better serve teachers.

What is the purpose of this document?

To increase student achievement by ensuring educators understand specifically what the new standards mean a student must know, understand and be able to do.

What is in the document?

Descriptions of what each standard means a student will know, understand and be able to do. The "unpacking" of the standards done in this document is an effort to answer a simple question "What does this standard mean that a student must know and be able to do?" and to ensure the description is helpful, specific and comprehensive for educators.

How do I send Feedback?

We intend the explanations and examples in this document to be helpful and specific. That said, we believe that as this document is used, teachers and educators will find ways in which the unpacking can be improved and made ever more useful. Please send feedback to us at feedback@dpi.state.nc.us and we will use your input to refine our unpacking of the standards. Thank You!

Just want the standards alone?

You can find the standards alone at http://corestandards.org/the-standards

At A Glance

This page was added to give a snapshot of the mathematical concepts that are new or have been removed from this grade level as well as instructional considerations for the first year of implementation.

New to 6th Grade:

- Unit rate (6.RP.3b)
- Measurement unit conversions (6.RP 3d)
- Number line – opposites and absolute value (6.NS.6a, 6.NS.7c)
- Vertical and horizontal distances on the coordinate plane (6.NS.8)
- Distributive property and factoring (6.EE.3)
- Introduction of independent and dependent variables (6.NS.9)
- Volume of right rectangular prisms with fractional edges (6.G.2)
- Surface area with nets (only triangle and rectangle faces) (6.G.4)
- Dot plots, histograms, box plots (6.SP.4)
- Statistical variability (Mean Absolute Deviation (MAD) and Interquartile Range (IQR)) (6.G.5c)

Moved from 6th Grade:

- Multiplication of fractions (moved to 5th grade)
- Scientific notation (moved to 8th grade)
- Transformations (moved to 8th grade)
- Area and circumference of circles (moved to 7th grade)
- Probability (moved to 7th grade)
- Two-step equations (moved to 7th grade)
- Solving one- and two-step inequalities (moved to 7th grade)

Notes:

- Topics may appear to be similar between the CCSS and the 2003 NCSCOS; however, the CCSS may be presented at a higher cognitive demand.
- Equivalent fractions, decimals and percents are in 6th grade but as conceptual representations (see 6.RP.2c). Use of the number line (building on elementary foundations) is also encouraged.
- For more detailed information, see the crosswalks.
- 6.NS. 2 is the final check for student understanding of place value.
- **For more detailed information, see the crosswalks (http://www.ncpublicschools.org/acre/standards/common-core-tools)**

Instructional considerations for CCSS implementation in 2012 – 2013:

- Multiplication of fractions (reference 5.NF.3, 5.NF.4a, 5.NF.4b, 5.NF.5a, 5.NF.5b, 5.NF.6)
- Division of whole number by unit fractions and division of unit fractions by whole numbers (reference 5.NF.7a, 5.NF.7b, 5.NF.7c)
- Multiplication and division of decimals (reference 5.NBT.7)
- Volume with whole number (reference 5.MD.3, 5.MD.4, 5.MD.5)
- Classification of two-dimensional figures based on their properties (reference 5.G.3, 5,G.4)

Standards for Mathematical Practice

The Common Core State Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that students complete.

Standards for Mathematical Practice	**Explanations and Examples**
1. Make sense of problems and persevere in solving them.	In grade 6, students solve real world problems through the application of algebraic and geometric concepts. These problems involve ratio, rate, area and statistics. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?". Students can explain the relationships between equations, verbal descriptions, tables and graphs. Mathematically proficient students check answers to problems using a different method.
2. Reason abstractly and quantitatively.	In grade 6, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3. Construct viable arguments and critique the reasoning of others.	In grade 6, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.
4. Model with mathematics.	In grade 6, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students begin to explore covariance and represent two quantities simultaneously. Students use number lines to compare numbers and represent inequalities. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences about and make comparisons between data sets. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
5. Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 6 may decide to represent figures on the coordinate plane to calculate area. Number lines are used to understand division and to create dot plots, histograms and box plots to visually compare the center and variability of the data. Additionally, students might use physical objects or applets to construct nets and calculate the surface area of three-dimensional figures.
6. Attend to precision.	In grade 6, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations or inequalities.

Standards for Mathematical Practice	**Explanations and Examples**
7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 3 (2 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality, c=6 by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving area and volume.
8. Look for and express regularity in repeated reasoning.	In grade 6, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. Students connect place value and their prior work with operations to understand algorithms to fluently divide multi-digit numbers and perform all operations with multi-digit decimals. Students informally begin to make connections between covariance, rates, and representations showing the relationships between quantities.

Grade 6 Critical Areas (from CCSS pgs. 39 – 40)

The Critical Areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction. The Critical Areas for sixth grade can be found beginning on page 39 in the *Common Core State Standards for Mathematics.*

1. **Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems.**
   Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

2. **Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers.**
   Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

3. **Writing, interpreting, and using expressions and equations.**
   Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

4. **Developing understanding of statistical thinking.**
   Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Grade 6 Critical Areas (from CCSS pgs. 39 – 40)

5. **Reasoning about relationships among shapes to determine area, surface area, and volume.**
   Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

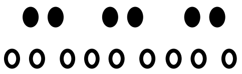
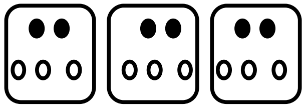
Ratios and Proportional Relationships 6.RP

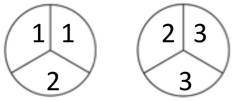
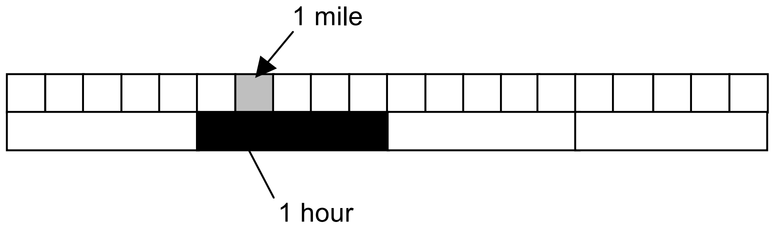
Common Core Cluster

Understand ratio concepts and use ratio reasoning to solve problems.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **ratio, equivalent ratios, tape diagram, unit rate, part-to-part, part-to-whole, percent**

A detailed progression of the Ratios and Proportional Relationships domain with examples can be found at http://commoncoretools.wordpress.com/

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*	**6.RP.1** A ratio is the comparison of two quantities or measures. The comparison can be part-to-whole (ratio of guppies to all fish in an aquarium) or part-to-part (ratio of guppies to goldfish). Example 1: A comparison of 6 guppies and 9 goldfish could be expressed in any of the following forms: $\frac{6}{9}$, 6 to 9 *or* 6:9. If the number of guppies is represented by black circles and the number of goldfish is represented by white circles, this ratio could be modeled as  These values can be regrouped into 2 black circles (goldfish) to 3 white circles (guppies), which would reduce the ratio to, $\frac{2}{3}$, 2 to 3 or 2:3.  Students should be able to identify and describe any ratio using "For every _____ ,there are _____" In the example above, the ratio could be expressed saying, "For every 2 goldfish, there are 3 guppies".
6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the	**6.RP.2** A unit rate expresses a ratio as part-to-one, comparing a quantity in terms of one unit of another quantity. Common unit rates are cost per item or distance per time.

context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1] [1] Expectations for unit rates in this grade are limited to non-complex fractions.	Students are able to name the amount of either quantity in terms of the other quantity. Students will begin to notice that related unit rates (i.e. miles / hour and hours / mile) are reciprocals as in the second example below. At this level, students should use reasoning to find these unit rates instead of an algorithm or rule. In 6th grade, students are not expected to work with unit rates expressed as complex fractions. Both the numerator and denominator of the original ratio will be whole numbers. <u>Example 1:</u> There are 2 cookies for 3 students. What is the amount of cookie each student would receive? (i.e. the unit rate) *Solution:* This can be modeled as shown below to show that there is $\frac{2}{3}$ of a cookie for 1 student, so the unit rate is $\frac{2}{3}$: 1.  <u>Example 2:</u> On a bicycle Jack can travel 20 miles in 4 hours. What are the unit rates in this situation, (the distance Jack can travel in 1 hour and the amount of time required to travel 1 mile)? *Solution:* Jack can travel 5 miles in 1 hour written as $\frac{5\,mi}{1\,hr}$ and it takes $\frac{1}{5}$ of a hour to travel each mile written as $\frac{\frac{1}{5}\text{hr}}{1\text{mi}}$. Students can represent the relationship between 20 miles and 4 hours. 

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

6.RP.3 Ratios and rates can be used in ratio tables and graphs to solve problems. Previously, students have used additive reasoning in tables to solve problems. To begin the shift to proportional reasoning, students need to begin using multiplicative reasoning. To aid in the development of proportional reasoning the cross-product algorithm is ***not*** expected at this level. When working with ratio tables and graphs, ***whole number*** measurements are the expectation for this standard.

Example 1:
At Books Unlimited, 3 paperback books cost $18. What would 7 books cost? How many books could be purchased with $54.

Solution: To find the price of 1 book, divide $18 by 3. One book costs $6. To find the price of 7 books, multiply $6 (the cost of one book times 7 to get $42. To find the number of books that can be purchased with $54, multiply $6 times 9 to get $54 and then multiply 1 book times 9 to get 9 books. Students use ratios, unit rates and multiplicative reasoning to solve problems in various contexts, including measurement, prices, and geometry. Notice in the table below, a multiplicative relationship exists between the numbers both horizontally (times 6) and vertically (ie. 1 • 7 = 7; 6 • 7 = 42). Red numbers indicate solutions.

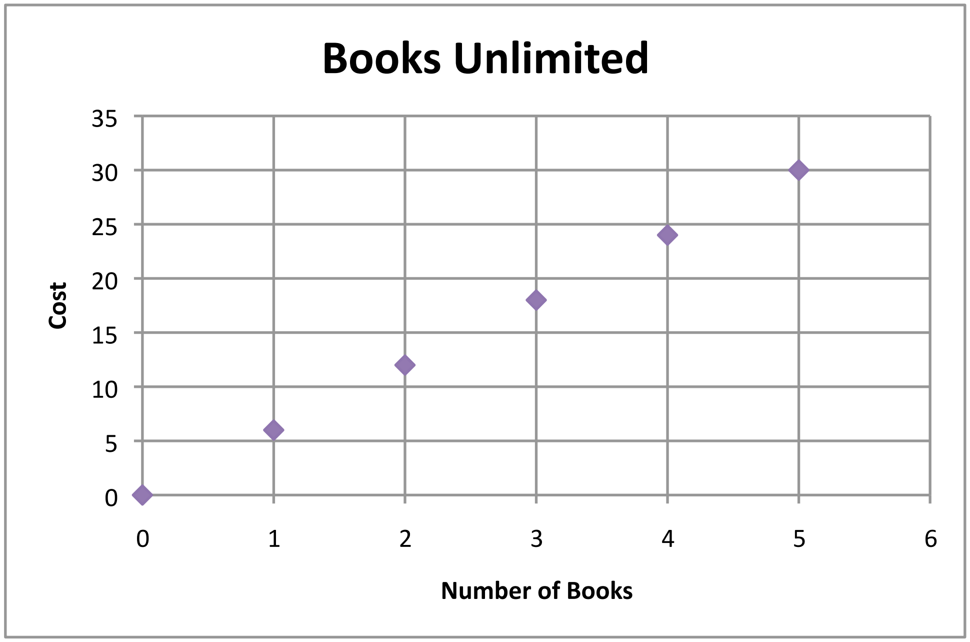
Number of Books (n)	**Cost (C)**
1	6
3	18
7	42
9	54

Students use tables to compare ratios. Another bookstore offers paperback books at the prices below. Which bookstore has the best buy? Explain your answer.

