Core Content

Cluster Title: Reason with shapes and their attributes.
Standard 1: Understand that shapes in different categories (e.g., rhombi, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombi, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
MASTERY Patterns of Reasoning:
Conceptual: Students will understand the attributes of different categories of quadrilaterals. Students will understand shapes that are examples and non-examples of quadrilaterals. Students will understand shared attributes can define a larger category of polygons. **Procedural:** Students can classify shapes based on the number of sides. Students can classify shapes based on length of sides. Students can classify shapes based on angles. Students can articulate proper vocabulary and details when describing the properties of quadrilaterals. Students can show examples of quadrilaterals that do not belong. **Representational:** Students can create or represent many varied and unusual squares, rectangles, rhombuses, parallelograms, rhombi, and trapezoids and explain them verbally or in written form. Students can sort geometric figures and identify squares, rectangles, rhombi, trapezoids, and parallelograms as quadrilaterals based on their attributes. Students can draw quadrilaterals that do not belong to these subcategories (squares, rectangles, rhombus, trapezoid, and parallelograms).

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will know basic shapes and their attributes.

Code: 3G1

Students will show a basic understanding of angles, sides, and faces.
Students will know closed shapes.

Procedural:

Students can identify basic shapes.
Students can sort shapes based on attributes.

Representational:

Students can draw basic shapes.
Students can draw a representation of a shape sort.
Students can make a list of common attributes of different shapes.

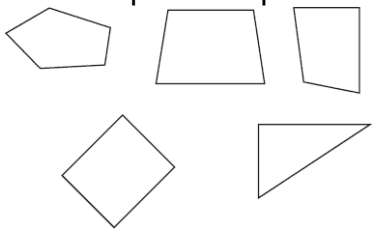
Academic Vocabulary and Notation

attribute, angle, closed figure, faces, polygon, rhombus, rectangle, side, square, parallel, parallelogram, quadrilateral, trapezoid, vertex, ||, right angle, 

** It would be helpful for the teacher to understand that rectilinear figures must have 4 right angles.
** Teach correct plural forms. Rhombus=Rhombi, Vertex= Vertices

Instructional Strategies Used	Resources Used
Provide many different and varied examples of quadrilaterals. Do a shape scavenger hunt to find quadrilaterals around the classroom or school. Have the students sort them into different groups. Students will explain how they grouped the quadrilaterals. Choose a quadrilateral and describe how you would change it to make it fit another group. Use geoboards, geo dot paper or other manipulatives to build many and varied examples of quadrilaterals.	Blaisdell, Molly. *If You Were a Quadrilateral.* Picture Window Books, 2009. Wentz, Stephen. *Square and the Missing Quadrilaterals.* Bookemon, 2011. http://www.Bookemon.com/book_read_flip.php?book_id=97264&size=1.4&style=popup2 http://ww.mathsisfun.com/quadrilaterals.html http://www.teachervision.fen.com/math-instruments/grapic-organizers/44643.html

Code: 3G1

Draw non-examples of quadrilaterals that do fit the subcategories (e.g., convex quadrilaterals).	http://www.nlvm.usu.edu/en/nav/frames_asid_172_g_2_t_3.html
Assessment Tasks Used	
Skill-Based Task: • Which of these shapes is a quadrilateral?  • Sort many, varied quadrilaterals based on their attributes.	**Problem Task:** Compare two quadrilaterals. How are they alike and how are they different? List an example of each quadrilateral that you can see in the classroom.