Number of Books (n)	**Cost (C)**
4	20
8	40

To help understand the multiplicative relationship between the number of books and cost, students write equations to express the cost of any number of books. Writing equations is foundational for work in 7th grade. For example, the equation for the first table would be $C = 6n$, while the equation for the second bookstore is $C = 5n$.
The numbers in the table can be expressed as ordered pairs (number of books, cost) and plotted on a coordinate plane.

Students are able to plot ratios as ordered pairs. For example, a graph of Books Unlimited would be:



Example 2:
Ratios can also be used in problem solving by thinking about the total amount for each ratio unit.
The ratio of cups of orange juice concentrate to cups of water in punch is 1: 3. If James made 32 cups of punch, how many cups of orange did he need?

Solution: Students recognize that the total ratio would produce 4 cups of punch. To get 32 cups, the ratio would need to be duplicated 8 times, resulting in 8 cups of orange juice concentrate.

Example 3:
Using the information in the table, find the number of yards in 24 feet.

Feet	3	6	9	15	24
Yards	1	2	3	5	?

Solution:

There are several strategies that students could use to determine the solution to this problem:

- Add quantities from the table to total 24 feet (9 feet and 15 feet); therefore the number of yards in 24 feet must be 8 yards (3 yards and 5 yards).
- Use multiplication to find 24 feet: 1) 3 feet x 8 = 24 feet; therefore 1 yard x 8 = 8 yards, or 2) 6 feet x 4 = 24 feet; therefore 2 yards x 4 = 8 yards.

<u>Example 4:</u>

Compare the number of black circles to white circles. If the ratio remains the same, how many black circles will there be if there are 60 white circles?

● ● ● ● ○ ○ ○

Black	4	40	20	60	?
White	3	30	15	45	60

Solution:

There are several strategies that students could use to determine the solution to this problem

- Add quantities from the table to total 60 white circles (15 + 45). Use the corresponding numbers to determine the number of black circles (20 + 60) to get 80 black circles.
- Use multiplication to find 60 white circles (one possibility 30 x 2). Use the corresponding numbers and operations to determine the number of black circles (40 x 2) to get 80 black circles.

b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

Students recognize the use of ratios, unit rate and multiplication in solving problems, which could allow for the use of fractions and decimals.

<u>Example 1:</u>

In trail mix, the ratio of cups of peanuts to cups of chocolate candies is 3 to 2. How many cups of chocolate candies would be needed for 9 cups of peanuts.

Peanuts	**Chocolate**
3	2

Solution:
One possible solution is for students to find the number of cups of chocolate candies for 1 cup of peanuts by dividing both sides of the table by 3, giving $\frac{2}{3}$ cup of chocolate for each cup of peanuts. To find the amount of chocolate needed for 9 cups of peanuts, students multiply the unit rate by nine $(9 \bullet \frac{2}{3})$, giving 6 cups of chocolate.

<u>Example 2:</u>
If steak costs $2.25 per pound, how much does 0.8 pounds of steak cost? Explain how you determined your answer.

Solution:
The unit rate is $2.25 per pound so multiply $2.25 x 0.8 to get $1.80 per 0.8 lb of steak.

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c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

This is the students' first introduction to percents. Percentages are a rate per 100. Models, such as percent bars or 10 x 10 grids should be used to model percents.

Students use ratios to identify percents.

<u>Example 1:</u>
What percent is 12 out of 25?
Solution: One possible solution method is to set up a ratio table: Multiply 25 by 4 to get 100. Multiplying 12 by 4 will give 48, meaning that 12 out of 25 is equivalent to 48 out of 100 or 48%.

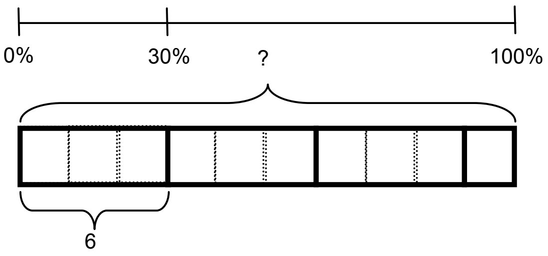
Part	**Whole**
12	25
?	100

Students use percentages to find the part when given the percent, by recognizing that the whole is being divided into 100 parts and then taking a part of them (the percent).

<u>Example 2:</u>
What is 40% of 30?
Solution: There are several methods to solve this problem. One possible solution using rates is to use a 10 x 10 grid to represent the whole amount (or 30). If the 30 is divided into 100 parts, the rate for one block is 0.3. Forty percent would be 40 of the blocks, or 40 x 0.3, which equals 12.
See the weblink below for more information.
http://illuminations.nctm.org/LessonDetail.aspx?id=L249

Students also determine the whole amount, given a part and the percent.

<u>Example 3:</u>
If 30% of the students in Mrs. Rutherford's class like chocolate ice cream, then how many students are in Mrs. Rutherford's class if 6 like chocolate ice cream?



(*Solution:* 20)

Example 4:
A credit card company charges 17% interest fee on any charges not paid at the end of the month. Make a ratio table to show how much the interest would be for several amounts. If the bill totals $450 for this month, how much interest would you have to be paid on the balance?
Solution:

Charges	$1	$50	$100	$200	$450
Interest	$0.17	$8.50	$17	$34	?

One possible solution is to multiply 1 by 450 to get 450 and then multiply 0.17 by 450 to get $76.50.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

A ratio can be used to compare measures of two different types, such as inches per foot, milliliters per liter and centimeters per inch. Students recognize that a conversion factor is a fraction equal to 1 since the numerator and denominator describe the same quantity. For example, $\frac{12\text{ inches}}{1\text{ foot}}$ is a conversion factor since the numerator and denominator equal the same amount. Since the ratio is equivalent to 1, the identity property of multiplication allows an amount to be multiplied by the ratio. Also, the value of the ratio can also be expressed as $\frac{1\text{ foot}}{12\text{ inches}}$ allowing for the conversion ratios to be expressed in a format so that units will "cancel".

Students use ratios as conversion factors and the identity property for multiplication to convert ratio units.

Example 1:
How many centimeters are in 7 feet, given that 1 inch ≈ 2.54 cm.
Solution:

$$7\text{ feet} \times \frac{12\text{ inches}}{1\text{ foot}} \times \frac{2.54\text{ cm}}{1\text{ inch}} = 7\,\cancel{\text{feet}} \times \frac{12\,\cancel{\text{inches}}}{1\,\cancel{\text{foot}}} \times \frac{2.54\text{ cm}}{1\,\cancel{\text{inch}}} = 7 \times 12 \times 2.54\text{ cm} = 213.36\text{ cm}$$

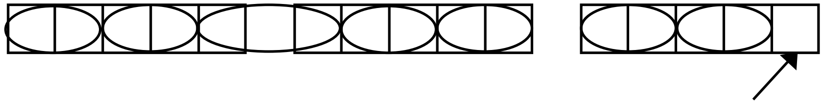
Note: Conversion factors will be given. Conversions can occur both between and across the metric and English system. Estimates are not expected.

The Number System 6.NS

Common Core Cluster

Apply and extend previous understands of multiplication and division to divide fractions by fractions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **reciprocal, multiplicative inverses, visual fraction model**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*	**6.NS.1** In 5th grade students divided whole numbers by unit fractions and divided unit fractions by whole numbers. Students continue to develop this concept by using visual models and equations to divide whole numbers by fractions and fractions by fractions to solve word problems. Students develop an understanding of the relationship between multiplication and division. Example 1: Students understand that a division problem such as $3 \div \frac{2}{5}$ is asking, "how many $\frac{2}{5}$ are in 3?" One possible visual model would begin with three whole and divide each into fifths. There are 7 groups of two-fifths in the three wholes. However, one-fifth remains. Since one-fifth is half of a two-fifths group, there is a remainder of $\frac{1}{2}$. Therefore, $3 \div \frac{2}{5} = 7\frac{1}{2}$, meaning there are $7\frac{1}{2}$ groups of two-fifths. Students interpret the solution, explaining how division by fifths can result in an answer with halves.  This section represents one-half of two-fifths Students also write contextual problems for fraction division problems. For example, the problem, $\frac{2}{3} \div \frac{1}{6}$ can be illustrated with the following word problem:

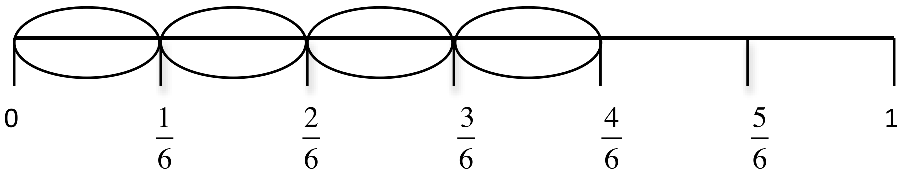
<u>Example 2:</u>

Susan has $\frac{2}{3}$ of an hour left to make cards. It takes her about $\frac{1}{6}$ of an hour to make each card. About how many can she make?

This problem can be modeled using a number line.

a. Start with a number line divided into thirds.

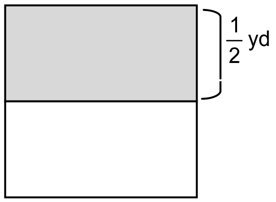
b. The problem wants to know how many sixths are in two-thirds. Divide each third in half to create sixths.

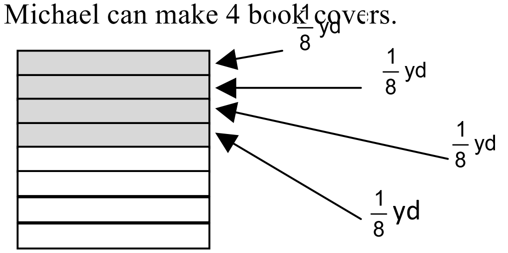


c. Each circled part represents $\frac{1}{6}$. There are four sixths in two-thirds; therefore, Susan can make 4 cards.

Example 3:

Michael has $\frac{1}{2}$ of a yard of fabric to make book covers. Each book cover is made from $\frac{1}{8}$ of a yard of fabric. How many book covers can Michael make? Solution: Michael can make 4 book covers.





Example 4:

Represent $\frac{1}{2} \div \frac{2}{3}$ in a problem context and draw a model to show your solution.

Context: A recipe requires $\frac{2}{3}$ of a cup of yogurt. Rachel has $\frac{1}{2}$ of a cup of yogurt from a snack pack. How much of the recipe can Rachel make?

Explanation of Model:

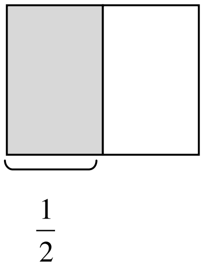
The first model shows $\frac{1}{2}$ cup. The shaded squares in all three models show the $\frac{1}{2}$ cup.

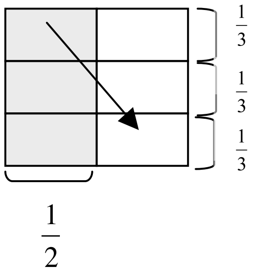
The second model shows $\frac{1}{2}$ cup and also shows $\frac{1}{3}$ cups horizontally.

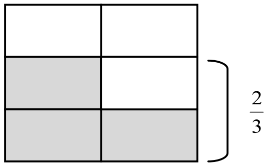
The third model shows $\frac{1}{2}$ cup moved to fit in only the area shown by $\frac{2}{3}$ of the model.

$\frac{2}{3}$ is the new referent unit (whole) .

3 out of the 4 squares in the $\frac{2}{3}$ portion are shaded. A $\frac{1}{2}$ cup is only $\frac{3}{4}$ of a $\frac{2}{3}$ cup portion, so only ¾ of the recipe can be made.







The Number System	**6.NS**
Common Core Cluster	
Compute fluently with multi-digit numbers and find common factors and multiples.	
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **multi-digit**	
Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.	**6.NS.2** In the elementary grades, students were introduced to division through concrete models and various strategies to develop an understanding of this mathematical operation (limited to 4-digit numbers divided by 2-digit numbers). In 6th grade, students become fluent in the use of the standard division algorithm, continuing to use their understanding of place value to describe what they are doing. Place value has been a major emphasis in the elementary standards. This standard is the end of this progression to address students' understanding of place value. Example 1: When dividing 32 into 8456, students should say, "there are 200 thirty-twos in 8456" as they write a 2 in the quotient. They could write 6400 beneath the 8456 rather than only writing 64.

$\begin{array}{r} 2 \\ 32\overline{)8456} \end{array}$	There are 200 thirty twos in 8456.
$\begin{array}{r} 2 \\ 32\overline{)8456} \\ \underline{6400} \\ 2056 \end{array}$	200 times 32 is 6400. 8456 minus 6400 is 2056.
$\begin{array}{r} 26 \\ 32\overline{)8456} \\ \underline{6400} \\ 2056 \end{array}$	There are 60 thirty twos in 2056.
$\begin{array}{r} 26 \\ 32\overline{)8456} \\ \underline{6400} \\ 2056 \\ \underline{1920} \\ 136 \end{array}$	60 times 32 is 1920. 2056 minus 1920 is 136.
$\begin{array}{r} 264 \\ 32\overline{)8456} \\ \underline{6400} \\ 2056 \\ \underline{1920} \\ 136 \\ \underline{128} \end{array}$	There are 4 thirty twos in 136. 4 times 32 is 128.
$\begin{array}{r} 264 \\ 32\overline{)8456} \\ \underline{6400} \\ 2056 \\ \underline{1920} \\ 136 \\ \underline{128} \\ 8 \end{array}$	The remainder is 8. There is not a full thirty two in 8; there is only part of a thirty two in 8. This can also be written as $\frac{8}{32}$ or $\frac{1}{4}$. There is ¼ of a thirty two in 8. 8456 = 264 * 32 + 8

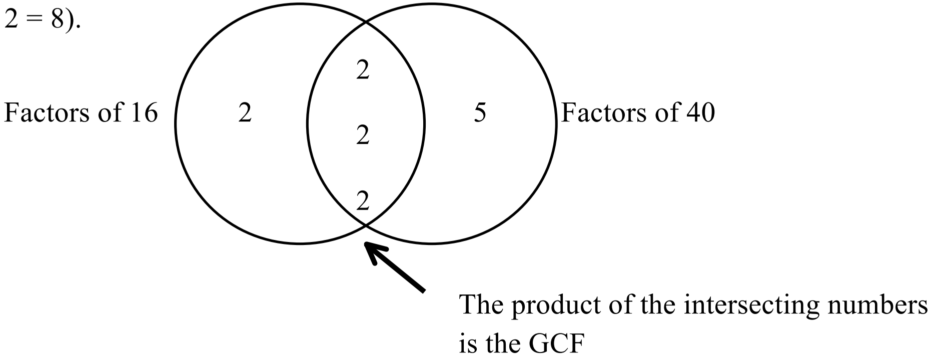
6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.	**6.NS.3** Procedural fluency is defined by the Common Core as "skill in carrying out procedures flexibly, accurately, efficiently and appropriately". In 4th and 5th grades, students added and subtracted decimals. Multiplication and division of decimals were introduced in 5th grade (decimals to the hundredth place). At the elementary level, these operations were based on concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. In 6th grade, students become fluent in the use of the standard algorithms of each of these operations. The use of estimation strategies supports student understanding of decimal operations. <u>Example 1:</u> First estimate the sum of 12.3 and 9.75. *Solution:* An estimate of the sum would be 12 + 10 or 22. Student could also state if their estimate is high or low. Answers of 230.5 or 2.305 indicate that students are not considering place value when adding.

The Number System 6.NS

Common Core Cluster

Compute fluently with multi-digit numbers and find common factors and multiples.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **greatest common factor, least common multiple, prime numbers, composite numbers, relatively prime, factors, multiples, distributive property, prime factorization**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).	In elementary school, students identified primes, composites and factor pairs (4.OA.4). In 6th grade students will find the greatest common factor of two whole numbers less than or equal to 100. For example, the greatest common factor of 40 and 16 can be found by 1) listing the factors of 40 (1, 2, 4, 5, 8, 10, 20, 40) and 16 (1, 2, 4, 8, 16), then taking the greatest common factor (8). Eight (8) is also the largest number such that the other factors are **relatively prime** (two numbers with no common factors other than one). For example, 8 would be multiplied by 5 to get 40; 8 would be multiplied by 2 to get 16. Since the 5 and 2 are relatively prime, then 8 is the greatest common factor. If students think 4 is the greatest, then show that 4 would be multiplied by 10 to get 40, while 16 would be 4 times 4. Since the 10 and 4 are not relatively prime (have 2 in common), the 4 cannot be the greatest common factor. 2) listing the prime factors of 40 (2 • 2 • 2 • 5) and 16 (2 • 2 • 2 • 2) and then multiplying the common factors (2 • 2 • 2 = 8).  The product of the intersecting numbers is the GCF Students also understand that the greatest common factor of two prime numbers is 1. Example 1: What is the greatest common factor (GCF) of 18 and 24? *Solution:* $2 \ast 3^2 = 18$ and $2^3 \ast 3 = 24$. Students should be able to explain that both 18 and 24 will have at least one factor of 2 and at least one factor of 3 in common, making $2 \ast 3$ or 6 the GCF.

Given various pairs of addends using whole numbers from 1-100, students should be able to identify if the two numbers have a common factor. If they do, they identify the common factor and use the distributive property to rewrite the expression. They prove that they are correct by simplifying both expressions.

Example 2:

Use the greatest common factor and the distributive property to find the sum of 36 and 8.

36 + 8 = 4 (9) + 4(2)

44 = 4 (9 + 2)

44 = 4 (11)

44 = 44✓

Example 3:

Ms. Spain and Mr. France have donated a total of 90 hot dogs and 72 bags of chips for the class picnic. Each student will receive the same amount of refreshments. All refreshments must be used.

a. What is the greatest number of students that can attend the picnic?
b. How many bags of chips will each student receive?
c. How many hotdogs will each student receive?

Solution:

a. Eighteen (18) is the greatest number of students that can attend the picnic (GCF).
b. Each student would receive 4 bags of chips.
c. Each student would receive 5 hot dogs.

Students find the least common multiple of two whole numbers less than or equal to twelve. For example, the least common multiple of 6 and 8 can be found by

1) listing the multiplies of 6 (6, 12, 18, 24, 30, …) and 8 (8, 26, 24, 32, 40…), then taking the least in common from the list (24); or
2) using the prime factorization.
   Step 1: find the prime factors of 6 and 8.
   6 = 2 • 3
   8 = 2 • 2 • 2

Step 2: Find the common factors between 6 and 8. In this example, the common factor is 2

Step 3: Multiply the common factors and any extra factors: 2 • 2 • 2 • 3 or 24 (one of the twos is in common; the other twos and the three are the extra factors.

Example 4:

The elementary school lunch menu repeats every 20 days; the middle school lunch menu repeats every 15 days. Both schools are serving pizza today. In how may days will both schools serve pizza again?

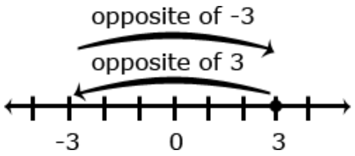
	Solution: The solution to this problem will be the least common multiple (LCM) of 15 and 20. Students should be able to explain that the least common multiple is the smallest number that is a multiple of 15 and a multiple of 20. One way to find the least common multiple is to find the prime factorization of each number: 2^2 * 5 = 20 and 3 * 5 = 15. To be a multiple of 20, a number must have 2 factors of 2 and one factor of 5 (2 * 2 * 5). To be a multiple of 15, a number must have factors of 3 and 5. The least common multiple of 20 and 15 must have 2 factors of 2, one factor of 3 and one factor of 5 (2 * 2 * 3 * 5) or 60.