Core Content

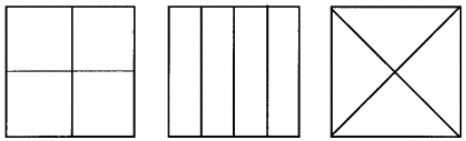
Cluster Title: Reason with shapes and their attributes.
Standard 2: Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. (For example, partition a shape into 4 parts with equal areas, and describe the area of each part as 1/4 of the area of the shape.) **Note: This domain is limited to fractions with denominators 2, 3, 4, 6, and 8.**
MASTERY Patterns of Reasoning:
Conceptual: Students will understand that shapes can be divided into equal parts. Students will understand that each part is a fraction of the whole. **Procedural:** Students can divide a variety of shapes into equal parts. Students can label each part as a fraction. Students can explain their reasoning verbally or in written form. (e.g., 1/2 is 1 out of 2 equal parts.). **Representational:** Students can model the division of shapes into equal parts.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand that rectangles can be divided into two, three, or four equal shares. Students will understand that equal shares of identical wholes do not have to have the same shape. Students will understand what a numerator and a denominator represent. Students will understand that area is additive. **Procedural**: Students can divide circles and rectangles into two, three, or four equal shares. Students can label the parts of the whole.

Code: 3G2

Students can describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths, etc.
Students recognize that equal shares of identical wholes need not have the same shape.



Representational:
Students can model the division of a square into 4 equal parts 3 different ways.

Academic Vocabulary and Notation

halves (1⁄2), thirds (1/3), fourths (1⁄4), sixths (1/6), eighths (1⁄8), equal parts, area

Instructional Strategies Used	Resources Used
Model shapes and then divide them in to equal parts. Now see if you can take the same shape and divide it equally but in a different way than before. Explain that even though the pieces may look very different, they are still the same unit fraction of the whole. Explore how many triangles, rhombi, or trapezoids will fit on a variety of pattern block shapes. Fold and cut paper into different fractional pieces. Use the following manipulatives: pattern blocks, graph paper, geoboards, colored paper.	See books from the third grade Numbers and Operations–Fractions domain. http://nlvm.usu.edu/en/nav/category_g_2_t_3.html

Assessment Tasks Used	
Skill-Based Task: What fraction of the shape is missing—1/2, 1/3, 1/4, or 3/4? 	**Problem Task:** What are all the ways you can divide a rectangular birthday cake into 8 equal parts? Explain which way you would like the cake to be cut for your birthday.

Core Content

Cluster Title: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

Standard 1: Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand how many minutes are in an hour.
Students will understand that the clock can be divided into fifteen-minute intervals.
Students will understand that hour and minute hands move at different rates.
Students will understand the concept of elapsed time, including between a.m. and p.m.
Students will understand concepts of whole, half and quarter, as they relates to a number line.
Students will understand how to apply estimation of time for different tasks (e.g., about how long is your favorite T.V. show?).

Procedural:

Students can write time on a digital clock and draw hands on analog clock to a precise minute.
Students can accurately compute elapsed time to the nearest minute.
Students can solve elapsed time word problems using addition and subtraction.
Students can figure elapsed time on a number line.

Representational:

Students can demonstrate a given time on an analog and digital clock to the nearest minute.
Students can demonstrate elapsed time on a number line.
Students can describe strategies used to solve elapsed time in story problems.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand the parts of a clock, and how they work. Students will know that there are 60 seconds in a minute, 60 minutes in an hour, 24 hours in a day, and the meaning of a.m. and p.m. Students will understand that the clock is divided into intervals (5 minutes, 10 minutes, 15 minutes, 30 minutes, etc.). Students will understand the concept of a number line. Students will understand the terms before, after, ago, from now, start, end, begin, half past, o'clock. Students will understand how to write time, using the colon correctly. **Procedural:** Students can tell time from an analog and digital clock to within five minutes. Students can represent time in written form (e.g., 3:25). **Representational:** Students can manipulate a number line as a tool to solve simple addition and subtraction time problems. Students can manipulate a model clock to show time.
Academic Vocabulary and Notation
elapsed, quarter to/till, quarter of, quarter past, quarter after, midnight, noon

Instructional Strategies Used	Resources Used

Organize a year-long sunrise/sunset project with weekly data collection of the time of sunrise and sunset, figuring the length of day, and plotting it on a graph. Stop work intermittently during the day to record time on a representation of an analog clock, writing what time it is, and figuring the elapsed time from the previous recording. Teach students how to figure elapsed time on a number line. Have students estimate the time until the next subject, lunch, recess, etc., then figure out the actual elapsed time.	Harper, Don. *Telling Time with Big Mama Cat.* HMH Books, 1998. Murphy, Stuart. *Game Time (MathStart 3).* HarperCollins, 2000. Carle, Eric. *The Grouchy Ladybug.* HarperCollins, 1996. Axelrod, Amy. *Pigs on a Blanket.* Aladdin, 1998. Hutchins, Pat. *Clocks and More Clocks.* Aladdin, 1994. Wiesner, David. *Tuesday.* HMH Books, 2011. Cave, Kathryn. *Just in Time.* Clarkson Potter, 1984. http://www.ixl.com/math/grade-3 http://www.ixl.com/math/grade-2 http://www.softschools.com/time/tellingtime.jsp http://math.pppst.com/tellingtime.html http://wartgames.com/themes/math/tellingtime.html

Assessment Tasks Used	
Skill-Based Task: Tell and write time to the nearest minute, figuring elapsed time to the nearest minute both forward and backward, including between a.m. and p.m. Hannah is making brownies. They need to bake for 30 minutes. She put them in the oven at 4:30 p.m. At what time should she take them out? How long did Abby sleep if she went to bed at 8:45 p.m. and got up at 7:30 a.m.? Jason completed his bike tour 5 hours and 15 minutes after he started. If he finished at 2:30 p.m., what time did he start?	**Problem Task:** Create a schedule with set time parameters, including a given number of tasks. Figure the time necessary to complete a number of certain activities varying in length. Given a number line showing quarters and halves, figure elapsed time between two given times to the minute. John wants to play with his best friend. His mother said he can go play with his friend on Friday from 4:30 p.m. until 6:15 p.m., or he can play with his friend on Saturday from 11:30 a.m. until 1:45 p.m. Which day should he play with his friend? Justify your choice by using pictures, numbers and words.

Core Content

Cluster Title: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
Standard 2: Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[1] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[2] Notes: [1]Excludes compound units such as cm^3 and finding the geometric volume of a container. [2]Excludes multiplicative comparison problems (problems involving notions of “times as much”).
MASTERY Patterns of Reasoning:
Conceptual: Students will understand the concept of mass in relationship to weight. Students will understand the concept when a liquid takes up space it is measured by volume. Students will understand units of metric capacity (liter, gram, kilogram), their measuring tools and their application to real life (e.g., two liters of soda). Students will understand that masses and volumes can be added, subtracted, multiplied and divided. Students will understand how to estimate measurement of liquid volume and mass. Students will understand abbreviations used to represent units of measure. **Procedural:** Students can estimate the capacity of real-life items to the nearest liter. Students can accurately measure liquids using liters. Students can estimate the mass of real-life items to the nearest gram or kilogram. Students can measure mass using grams and kilograms. Students can choose appropriate units of measure for specific problems and solve. **Representational:** Students can solve story problems about metric capacity and mass by drawing pictures to represent the problem. Students can explain in writing the strategies used to solve. Students can estimate volume and mass accurately.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand the concept of measurement and how it relates to real life. Students will understand what liquids and solids are. Students will understand the concepts of *greater than* and *less than*, and heavier and lighter. Students will understand why someone would measure an item. Students will understand basic place value concepts, and the concepts of addition, subtraction, multiplying, and dividing. Students will understand sequential language (e.g., first, next, finally, etc.). Students will understand the structure of word problems. **Procedural:** Students can solve one-step story problems involving basic addition and subtraction. **Representational:** Students can represent addition and subtraction problems in pictures and draw items to approximate size labeling. Students can use sequential language accurately, both verbally and in writing
Academic Vocabulary and Notation
gram (g), kilogram (kg), liter (l), capacity, liquid volume, mass, weight, scale, estimate