The Number System 6.NS

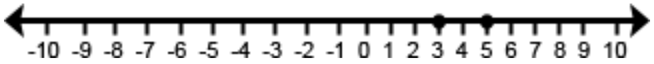
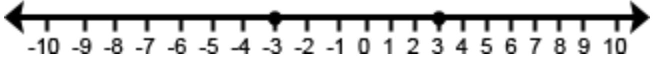
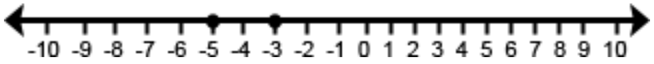
Common Core Cluster

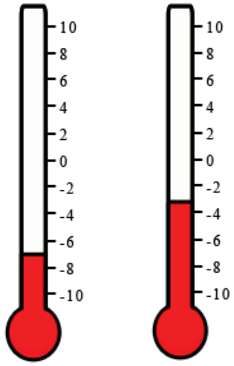
Apply and extend previous understandings of numbers to the system of rational numbers.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **rational numbers, opposites, absolute value, greater than, >, less than, <, greater than or equal to, ≥, less than or equal to, ≤, origin, quadrants, coordinate plane, ordered pairs, *x*-axis, *y*-axis, coordinates**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	**6.NS.5** Students use rational numbers (fractions, decimals, and integers) to represent real-world contexts and understand the meaning of 0 in each situation. Example 1: a. Use an integer to represent 25 feet below sea level b. Use an integer to represent 25 feet above sea level. c. What would 0 (zero) represent in the scenario above? *Solution:* a. -25 b. +25 c. 0 would represent sea level
6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., – (–3) = 3, and that 0 is its own opposite	**6.NS.6** In elementary school, students worked with positive fractions, decimals and whole numbers on the number line and in quadrant 1 of the coordinate plane. In 6th grade, students extend the number line to represent all rational numbers and recognize that number lines may be either horizontal or vertical (i.e. thermometer) which facilitates the movement from number lines to coordinate grids. Students recognize that a number and its opposite are equidistance from zero (reflections about the zero). The opposite sign (–) shifts the number to the opposite side of 0. For example, – 4 could be read as "the opposite of 4" which would be negative 4. In the example, – (–6.4) would be read as "the opposite of the opposite of 6.4" which would be 6.4. Zero is its own opposite.  Example 1: What is the opposite of $2\frac{1}{2}$? Explain your answer? *Solution:* $- 2\frac{1}{2}$ because it is the same distance from 0 on the opposite side.

b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	Students worked with Quadrant I in elementary school. As the *x*-axis and *y*-axis are extending to include negatives, students begin to with the Cartesian Coordinate system. Students recognize the point where the *x*-axis and *y*-axis intersect as the origin. Students identify the four quadrants and are able to identify the quadrant for an ordered pair based on the signs of the coordinates. For example, students recognize that in Quadrant II, the signs of all ordered pairs would be (–, +). Students understand the relationship between two ordered pairs differing only by signs as reflections across one or both axes. For example, in the ordered pairs (-2, 4) and (-2, -4), the *y*-coordinates differ only by signs, which represents a reflection across the *x*-axis. A change is the *x*-coordinates from (-2, 4) to (2, 4), represents a reflection across the *y*-axis. When the signs of both coordinates change, [(2, -4) changes to (-2, 4)], the ordered pair has been reflected across both axes. <u>Example1:</u> Graph the following points in the correct quadrant of the coordinate plane. If the point is reflected across the *x*-axis, what are the coordinates of the reflected points? What similarities are between coordinates of the original point and the reflected point? $\left(\frac{1}{2}, -3\frac{1}{2}\right)$ $\left(-\frac{1}{2}, -3\right)$ $(0.25, 0.75)$ *Solution:* The coordinates of the reflected points would be $\left(\frac{1}{2}, 3\frac{1}{2}\right)$ $\left(-\frac{1}{2}, 3\right)$ $(0.25, 0.75)$. Note that the *y*-coordinates are opposites. <u>Example 2:</u> Students place the following numbers would be on a number line: –4.5, 2, 3.2, $-3\frac{3}{5}$, 0.2, –2, $\frac{11}{2}$. Based on number line placement, numbers can be placed in order. *Solution:* The numbers in order from least to greatest are: –4.5, $-3\frac{3}{5}$, –2, 0.2, 2, 3.2, $\frac{11}{2}$ Students place each of these numbers on a number line to justify this order.

6.NS.7 Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*	**6.NS.7** Students use inequalities to express the relationship between two rational numbers, understanding that the value of numbers is smaller moving to the left on a number line. Common models to represent and compare integers include number line models, temperature models and the profit-loss model. On a number line model, the number is represented by an arrow drawn from zero to the location of the number on the number line; the absolute value is the length of this arrow. The number line can also be viewed as a thermometer where each point of on the number line is a specific temperature. In the profit-loss model, a positive number corresponds to profit and the negative number corresponds to a loss. Each of these models is useful for examining values but can also be used in later grades when students begin to perform operations on integers. **Operations with integers are not the expectation at this level.** In working with number line models, students internalize the order of the numbers; larger numbers on the right (horizontal) or top (vertical) of the number line and smaller numbers to the left (horizontal) or bottom (vertical) of the number line. They use the order to correctly locate integers and other rational numbers on the number line. By placing two numbers on the same number line, they are able to write inequalities and make statements about the relationships between two numbers. Case 1: Two positive numbers  5 > 3 5 is greater than 3 3 is less than 5 Case 2: One positive and one negative number  3 > -3 positive 3 is greater than negative 3 negative 3 is less than positive 3 Case 3: Two negative numbers  -3 > -5 negative 3 is greater than negative 5 negative 5 is less than negative 3

	Example 1: Write a statement to compare – 4 ½ and –2. Explain your answer. *Solution:* – 4 ½ < –2 because – 4 ½ is located to the left of –2 on the number line Students recognize the distance from zero as the absolute value or magnitude of a rational number. Students need multiple experiences to understand the relationships between numbers, absolute value, and statements about order.
b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3°C > –7°C to express the fact that –3°C is warmer than –7°C.*	Students write statements using < or > to compare rational number in context. However, explanations should reference the context rather than "less than" or "greater than". Example 1: The balance in Sue's checkbook was –\$12.55. The balance in John's checkbook was –\$10.45. Write an inequality to show the relationship between these amounts. Who owes more? *Solution:* –12.55 < –10.45, Sue owes more than John. The interpretation could also be "John owes less than Sue". Example 2: One of the thermometers shows -3°C and the other shows -7°C. Which thermometer shows which temperature? Which is the colder temperature? How much colder? Write an inequality to show the relationship between the temperatures and explain how the model shows this relationship.  *Solution:* • The thermometer on the left is -7; right is -3 • The left thermometer is colder by 4 degrees • Either -7 < -3 or -3 > -7 Although 6.NS.7a is limited to two numbers, this part of the standard expands the ordering of rational numbers to more than two numbers in context. Example 3: A meteorologist recorded temperatures in four cities around the world. List these cities in order from coldest temperature to warmest temperature: Albany 5° Anchorage -6° Buffalo -7° Juneau -9° Reno 12°

<table>
<tr><td></td><td>*Solution:*
Juneau -9°
Buffalo -7°
Anchorage -6°
Albany 5°
Reno 12°</td></tr>
<tr><td>c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*</td><td>Students understand absolute value as the distance from zero and recognize the symbols | | as representing absolute value.
Example 1:
Which numbers have an absolute value of 7
Solution: 7 and –7 since both numbers have a distance of 7 units from 0 on the number line.

Example 2:
What is the | $-3\frac{1}{2}$ |?
Solution: $3\frac{1}{2}$

In real-world contexts, the absolute value can be used to describe size or magnitude. For example, for an ocean depth of 900 feet, write | –900| = 900 to describe the distance below sea level.</td></tr>
<tr><td>d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*</td><td>When working with positive numbers, the absolute value (distance from zero) of the number and the value of the number is the same; therefore, ordering is not problematic. However, negative numbers have a distinction that students need to understand. As the negative number increases (moves to the left on a number line), the value of the number decreases. For example, –24 is less than –14 because –24 is located to the left of –14 on the number line. However, absolute value is the distance from zero. In terms of absolute value (or distance) the absolute value of –24 is greater than the absolute value of –14. For negative numbers, as the absolute value increases, the value of the negative number decreases.</td></tr>
<tr><td>**6.NS.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.</td><td>**6.NS.8** Students find the distance between points when ordered pairs have the same x-coordinate (vertical) or same y-coordinate (horizontal).

Example 1:
What is the distance between (–5, 2) and (–9, 2)?

Solution: The distance would be 4 units. This would be a horizontal line since the *y*-coordinates are the same. In this scenario, both coordinates are in the same quadrant. The distance can be found by using a number line to find the distance between –5 and –9. Students could also recognize that –5 is 5 units from 0 (absolute value) and that –9 is 9 units from 0 (absolute value). Since both of these are in the same quadrant, the distance can be found by finding the difference between the distances 9 and 5. (| 9 | - | 5 |).</td></tr>
</table>

Coordinates could also be in two quadrants and include rational numbers.

Example 2:

What is the distance between $(3, -5\frac{1}{2})$ and $(3, 2\frac{1}{4})$?

Solution: The distance between $(3, -5\frac{1}{2})$ and $(3, 2\frac{1}{4})$ would be $7\frac{3}{4}$ units. This would be a vertical line since the *x*-coordinates are the same. The distance can be found by using a number line to count from $-5\frac{1}{2}$ to $2\frac{1}{4}$ or by recognizing that the distance (absolute value) from $-5\frac{1}{2}$ to 0 is $5\frac{1}{2}$ units and the distance (absolute value) from 0 to $2\frac{1}{4}$ is $2\frac{1}{4}$ units so the total distance would be $5\frac{1}{2} + 2\frac{1}{4}$ or $7\frac{3}{4}$ units.

Students graph coordinates for polygons and find missing vertices based on properties of triangles and quadrilaterals.

Expressions and Equations 6.EE

Common Core Cluster

Apply and extend previous understanding of arithmetic to algebraic expressions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **exponents, base, numerical expressions, algebraic expressions, evaluate, sum, term, product, factor, quantity, quotient, coefficient, constant, like terms, equivalent expressions, variables**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.	**6.EE.1** Students demonstrate the meaning of exponents to write and evaluate numerical expressions with whole number exponents. The base can be a whole number, positive decimal or a positive fraction (i.e. $\frac{1}{2}^{5}$ can be written $\frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2}$ which has the same value as $\frac{1}{32}$). Students recognize that an expression with a variable represents the same mathematics (ie. x^5 can be written as $x \bullet x \bullet x \bullet x \bullet x$) and write algebraic expressions from verbal expressions. Order of operations is introduced throughout elementary grades, including the use of grouping symbols, ( ), { }, and [ ] in 5th grade. Order of operations with exponents is the focus in 6th grade. <u>Example 1:</u> What is the value of: • 0.2^3 *Solution:* 0.008 • $5 + 2^4 \bullet 6$ *Solution:* 101 • $7^2 - 24 \div 3 + 26$ *Solution:* 67 <u>Example 2:</u> What is the area of a square with a side length of $3x$? *Solution:* $3x \bullet 3x = 9x^2$ <u>Example 3:</u> $4^x = 64$ *Solution:* $x = 3$ because $4 \bullet 4 \bullet 4 = 64$

6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.

a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as 5 – y.

b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*

6.EE.2 Students write expressions from verbal descriptions using letters and numbers, understanding order is important in writing subtraction and division problems. Students understand that the expression "5 times any number, *n*" could be represented with 5*n* and that a number and letter written together means to multiply. All rational numbers may be used in writing expressions when operations are not expected. Students use appropriate mathematical language to write verbal expressions from algebraic expressions. It is ***important*** for students to read algebraic expressions in a manner that reinforces that the variable represents a number.

Example Set 1:

Students read algebraic expressions:

- r + 21 as "some number plus 21" as well as "r plus 21"
- n • 6 as "some number times 6" as well as "n times 6"
- $\frac{s}{6}$ and s ÷ 6 as "as some number divided by 6" as well as "s divided by 6"

Example Set 2:

Students write algebraic expressions:

- 7 less than 3 times a number
  Solution: $3x - 7$
- 3 times the sum of a number and 5
  Solution: $3\ (x + 5)$
- 7 less than the product of 2 and a number
  Solution: $2x - 7$
- Twice the difference between a number and 5
  Solution: $2(z - 5)$
- The quotient of the sum of *x* plus 4 and 2
  Solution: $\frac{x + 4}{2}$

Students can describe expressions such as 3 (2 + 6) as the product of two factors: 3 and (2 + 6). The quantity (2 + 6) is viewed as one factor consisting of two terms.