Instructional Strategies Used	Resources Used
Review centimeters and meters, and their relationship to one another. Review that there are also measurements for length and for capacity. Relate the concept to grams, kilograms, and liters. Brainstorm events where exact measurement is necessary and times when an estimate is sufficient. Share items from daily routine that are measured in grams, kilograms, and liters. Demonstrate objects, such as paper clips, that weigh a gram or kilogram for students to manipulate. Show products such as soda bottles that contain a whole number of liters. Hold student scavenger hunts to find classroom items (or items from home) that weigh close to a gram or kilogram. Have students find vessels that are close to a liter. Students can fill larger containers with only a liter of liquid. Have students estimate volume or mass, then check. (You can make it into a game—e.g., who gets closest to the true amount.) Use think-pair-share strategies for solving one-step word problems involving mass or liquid volume given in the same units, such as: “Charlie had six pieces of candy that had a mass of 10 grams each. What was the mass of the candy?”	Trumbauer, Lisa. *What Is Volume*? Children’s Press, 2006. Pluckrose, Henry. *Capacity.* Children’s Press, 1995. Murphy, Stuart J. *Room for Ripley (Mathstart: Level 3).* HarperCollins, 1999. Schwarz, David M. *Millions to Measure.* HarperCollins, 2006. http://www.ixl.com/math/grade-2 http://www.ixl.com/math/grade-3 http://www.bbc.co.uk/skillswise/topic/capacity http://www.bbc.co.uk/skillswise/topic/weight

Assessment Tasks Used	
Skill-Based Task: Students will precisely measure and label a required amount of liquid in liters using measuring tools accurately. Students will add, subtract, multiply and divide amounts of the same unit accurately. Students will draw a representation of multiple amounts of the same unit, and figure the total accurately.	**Problem Task:** Lisa has a wading pool that has 10 liter measuring marks on the side. It will hold 120 liters of water. Draw a picture of the pool. Right now it has 50 liters of water in it. How many more liters can she pour in? Angela is doing an experiment with paper clips. She discovered that 90 paper clips have a mass of 450 grams. What is the mass of each paper clip?

Core Content

Cluster Title: Represent and interpret data.
Standard 3: Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent five pets.
MASTERY Patterns of Reasoning:
Conceptual: Students will understand how to present data on a scaled picture graph and a scaled bar graph with several categories. Students will understand that each picture on a picture graph represents a set number of items. Students will understand that the lines on a bar graph represent a specified increment, and can identify the amount represented. Students will understand that categories on a bar graph can be compared to each other to determine how many more or less. . **Procedural:** Students can accurately solve two step problems (how many more, how many less) relating to a picture or bar graph. Students can compute, with repeated addition or multiplication of given symbols, the totals shown in each category on a picture graph. Students can compute, with repeated addition or multiplication, the totals shown in given categories on a bar graph. Students can convert data, such as tally marks, to numbers to show totals in each category. Students can use the scale given to accurately show data on the graph. Students can collect data through a survey in order to create a graph. **Representational:** Students can create a picture graph and a bar graph showing a data set, using varied scales appropriate to the data set, and verbally defend the graph created. Students can model information shown on a graph with manipulatives. Students can explain strategies for solving one- and two-step “how many more” and “how many less” problems, both verbally and in writing.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand that bar graphs and picture graphs are pictures that show data. Students will understand data when presented in single units, with up to four categories. Students will skip count by twos, fives, and tens. Students will understand the concept of compare and contrast, and how to verbalize differences. Students will know the difference between a bar graph and a picture graph. Students will know that graphs have a key and what the key is used for. Students will know that picture graphs and bar graphs are used to count objects. **Procedural:** Students can answer simple put-together (addition), take-apart (subtraction), and comparison problems using information presented on a picture graph or bar graph. **Representational:** Using given data, students can create picture graphs and bar graphs when key and scale are in single units (whole numbers) and the graph has no more than four categories.
Academic Vocabulary and Notation
data, picture graph, pictograph, symbol, key, scale, category, title labels, compare, how many more/less, tally marks, chart, survey

Instructional Strategies Used	Resources Used
Use literature to introduce and pique interest in graphs. For example, have students use *The Great Graph Contest* by Loreen Leedy, or find graphs in newspapers or magazines, or survey the class to collect data of personal interest. Present clear data sets for students to create bar and picture graphs. Make available multiple art media for creating eye catching graphs. Explain these graphs verbally with a partner/group.	Nagda, Ann Whitehead. *Tiger Math: Learning to Graph from a Baby Tiger.* Henry Holt, 2002. Ochiltree, Diane. *Bart's Amazing Charts.* Demco Media, 2000. Smith, David J. *If the World Were a Village.* Kids Can Press, 2011. Murphy, Stuart J. *Lemonade for Sale (MathStart 3).* HarperCollins, 1997. Leedy, Loreen. *The Great Graph Contest.* Holiday House, 2005. **http://nces.ed.gov/nceskids/createagraph/**

Assessment Tasks Used	
Skill-Based Task: Given a piece of graph paper, students create picture and bar graphs for a given set of data with several categories, and with a scale or key of 2, 5, or 10.	**Problem Task:** Josey wanted to know how many picture books certain authors have written. She found the information online and then created the scaled picture graph below. Each picture represents five books. What does the graph tell you? Justify your answers. **Number of Books by Children's Authors** 

Core Content

Cluster Title: Represent and interpret data.

Standard 4: Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand when it's important to measure precisely to a half or quarter inch.
Students will understand that measurement data can be shown through the use of a line plot.
Students will understand the concept of equivalence—that 2/2 = a whole, that 4/4 = one whole, and that 2/4 = 1/2—and understand the markings on a ruler.
Students will understand how to gather data and graph the data on a line plot.
Students will understand what a line plot looks like and how it represents data.
Students will understand the concept of fractional parts of an inch, especially whole, halves, and quarters.

Procedural:

Students can demonstrate accurate measurement to the nearest half inch and quarter inch using a ruler.
Students can collect a linear measurement data set and plot the data on a line plot marked with whole, half and quarter inches.

Representational:

Students can generate data by measuring and create a line plot to display findings.
Students can explain, both verbally and in writing, how to accurately measure to the nearest half-inch and/or quarter inch, and how to put measurement data on a line plot.