Terms are the parts of a sum. When the term is an explicit number, it is called a constant. When the term is a product of a number and a variable, the number is called the coefficient of the variable.

Students should identify the parts of an algebraic expression including variables, coefficients, constants, and the names of operations (sum, difference, product, and quotient). Variables are letters that represent numbers. There are various possibilities for the number they can represent.

	Consider the following expression: $x^2 + 5y + 3x + 6$ The variables are x and y. There are 4 terms, x^2, 5y, 3x, and 6. There are 3 variable terms, x^2, 5y, 3x. They have coefficients of 1, 5, and 3 respectively. The coefficient of x^2 is 1, since $x^2 = 1x^2$. The term 5y represent 5y's or 5 • y. There is one constant term, 6. The expression represents a sum of all four terms.
c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole- number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6\ s^2$ to find the volume and surface area of a cube with sides of length s = ½.*	Students evaluate algebraic expressions, using order of operations as needed. Problems such as example 1 below require students to understand that multiplication is understood when numbers and variables are written together and to use the order of operations to evaluate. Order of operations is introduced throughout elementary grades, including the use of grouping symbols, ( ), { }, and [ ] in 5th grade. Order of operations with exponents is the focus in 6th grade. <u>Example 1:</u> Evaluate the expression $3x + 2y$ when x is equal to 4 and y is equal to 2.4. *<u>Solution:</u>* 3 • 4 + 2 • 2.4 12 + 4.8 16.8 <u>Example 2:</u> Evaluate $5(n + 3) – 7n$, when $n = \frac{1}{2}$. *Solution:* $5(\frac{1}{2} + 3) – 7(\frac{1}{2})$ $5\ (3\frac{1}{2})$ - $3\frac{1}{2}$ Note: $7(\frac{1}{2}) = \frac{7}{2} = 3\frac{1}{2}$ $17\frac{1}{2}$ - $3\frac{1}{2}$ Students may also reason that 5 groups of $3\frac{1}{2}$ take away 1 group of $3\frac{1}{2}$ would give 4 groups of $3\frac{1}{2}$. Multiply 4 times $3\frac{1}{2}$ to get 14. 14

Example 3:
Evaluate $7xy$ when $x = 2.5$ and $y = 9$

Solution: Students recognize that two or more terms written together indicates multiplication.
7 (2.5) (9)
157.5

In 5th grade students worked with the grouping symbols ( ), [ ], and { }. Students understand that the fraction bar can also serve as a grouping symbol (treats numerator operations as one group and denominator operations as another group) as well as a division symbol.

Example 4:
Evaluate the following expression when $x = 4$ and $y = 2$

$$\frac{x^2 + y^3}{3}$$

Solution:

$\frac{(4)^2 + (2)^3}{3}$ *substitute the values for x and y*

$\frac{16 + 8}{3}$ *raise the numbers to the powers*

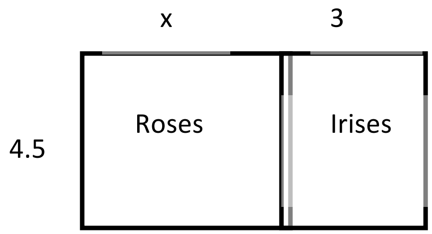
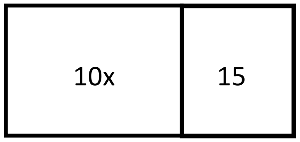
$\frac{24}{3}$ *divide 24 by 3*

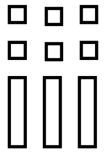
8

Given a context and the formula arising from the context, students could write an expression and then evaluate for any number.

Example 5:
It costs $100 to rent the skating rink plus $5 per person. Write an expression to find the cost for any number (n) of people. What is the cost for 25 people?

Solution:
The cost for any number (n) of people could be found by the expression, $100 + 5n$. To find the cost of 25 people substitute 25 in for n and solve to get 100 + 5 * 25 = 225.

	Example 6: The expression $c + 0.07c$ can be used to find the total cost of an item with 7% sales tax, where c is the pre-tax cost of the item. Use the expression to find the total cost of an item that cost $25. *Solution*: Substitute 25 in for c and use order of operations to simplify $c + 0.07c$ $25 + 0.07\ (25)$ $25 + 1.75$ 26.75
6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*	**6.EE.3** Students use the distributive property to write equivalent expressions. Using their understanding of area models from elementary students illustrate the distributive property with variables. Properties are introduced throughout elementary grades (3.OA.5); however, there has not been an emphasis on recognizing and naming the property. In 6th grade students are able to use the properties and identify by name as used when justifying solution methods (see example 4). Example 1: Given that the width is 4.5 units and the length can be represented by x + 2, the area of the flowers below can be expressed as 4.5(x + 3) or 4.5x + 13.5.  When given an expression representing area, students need to find the factors. Example 2: The expression 10x + 15 can represent the area of the figure below. Students find the greatest common factor (5) to represent the width and then use the distributive property to find the length (2x + 3). The factors (dimensions) of this figure would be 5(2x + 3). 

	Example 3: Students use their understanding of multiplication to interpret *3 (2 + x) as 3 groups of (2 + x)*. They use a model to represent x, and make an array to show the meaning of *3(2 + x)*. They can explain why it makes sense that *3(2 + x)* is equal to *6 + 3x*. An array with 3 columns and $x + 2$ in each column:  Students interpret *y* as referring to one *y*. Thus, they can reason that one *y* plus one *y* plus one *y* ***must be*** *3y*. They also use the distributive property, the multiplicative identity property of 1, and the commutative property for multiplication to prove that $y + y + y = 3y$: Example 4: Prove that $y + y + y = 3y$ *Solution:* $y + y + y$ $y \bullet 1 + y \bullet 1 + y \bullet 1$ Multiplicative Identity $y \bullet (1 + 1 + 1)$ Distributive Property $y \bullet 3$ $3y$ Commutative Property
6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*	**6.EE.4** Students demonstrate an understanding of like terms as quantities being added or subtracted with the same variables and exponents. For example, $3x + 4x$ are like terms and can be combined as $7x$; however, $3x + 4x^2$ are not like terms since the exponents with the *x* are not the same. This concept can be illustrated by substituting in a value for *x*. For example, $9x – 3x = 6x$ not 6. Choosing a value for *x*, such as 2, can prove non-equivalence. $9(2) – 3(2) = 6(2)$ however $9(2) – 3(2) \overset{?}{=} 6$ $18 – 6 = 12$ $18 – 6 \overset{?}{=} 6$ $12 = 12$ $12 \neq 6$ Students can also generate equivalent expressions using the associative, commutative, and distributive properties. They can prove that the expressions are equivalent by simplifying each expression into the same form.

Example 1:

Are the expressions equivalent? Explain your answer?

$4m + 8$ $4(m+2)$ $3m + 8 + m$ $2 + 2m + m + 6 + m$

Solution:

Expression	Simplifying the Expression	Explanation
$4m + 8$	$4m + 8$	Already in simplest form
$4(m+2)$	$4(m+2)$ $4m + 8$	*Distributive property*
$3m + 8 + m$	$3m + 8 + m$ $3m + m + 8$ $4m + 8$	*Combined like terms*
$2 + 2m + m + 6 + m$	$2m + m + m + 2 + 6$ $4m + 8$	*Combined like terms* *Combined like terms*

Expressions and Equations 6.EE

Common Core Cluster

Reason about and solve one-variable equations and inequalities.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **inequalities, equations, greater than, >, less than, <, greater than or equal to, ≥, less than or equal to, ≤, profit, exceed**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	In elementary grades, students explored the concept of equality. In 6th grade, students explore equations as expressions being set equal to a specific value. The solution is the value of the variable that will make the equation or inequality true. Students use various processes to identify the value(s) that when substituted for the variable will make the equation true.

Example 1:

Joey had 26 papers in his desk. His teacher gave him some more and now he has 100. How many papers did his teacher give him?

This situation can be represented by the equation $26 + n = 100$ where n is the number of papers the teacher gives to Joey. This equation can be stated as “some number was added to 26 and the result was 100.” Students ask themselves “What number was added to 26 to get 100?” to help them determine the value of the variable that makes the equation true. Students could use several different strategies to find a solution to the problem:

- Reasoning: 26 + 70 is 96 and 96 + 4 is 100, so the number added to 26 to get 100 is 74.
- Use knowledge of fact families to write related equations:
  $n + 26 = 100$, $100 - n = 26$, $100 - 26 = n$. Select the equation that helps to find n easily.
- Use knowledge of inverse operations: Since subtraction “undoes” addition then subtract 26 from 100 to get the numerical value of n
- Scale model: There are 26 blocks on the left side of the scale and 100 blocks on the right side of the scale. All the blocks are the same size. 74 blocks need to be added to the left side of the scale to make the scale balance.
- Bar Model: Each bar represents one of the values. Students use this visual representation to demonstrate that 26 and the unknown value together make 100.

100	
26	n

	Solution: Students recognize the value of 74 would make a true statement if substituted for the variable. $26 + n = 100$ $26 + 74 = 100$ $100 = 100$ ✓ Example 2: The equation $0.44\ s = 11$ where s represents the number of stamps in a booklet. The booklet of stamps costs 11 dollars and each stamp costs 44 cents. How many stamps are in the booklet? Explain the strategies used to determine the answer. Show that the solution is correct using substitution. *Solution:* There are 25 stamps in the booklet. I got my answer by dividing 11 by 0.44 to determine how many groups of 0.44 were in 11. By substituting 25 in for s and then multiplying, I get 11. $0.44(25) = 11$ $11 = 11$ ✓ Example 3: Twelve is less than 3 times another number can be shown by the inequality $12 < 3n$. What numbers could possibly make this a true statement? *Solution:* Since 3 • 4 is equal to 12 I know the value must be greater than 4. Any value greater than 4 will make the inequality true. Possibilities are 4.13, 6, $5\frac{3}{4}$, and 200. Given a set of values, students identify the values that make the inequality true.
6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	**6.EE.6**. Students write expressions to represent various real-world situations. Example Set 1: • Write an expression to represent Susan's age in three years, when a represents her present age. • Write an expression to represent the number of wheels, w, on any number of bicycles. • Write an expression to represent the value of any number of quarters, q. *Solutions:* • $a + 3$ • $2n$ • $0.25q$

Given a contextual situation, students define variables and write an expression to represent the situation.

Example 2:
The skating rink charges $100 to reserve the place and then $5 per person. Write an expression to represent the cost for any number of people.

n = the number of people
$100 + 5n$

No solving is expected with this standard; however, 6.EE.2c does address the evaluating of the expressions.

Students understand the inverse relationships that can exist between two variables. For example, if Sally has 3 times as many bracelets as Jane, then Jane has $\frac{1}{3}$ the amount of Sally. If S represents the number of bracelets Sally has, the $\frac{1}{3}s$ or $\frac{s}{3}$ represents the amount Jane has.