Supports for Teachers

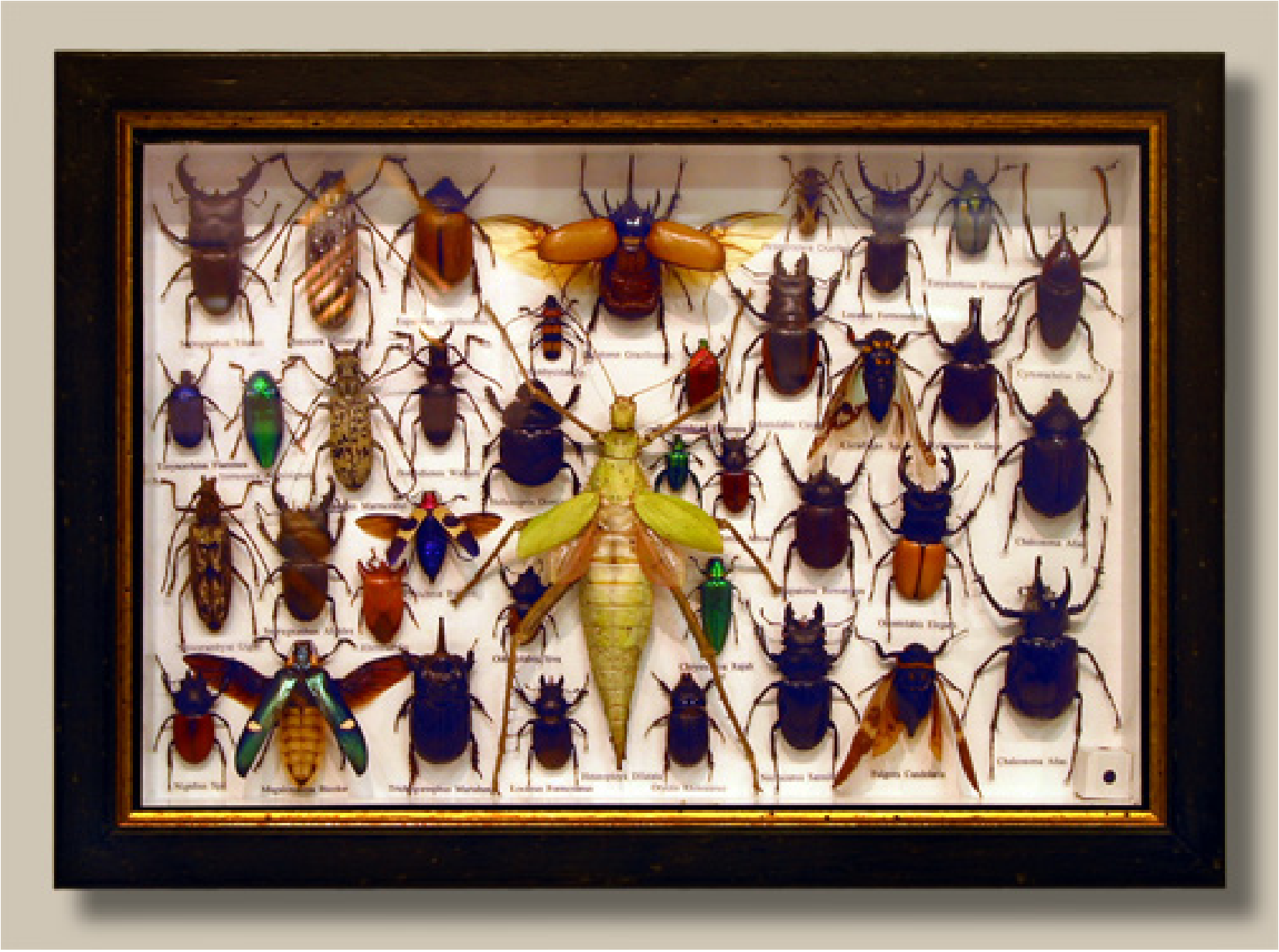
Critical Background Knowledge
Conceptual: Students will understand the concept of fractions as part of a whole, specifically halves and fourths. Students will understand the concept of measuring length to the nearest inch. Students will have a basic understanding of how data can be represented in graphs. **Procedural:** Students can use the tools of linear measure. Students can measure to the nearest unit. **Representational:** Students can explain, both verbally and in writing, how to measure to the nearest unit. Students can create a giant ruler
Academic Vocabulary and Notation
fraction, whole, half, quarter, fourths, line plot, data, linear, length, measure, “, 1/2 1/4, 2/4, 3/4, equivalent, length markings, scale

Instructional Strategies Used	Resources Used
Create a “giant inch” by taking a strip of paper (standard copy paper cut in half the long way), and folding it first in half, then in half again to show the fourths. With these “giant inches” it is easier to talk about the parts of an inch using a model that’s large enough to work with. Construction paper pieces can be cut in sizes to match the “giant 1/2-inch, and 1/4-inch” for manipulation. Have a scavenger hunt where you ask kids to find items in the classroom that are certain lengths (e.g., 4-1/4 inch, 8-3/4 inch, etc.). Have students cut a square blue piece of construction paper in half (rectangles), labeling each piece. Have them cut a second piece in half	Adler, David A. *How Tall, How Short, How Far Away.* Holiday House, 2000. Leedy, Loreen. *Measuring Penny.* Henry Holt, 2000. Briggs, Raymond. *Jim and the Beanstalk.* Putnam and Grosset, 1997. Murphy, Stuart. *Super Sandcastle Saturday.* HarperCollins, 1998.