Connecting writing expressions with story problems and/or drawing pictures will give students a context for this work. It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number.

Example Set 3:

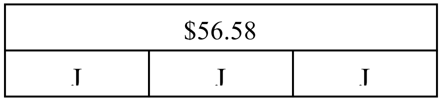
- Maria has three more than twice as many crayons as Elizabeth. Write an algebraic expression to represent the number of crayons that Maria has.
  Solution: $2c + 3$ where c represents the number of crayons that Elizabeth has

- An amusement park charges $28 to enter and $0.35 per ticket. Write an algebraic expression to represent the total amount spent.
  Solution: $28 + 0.35t$ where t represents the number of tickets purchased

- Andrew has a summer job doing yard work. He is paid $15 per hour and a $20 bonus when he completes the yard. He was paid $85 for completing one yard. Write an equation to represent the amount of money he earned.
  Solution: $15h + 20 = 85$ where h is the number of hours worked

- Describe a problem situation that can be solved using the equation $2c + 3 = 15$; where c represents the cost of an item
  Possible solution:
  Sarah spent $15 at a craft store.
    - She bought one notebook for $3.
    - She bought 2 paintbrushes for x dollars.

  If each paintbrush cost the same amount, what was the cost of one brush?

	- Bill earned \$5.00 mowing the lawn on Saturday. He earned more money on Sunday. Write an expression that shows the amount of money Bill has earned. *Solution:* \$5.00 + n
6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.	**6.EE.7** Students have used algebraic expressions to generate answers given values for the variable. This understanding is now expanded to equations where the value of the variable is unknown but the outcome is known. For example, in the expression, $x + 4$, any value can be substituted for the x to generate a numerical answer; however, in the equation $x + 4 = 6$, there is only one value that can be used to get a 6. Problems should be in context when possible and use only one variable. Students write equations from real-world problems and then use inverse operations to solve one-step equations based on real world situations. Equations may include fractions and decimals with non-negative solutions. Students recognize that dividing by 6 and multiplying by $\frac{1}{6}$ produces the same result. For example, $\frac{x}{6} = 9$ and $\frac{1}{6}x = 9$ will produce the same result. Beginning experiences in solving equations require students to understand the meaning of the equation and the solution in the context of the problem. <u>Example 1:</u> Meagan spent \$56.58 on three pairs of jeans. If each pair of jeans costs the same amount, write an algebraic equation that represents this situation and solve to determine how much one pair of jeans cost.  *Sample Solution:* Students might say: "I created the bar model to show the cost of the three pairs of jeans. Each bar labeled J is the same size because each pair of jeans costs the same amount of money. The bar model represents the equation $3J =$ \$56.58. To solve the problem, I need to divide the total cost of 56.58 between the three pairs of jeans. I know that it will be more than \$10 each because 10 x 3 is only 30 but less than \$20 each because 20 x 3 is 60. If I start with \$15 each, I am up to \$45. I have \$11.58 left. I then give each pair of jeans \$3. That's \$9 more dollars. I only have \$2.58 left. I continue until all the money is divided. I ended up giving each pair of jeans another \$0.86. Each pair of jeans costs \$18.86 (15+3+0.86). I double check that the jeans cost \$18.86 each because \$18.86 x 3 is \$56.58."

<table>
<tr>
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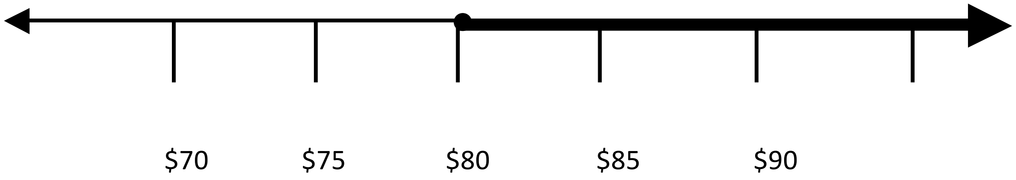
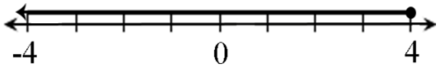
<u>Example 2:</u>
Julie gets paid $20 for babysitting. He spends $1.99 on a package of trading cards and $6.50 on lunch. Write and solve an equation to show how much money Julie has left.
<table>
<tr><td colspan="3">20</td></tr>
<tr><td>1.9</td><td>6.50</td><td>money left over (m)</td></tr>
</table>
<i>Solution</i>:   20 = 1.99 + 6.50 + x, x = $11.51
</td>
</tr>
<tr>
<td>6.EE.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.</td>
<td>
6.EE.8 Many real-world situations are represented by inequalities. Students write inequalities to represent real world and mathematical situations. Students use the number line to represent inequalities from various contextual and mathematical situations.

<u>Example 1:</u>
The class must raise at least $100 to go on the field trip. They have collected $20. Write an inequality to represent the amount of money, m, the class still needs to raise. Represent this inequality on a number line.

<i>Solution:</i>
The inequality $m \geq$ $80 represents this situation. Students recognize that possible values can include too many decimal values to name. Therefore, the values are represented on a number line by shading.


A number line diagram is drawn with an open circle when an inequality contains a < or > symbol to show solutions that are less than or greater than the number but not equal to the number. The circle is shaded, as in the example above, when the number is to be included. Students recognize that possible values can include fractions and decimals, which are represented on the number line by shading. Shading is extended through the arrow on a number line to show that an inequality has an infinite number of solutions.

<u>Example 2:</u>
Graph $x \leq 4$.

<i>Solution:</i>


<u>Example 3:</u>
The Flores family spent less than $200.00 last month on groceries. Write an inequality to represent this amount and graph this inequality on a number line.
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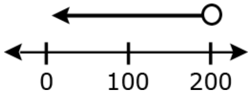
	Solution: $200 > x$, where x is the amount spent on groceries. 

Expressions and Equations 6.EE

Common Core Cluster

Represent and analyze quantitative relationships between dependent and independent variables.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **dependent variables, independent variables, discrete data, continuous data**

Common Core Standard

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Unpacking
What does this standard mean that a student will know and be able to do?

6.EE.9 The purpose of this standard is for students to understand the relationship between two variables, which begins with the distinction between dependent and independent variables. The independent variable is the variable that can be changed; the dependent variable is the variable that is affected by the change in the independent variable. Students recognize that the independent variable is graphed on the x-axis; the dependent variable is graphed on the y-axis.

Students recognize that not all data should be graphed with a line. Data that is discrete would be graphed with coordinates only. Discrete data is data that would not be represented with fractional parts such as people, tents, records, etc. For example, a graph illustrating the cost per person would be graphed with points since part of a person would not be considered. A line is drawn when both variables could be represented with fractional parts.

Students are expected to recognize and explain the impact on the dependent variable when the independent variable changes (As the x variable increases, how does the y variable change?) *Relationships should be proportional with the line passing through the origin.* Additionally, students should be able to write an equation from a word problem and understand how the coefficient of the dependent variable is related to the graph and /or a table of values.

Students can use many forms to represent relationships between quantities. Multiple representations include describing the relationship using language, a table, an equation, or a graph. Translating between multiple representations helps students understand that each form represents the same relationship and provides a different perspective.

Example 1:
What is the relationship between the two variables? Write an expression that illustrates the relationship.

x	1	2	3	4
y	2.5	5	7.5	10

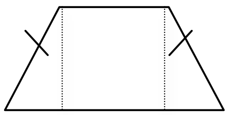
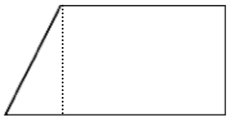
Solution:
$y = 2.5x$

Geometry 6.G

Common Core Cluster

Solve real-world and mathematical problems involving area, surface area, and volume.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **area, surface area, volume, decomposing, edges, dimensions, net, vertices, face, base, height, trapezoid, isosceles, right triangle, quadrilateral, rectangles, squares, parallelograms, trapezoids, rhombi, kites, right rectangular prism**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	**6.G.1** Students continue to understand that area is the number of squares needed to cover a plane figure. Students should know the formulas for rectangles and triangles. "Knowing the formula" does not mean memorization of the formula. To "know" means to have an understanding of ***why*** the formula works and how the formula relates to the measure (area) and the figure. This understanding should be for *all* students. Finding the area of triangles is introduced in relationship to the area of rectangles – a rectangle can be decomposed into two congruent triangles. Therefore, the area of the triangle is ½ the area of the rectangle. The area of a rectangle can be found by multiplying base x height; therefore, the area of the triangle is ½ bh or (b x h)/2. The following site helps students to discover the area formula of triangles. http://illuminations.nctm.org/LessonDetail.aspx?ID=L577 Students decompose shapes into rectangles and triangles to determine the area. For example, a trapezoid can be decomposed into triangles and rectangles (see figures below). Using the trapezoid's dimensions, the area of the individual triangle(s) and rectangle can be found and then added together. Special quadrilaterals include rectangles, squares, parallelograms, trapezoids, rhombi, and kites.  Isosceles trapezoid  Right trapezoid **Note:** Students recognize the marks on the isosceles trapezoid indicating the two sides have equal measure.

Example 1:

Find the area of a right triangle with a base length of three units, a height of four units, and a hypotenuse of 5.

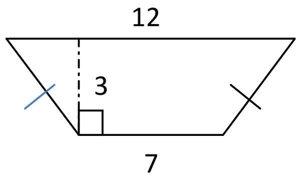
Solution:

Students understand that the hypotenuse is the longest side of a right triangle. The base and height would form the 90° angle and would be used to find the area using:

A = ½ bh
A = ½ (3 units)(4 units)
A = ½ 12 units^2
A = 6 units^2

Example 2:

Find the area of the trapezoid shown below using the formulas for rectangles and triangles.



Solution:

The trapezoid could be decomposed into a rectangle with a length of 7 units and a height of 3 units. The area of the rectangle would be 21 units^2.
The triangles on each side would have the same area. The height of the triangles is 3 units. After taking away the middle rectangle's base length, there is a total of 5 units remaining for both of the side triangles. The base length of each triangle is half of 5. The base of each triangle is 2.5 units. The area of one triangle would be ½ (2.5 units)(3 units) or 3.75 units^2.

Using this information, the area of the trapezoid would be:
21 units^2
3.75 units^2
$\underline{+3.75 \text{ units}^2}$
28.5 units^2

Example 3:

A rectangle measures 3 inches by 4 inches. If the lengths of each side double, what is the effect on the area?
Solution:
The new rectangle would have side lengths of 6 inches and 8 inches. The area of the original rectangle was 12 inches^2. The area of the new rectangle is 48 inches^2. The area increased 4 times (quadrupled).
Students may also create a drawing to show this visually.

Example 4:

The lengths of the sides of a bulletin board are 4 feet by 3 feet. How many index cards measuring 4 inches by 6 inches would be needed to cover the board?

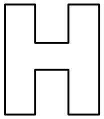
Solution:

Change the dimensions of the bulletin board to inches (4 feet = 48 inches; 3 feet = 36 inches). The area of the board would be 48 inches x 36 inches or 1728 inches^2. The area of one index card is 12 inches^2. Divide 1728 inches^2 by 24 inches^2 to get the number of index cards. 72 index cards would be needed.