(triangles), labeling each piece. Then have them cut red squares in fourths (repeat three times, 1 in squares, 1 in triangles, 1 in rectangles), labeling each piece. A third color square is cut into eighths (two ways, rectangles and triangles), and the pieces are labeled. Then the students are given a square template the same size as the construction paper squares. They place the pieces on the template, mixing and matching to see equivalences of a whole. Optional extensions include writing number sentences (e.g., 1/2 + 1/4 + 1/8 + 1/8 = 1). Students gather measurement data (e.g., the length of everyone's pencil to the nearest quarter inch). Have students create a line plot, marking the appropriate lengths on the horizontal line and plotting the data.	
Assessment Tasks Used	
Skill-Based Task: Students are able to precisely measure items to the nearest whole, half and quarter inch. Accurately interpret data presented on a line plot, having the horizontal scale marked off in whole, half, and quarter inches. Students can find items that measure a given length, (e.g., a 3-1/4 inch crayon). Given a data set, students can successfully draw a line plot with appropriate labels, whole, half and quarter inches, and accurately plot data.	**Problem Task:** Brock has a collection of insects. For a science fair project, he needs to create a line plot of their lengths. He measured them using a ruler marked in inches. On the following page is a picture of his collection. Measure the insects and create a line plot of their lengths to the nearest quarter inch.

Domain: Measurement and Data

Grade: 3



Code: 3MD4

Core Content

Cluster Title: Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

Standard 5: Recognize area as an attribute of plane figures and understand concepts of area measurement.

a. A square with side length one unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand that the area of a plane figure is dealing with the inside of the shape.
Students will understand what a square unit is and how it is used to measure area.
Students will understand that when using a unit square the entire surface of the plane figure must be measured without gaps or overlays.
Students will understand that area can be solved using *n* when the unit of measurement is unknown using repeated addition and multiplication.

Procedural:

Students can use manipulatives (unit blocks) to show area with no gaps or overlays.
Students can use repeated addition or multiplication to find the area of a plane figure.

Representational:

Students can draw pictures of plane figures and be able to show the area of that shape.
Students can take real-world objects (a book, table, etc.) and use manipulatives (unit blocks) to explain the area of the object without actually measuring it.
Using graph paper, students can draw a given figure and write the area.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Students will understand that a plane figure is has two dimensions.
Students will know the plane figures.
Students will know that repeated addition is the same as multiplication.
Students will know that shapes come in a variety of sizes.
Students will understand the concept of measurement.

Procedural:

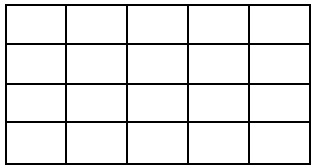
Students can measure the length of the side of a shape.
Students can solve basic multiplication problems.

Representational:

Students can draw plane figures on graph paper (arrays).
Students can show repeated addition with array drawing.
Students can represent repeated addition as a multiplication problem.
Students can show multiplication with array drawing.

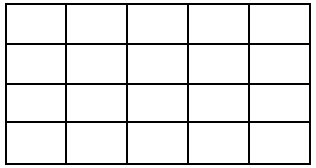
Academic Vocabulary and Notation

area, plane figure, square unit

Instructional Strategies Used	Resources Used
Define what a square unit is using graph paper.  ↖ (one unit square)	Burns, Marilyn. *Spaghetti and Meatballs for All: A Mathematical Story.* Scholastic Paperbacks, 2008. Ziefert, Harriet. *Squarehead.* Houghton Mifflin, 2001. Greene, Rhonda. *When a Line Bends A Shape Begins*. Sandpiper, 2001. http://www.shodor.org/interactive/activities/AreaExplorer/

Cover a plane figure with square units and then count them. Have students take a square piece of paper and fold the paper until it has many squares when opened. Students must count the square units to find the area.	http://www.ixl.com/math/grade-3 http://nlvm.usu.edu/en/nav/topic_t_4.html (geoboard) http://olc.spsd.sk.ca/de/math1-3/virtual%20manipulatives/areaGRID.html

Assessment Tasks Used

Skill-Based Task: Give students the following shape and have them color the square units and determine how many there are. Write a number sentence to show repeated addition and/or multiplication. 	**Problem Task:** Ask students to draw a rectangle on graph paper. The rectangle must follow the lines of the graph paper and keep all square units whole. Ask students to color and determine the area of the rectangle. On your graph paper, draw two different rectangular representations of 60 square units. Explain your illustration in writing.