Example 5:

The sixth grade class at Hernandez School is building a giant wooden H for their school. The "H" will be 10 feet tall and 10 feet wide and the thickness of the block letter will be 2.5 feet.

1. How large will the H be if measured in square feet?
2. The truck that will be used to bring the wood from the lumberyard to the school can only hold a piece of wood that is 60 inches by 60 inches. What pieces of wood (how many and which dimensions) will need to be bought to complete the project?



Solution:

1. One solution is to recognize that, if filled in, the area would be 10 feet tall and 10 feet wide or 100 ft^2. The size of one piece removed is 5 feet by 3.75 feet or 18.75 ft^2. There are two of these pieces.
   The area of the "H" would be 100 ft^2 – 18.75 ft^2 – 18.75 ft^2, which is 62.5ft^2.
   A second solution would be to decompose the "H" into two tall rectangles measuring 10 ft by 2.5 ft and one smaller rectangle measuring 2.5 ft by 5 ft. The area of each tall rectangle would be 25 ft^2 and the area of the smaller rectangle would be 12.5 ft^2. Therefore the area of the "H" would be 25 ft^2 + 25 ft^2 + 12.5 ft^2 or 62.5ft^2.
2. Sixty inches is equal to 5 feet, so the dimensions of each piece of wood are 5ft by 5ft. Cut **two pieces of wood in half to create four pieces 5 ft. by 2.5 ft.** These pieces will make the two taller rectangles. A **third piece would be cut to measure 5ft. by 2.5 ft.** to create the middle piece.

Example 6:

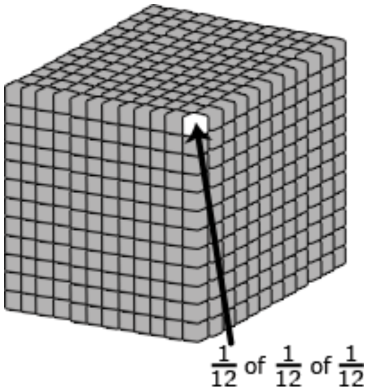
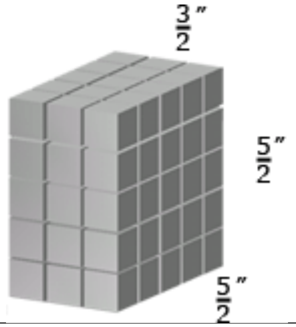
A border that is 2 ft wide surrounds a rectangular flowerbed 3 ft by 4 ft. What is the area of the border?

Solution:

Two sides 4 ft. by 2 ft. would be 8ft^2 x 2 or 16 ft^2
Two sides 3 ft. by 2 ft. would be 6ft^2 x 2 or 12 ft^2
Four corners measuring 2 ft. by 2 ft. would be 4ft^2 x 4 or 16 ft^2

The total area of the border would be 16 ft^2 + 12 ft^2 + 16 ft^2 or **44ft^2**

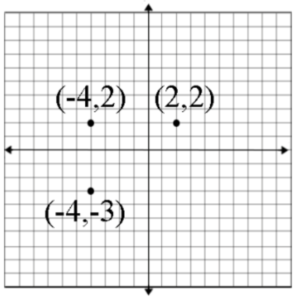
6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	**6.G.2** Previously students calculated the volume of right rectangular prisms (boxes) using whole number edges. The use of models was emphasized as students worked to derive the formula V = Bh (5.MD.3, 5.MD.4, 5.MD.5) The unit cube was 1 x 1 x 1. In 6th grade the unit cube will have fractional edge lengths. (ie. ½ • ½ • ½) Students find the volume of the right rectangular prism with these unit cubes. Students need multiple opportunities to measure volume by filling rectangular prisms with blocks and looking at the relationship between the total volume and the area of the base. Through these experiences, students *derive* the volume formula (volume equals the area of the base times the height). Students can explore the connection between filling a box with unit cubes and the volume formula using interactive applets such as the Cubes Tool on NCTM's Illuminations (http://illuminations.nctm.org/ActivityDetail.aspx?ID=6). In addition to filling boxes, students can draw diagrams to represent fractional side lengths, connecting with multiplication of fractions. This process is similar to composing and decomposing two-dimensional shapes. <u>Example 1:</u> A right rectangular prism has edges of $1\frac{1}{4}$”, 1” and $1\frac{1}{2}$”. How many cubes with side lengths of $\frac{1}{4}$ would be needed to fill the prism? What is the volume of the prism? *Solution:* The number of $\frac{1}{4}$” cubes can be found by recognizing the smaller cubes would be $\frac{1}{4}$” on all edges, changing the dimensions to $\frac{5}{4}$”, $\frac{4}{4}$” and $\frac{6}{4}$”. The number of one-fourth inch unit cubes making up the prism is 120 (5 x 4 x 6). Each smaller cube has a volume of $\frac{1}{64}$ ($\frac{1}{4}$” x $\frac{1}{4}$” x $\frac{1}{4}$”), meaning 64 small cubes would make up the unit cube. Therefore, the volume is $\frac{5}{4}$ x $\frac{6}{4}$ x $\frac{4}{4}$ or $\frac{120}{64}$ (120 smaller cubes with volumes of $\frac{1}{64}$ or $1\frac{56}{64}$ →1 unit cube with 56 smaller cubes with a volume of $\frac{1}{64}$.

	Example 2: The model shows a cubic foot filled with cubic inches. The cubic inches can also be labeled as a fractional cubic unit with dimensions of $\frac{1}{12}$ft^3.  Example 3: The model shows a rectangular prism with dimensions $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{5}{2}$ inches. Each of the cubic units in the model is $\frac{1}{2}$in. on each side. Students work with the model to illustrate $\frac{3}{2} x \frac{5}{2} x \frac{5}{2} = (3 x 5 x 5) x \frac{1}{8}$. Students reason that a small cube has volume of $\frac{1}{8}$ in^3 because 8 of them fit in a unit cube. 
6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	**6.G.3** Students are given the coordinates of polygons to draw in the coordinate plane. If both *x*-coordinates are the same (2, -1) and (2, 4), then students recognize that a vertical line has been created and the distance between these coordinates is the distance between -1 and 4, or 5. If both the *y*-coordinates are the same (-5, 4) and (2, 4), then students recognize that a horizontal line has been created and the distance between these coordinates is the distance between -5 and 2, or 7. Using this understanding, student solve real-world and mathematical problems, including finding the area and perimeter of geometric figures drawn on a coordinate plane.

This standard can be taught in conjunction with **6.G.1** to help students develop the formula for the triangle by using the squares of the coordinate grid. Given a triangle, students can make the corresponding square or rectangle and realize the triangle is ½.

Students progress from counting the squares to making a rectangle and recognizing the triangle as ½ to the development of the formula for the area of a triangle.

Example 1:
If the points on the coordinate plane below are the three vertices of a rectangle, what are the coordinates of the fourth vertex? How do you know? What are the length and width of the rectangle? Find the area and the perimeter of the rectangle.



Solution:
To determine the distance along the *x*-axis between the point (-4, 2) and (2, 2) a student must recognize that -4 is |-4| or 4 units to the left of 0 and 2 is |2| or 2 units to the right of zero, so the two points are total of 6 units apart along the x-axis. Students should represent this on the coordinate grid and numerically with an absolute value expression, |-4| + |2| . The length is 6 and the width is 5.

The fourth vertex would be (2, -3).
The area would be 5 x 6 or 30 units^2.
The perimeter would be 5 + 5 + 6 + 6 or 22 units.

Example 2:
On a map, the library is located at (-2, 2), the city hall building is located at (0,2), and the high school is located at (0,0). Represent the locations as points on a coordinate grid with a unit of 1 mile.

1. What is the distance from the library to the city hall building? The distance from the city hall building to the high school? How do you know?
2. What shape does connecting the three locations form? The city council is planning to place a city park in this area. How large is the area of the planned park?

Solution:

1. The distance from the library to city hall is 2 miles. The coordinates of these buildings have the same *y*-coordinate. The distance between the *x*-coordinates is 2 (from -2 to 0).
2. The three locations form a right triangle. The area is 2 mi^2.

6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

6.G.4 A net is a two-dimensional representation of a three-dimensional figure. Students represent three-dimensional figures whose nets are composed of rectangles and triangles. Students recognize that parallel lines on a net are congruent. Using the dimensions of the individual faces, students calculate the area of each rectangle and/or triangle and add these sums together to find the surface area of the figure.

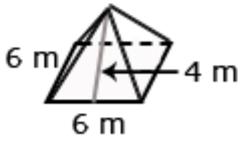
Students construct models and nets of three-dimensional figures, describing them by the number of edges, vertices, and faces. Solids include rectangular and triangular prisms. Students are expected to use the net to calculate the surface area.

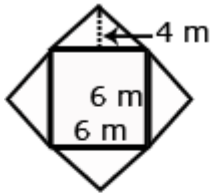
Students can create nets of 3D figures with specified dimensions using the Dynamic Paper Tool on NCTM's Illuminations (http://illuminations.nctm.org/ActivityDetail.aspx?ID=205).

Students also describe the types of faces needed to create a three-dimensional figure. Students make and test conjectures by determining what is needed to create a specific three-dimensional figure.

Example 1:
Describe the shapes of the faces needed to construct a rectangular pyramid. Cut out the shapes and create a model. Did your faces work? Why or why not?

Example 2:
Create the net for a given prism or pyramid, and then use the net to calculate the surface area.



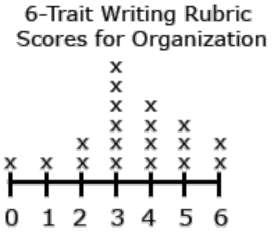


Statistics and Probability 6.SP

Common Core Cluster

Develop understanding of statistical variability.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **statistics, data, variability, distribution, dot plot, histograms, box plots, median, mean**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*	**6.SP.1** Statistics are numerical data relating to a group of individuals; statistics is also the name for the science of collecting, analyzing and interpreting such data. A statistical question anticipates an answer that varies from one individual to the next and is written to account for the variability in the data. Data are the numbers produced in response to a statistical question. Data are frequently collected from surveys or other sources (i.e. documents). Students differentiate between statistical questions and those that are not. A statistical question is one that collects information that addresses differences in a population. The question is framed so that the responses will allow for the differences. For example, the question, "How tall am I?" is not a statistical question because there is only one response; however, the question, "How tall are the students in my class?" is a statistical question since the responses anticipates variability by providing a variety of possible anticipated responses that have numerical answers. Questions can result in a narrow or wide range of numerical values. Students might want to know about the fitness of the students at their school. Specifically, they want to know about the exercise habits of the students. So rather than asking "Do you exercise?" they should ask about the amount of exercise the students at their school get per week. A statistical question for this study could be: "How many hours per week on average do students at Jefferson Middle School exercise?"
6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution, which can be described by its center, spread, and overall shape.	**6.SP.2** The distribution is the arrangement of the values of a data set. Distribution can be described using center (median or mean), and spread. Data collected can be represented on graphs, which will show the shape of the distribution of the data. Students examine the distribution of a data set and discuss the center, spread and overall shape with dot plots, histograms and box plots. <u>Example 1:</u> The dot plot shows the writing scores for a group of students on organization. Describe the data. 

Solution:
The values range from 0 – 6. There is a peak at 3. The median is 3, which means 50% of the scores are greater than or equal to 3 and 50% are less than or equal to 3. The mean is 3.68. If all students scored the same, the score would be 3.68.

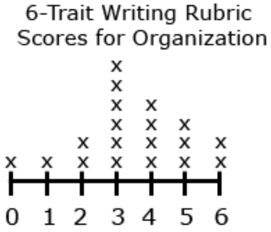
NOTE: Mode as a measure of center and range as a measure of variability are not addressed in the CCSS and as such are not a focus of instruction. These concepts can be introduced during instruction as needed.

6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

6.SP.3 Data sets contain many numerical values that can be summarized by one number such as a measure of center. The measure of center gives a numerical value to represent the center of the data (ie. midpoint of an ordered list or the balancing point). Another characteristic of a data set is the variability (or spread) of the values. Measures of variability are used to describe this characteristic.

Example 1:
Consider the data shown in the dot plot of the six trait scores for organization for a group of students.

- How many students are represented in the data set?
- What are the mean and median of the data set? What do these values mean? How do they compare?
- What is the range of the data? What does this value mean?



Solution:
- 19 students are represented in the data set.
- The mean of the data set is 3.5. The median is 3. The mean indicates that is the values were equally distributed, all students would score a 3.5. The median indicates that 50% of the students scored a 3 or higher; 50% of the students scored a 3 or lower.
- The range of the data is 6, indicating that the values vary 6 points between the lowest and highest scores.

Statistics and Probability 6.SP

Common Core Cluster

Summarize and describe distributions.

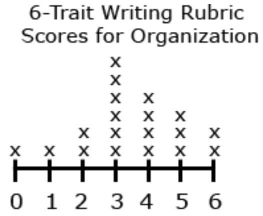
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **box plots, dot plots, histograms, frequency tables, cluster, peak, gap, mean, median, interquartile range, measures of center, measures of variability, data, Mean Absolute Deviation (M.A.D.), quartiles, lower quartile (1st quartile or Q_1), upper quartile (3rd quartile or Q_3), symmetrical, skewed, summary statistics, outlier**

Common Core Standard	**Unpacking** What does this standard mean that a student will know and be able to do?
6.SP.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	**6.SP.4** Students display data graphically using number lines. Dot plots, histograms and box plots are three graphs to be used. Students are expected to determine the appropriate graph as well as read data from graphs generated by others. Dot plots are simple plots on a number line where each dot represents a piece of data in the data set. Dot plots are suitable for small to moderate size data sets and are useful for highlighting the distribution of the data including clusters, gaps, and outliers. A histogram shows the distribution of continuous data using intervals on the number line. The height of each bar represents the number of data values in that interval. In most real data sets, there is a large amount of data and many numbers will be unique. A graph (such as a dot plot) that shows how many ones, how many twos, etc. would not be meaningful; however, a histogram can be used. Students group the data into convenient ranges and use these intervals to generate a frequency table and histogram. Note that changing the size of the bin changes the appearance of the graph and the conclusions may vary from it. A box plot shows the distribution of values in a data set by dividing the set into quartiles. It can be graphed either vertically or horizontally. The box plot is constructed from the five-number summary (minimum, lower quartile, median, upper quartile, and maximum). These values give a summary of the shape of a distribution. Students understand that the size of the box or whiskers represents the middle 50% of the data. Students can use applets to create data displays. Examples of applets include the Box Plot Tool and Histogram Tool on NCTM's Illuminations. Box Plot Tool - http://illuminations.nctm.org/ActivityDetail.aspx?ID=77 Histogram Tool -- http://illuminations.nctm.org/ActivityDetail.aspx?ID=78

Example 1:

Nineteen students completed a writing sample that was scored on organization. The scores for organization were 0, 1, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6. Create a data display. What are some observations that can be made from the data display?

Solution:



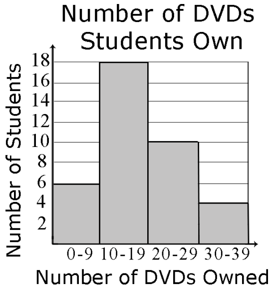
Example 2:

Grade 6 students were collecting data for a math class project. They decided they would survey the other two grade 6 classes to determine how many DVDs each student owns. A total of 48 students were surveyed. The data are shown in the table below in no specific order. Create a data display. What are some observations that can be made from the data display?

11	21	5	12	10	31	19	13	23	33
10	11	25	14	34	15	14	29	8	5
22	26	23	12	27	4	25	15	7	
2	19	12	39	17	16	15	28	16	

Solution:

A histogram using 5 intervals (bins) 0-9, 10-19, …30-39) to organize the data is displayed below.



Most of the students have between 10 and 19 DVDs as indicated by the peak on the graph. The data is pulled to the right since only a few students own more than 30 DVDs.

Example 3:

Ms. Wheeler asked each student in her class to write their age in months on a sticky note. The 28 students in the class brought their sticky note to the front of the room and posted them in order on the white board. The data set is listed below in order from least to greatest. Create a data display. What are some observations that can be made from the data display?

130	130	131	131	132	132	132	133	134	136
137	137	138	139	139	139	140	141	142	142
142	143	143	144	145	147	149	150		

Solution:

Five number summary

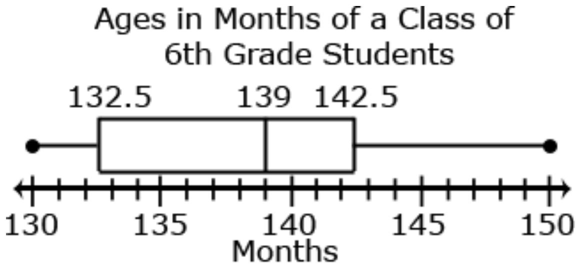
Minimum – 130 months

Quartile 1 (Q1) – (132 + 133) ÷ 2 = 132.5 months

Median (Q2) – 139 months

Quartile 3 (Q3) – (142 + 143) ÷ 2 = 142.5 months

Maximum – 150 months



This box plot shows that

- ¼ of the students in the class are from 130 to 132.5 months old
- ¼ of the students in the class are from 142.5 months to 150 months old
- ½ of the class are from 132.5 to 142.5 months old
- The median class age is 139 months.

6.SP.5 Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	**6.SP.5** Students summarize numerical data by providing background information about the attribute being measured, methods and unit of measurement, the context of data collection activities (addressing random sampling), the number of observations, and summary statistics. Summary statistics include quantitative measures of center (median and median) and variability (interquartile range and mean absolute deviation) including extreme values (minimum and maximum), mean, median, mode, range, and quartiles. Students record the number of observations. Using histograms, students determine the number of values between specified intervals. Given a box plot and the total number of data values, students identify the number of data points that are represented by the box. Reporting of the number of observations must consider the attribute of the data sets, including units (when applicable). Measures of Center Given a set of data values, students summarize the measure of center with the median or mean. The median is the value in the middle of a ordered list of data. This value means that 50% of the data is greater than or equal to it and that 50% of the data is less than or equal to it. The mean is the arithmetic average; the sum of the values in a data set divided by how many values there are in the data set. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students develop these understandings of what the mean represents by redistributing data sets to be level or fair (equal distribution) and by observing that the total distance of the data values above the mean is equal to the total distance of the data values below the mean (balancing point). Students use the concept of mean to solve problems. Given a data set represented in a frequency table, students calculate the mean. Students find a missing value in a data set to produce a specific average. Example 1: Susan has four 20-point projects for math class. Susan's scores on the first 3 projects are shown below: Project 1: 18 Project 2: 15 Project 3: 16 Project 4: ?? What does she need to make on Project 4 so that the average for the four projects is 17? Explain your reasoning. *Solution:* One possible solution to is calculate the total number of points needed (17 x 4 or 68) to have an average of 17. She has earned 49 points on the first 3 projects, which means she needs to earn 19 points on Project 4 (68 – 49 = 19).

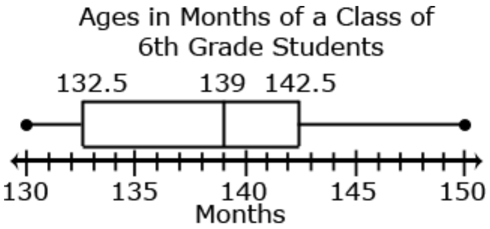
Measures of Variability

Measures of variability/variation can be described using the interquartile range or the Mean Absolute Deviation. The interquartile range (IQR) describes the variability between the middle 50% of a data set. It is found by subtracting the lower quartile from the upper quartile. It represents the length of the box in a box plot and is not affected by outliers.

Students find the IQR from a data set by finding the upper and lower quartiles and taking the difference or from reading a box plot.

Example 1:

What is the IQR of the data below:



Solution:

The first quartile is 132.5; the third quartile is 142.5. The IQR is 10 (142.5 – 132.5). This value indicates that the values of the middle 50% of the data vary by 10.

Mean Absolute Deviation (MAD) describes the variability of the data set by determining the absolute deviation (the distance) of each data piece from the mean and then finding the average of these deviations.

Both the interquartile range and the Mean Absolute Deviation are represented by a single numerical value. Higher values represent a greater variability in the data.

Example 2:

The following data set represents the size of 9 families:

3, 2, 4, 2, 9, 8, 2, 11, 4.

What is the MAD for this data set?

Solution:

The mean is 5. The MAD is the average variability of the data set. To find the MAD:

1. Find the deviation from the mean.
2. Find the absolute deviation for each of the values from step 1
3. Find the average of these absolute deviations.

The table below shows these calculations:

Data Value	Deviation from Mean	Absolute Deviation
3	-2	2
2	-3	3
4	-1	1
2	-3	3
9	4	4
8	3	3
2	-3	3
11	6	6
4	-1	1
MAD		26/9 = 2.89

This value indicates that on average family size varies 2.89 from the mean of 5.

Students understand how the measures of center and measures of variability are represented by graphical displays.

Students describe the context of the data, using the shape of the data and are able to use this information to determine an appropriate measure of center and measure of variability. The measure of center that a student chooses to describe a data set will depend upon the shape of the data distribution and context of data collection. The mode is the value in the data set that occurs most frequently. The mode is the least frequently used as a measure of center because data sets may not have a mode, may have more than one mode, or the mode may not be descriptive of the data set. The mean is a very common measure of center computed by adding all the numbers in the set and dividing by the number of values. The mean can be affected greatly by a few data points that are very low or very high. In this case, the median or middle value of the data set might be more descriptive. In data sets that are symmetrically distributed, the mean and median will be very close to the same. In data sets that are skewed, the mean and median will be different, with the median frequently providing a better overall description of the data set.

We would like to acknowledge the Arizona Department of Education for allowing us to use some of their examples and graphics.