Core Content

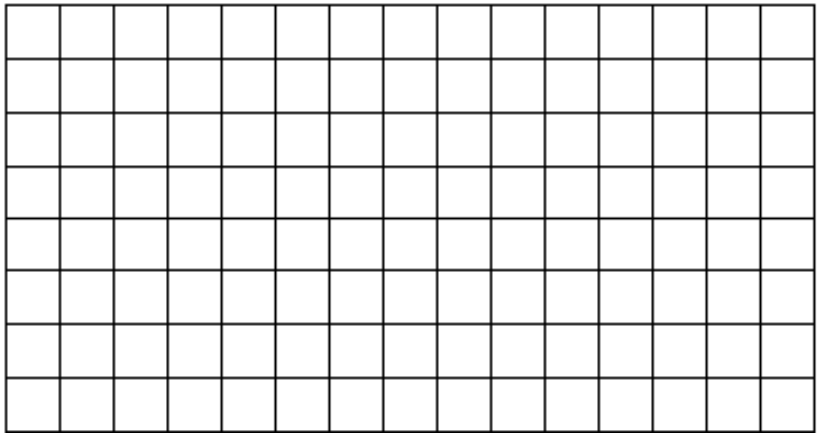
Cluster Title: Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
Standard 6: Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
MASTERY Patterns of Reasoning:
Conceptual: Students will understand that area is measured in square units. Students will understand square units can include customary and metric units of length. **Procedural:** Students can measure areas by counting unit squares. **Representational:** Students can model various areas with square tiles.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand the difference between customary and metric units of length. Students will understand meaning of area. Students will understand arrays. **Procedural:** Students can count and add like objects. **Representational:** Students can model rectangles and squares.
Academic Vocabulary and Notation
area, array, square unit, square, square cm, square inch, square meter, square feet, ft, m, in, square unit.

Code: 3MD6

Domain: Measurement and Data Grade: 3

Instructional Strategies Used	Resources Used
Make a model using different sizes of graph paper and then count up the square units to find the area of that model. Using 12 x 12-inch paper, give each student a square foot, make an array (no gaps or overlays), and then count the square feet to find the area.	Various sizes of graph paper Square tiles http://www.wartgames.com/themes/math/areaandperimeter.html http://math.pppst.com/perimeter.html
Assessment Tasks Used	
Skill-Based Task: Find the area of the rectangle. 	**Problem Task:** John built a rectangle using square meter units. He made four rows with six square meter units in each. Draw a picture to represent the rectangle John built and then find the area of his shape.

Code: 3MD6

Core Content

Cluster Title: Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

Standard 7: Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand the relationship of multiplication and addition to area.
Students will know the area algorithm to solve mathematical and real world problems.
Students will understand that rectilinear shapes can be broken down into rectangles.
Students will know that area is additive.

Procedural:

Students know that area equals length x width.
Students can work backwards to find the possible lengths and widths when given the area of a rectangle.
Students can divide a rectangle into two parts then using the distributive property find the area of the rectangle.
Students can determine the lengths for each side, and find the area for each rectangle.
Students can add the areas from each rectangle together to find the area of an original rectilinear shape.

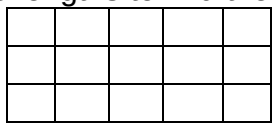
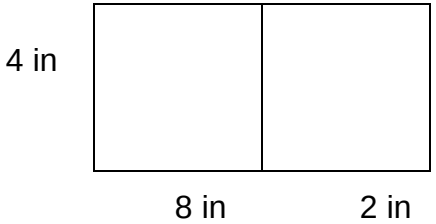
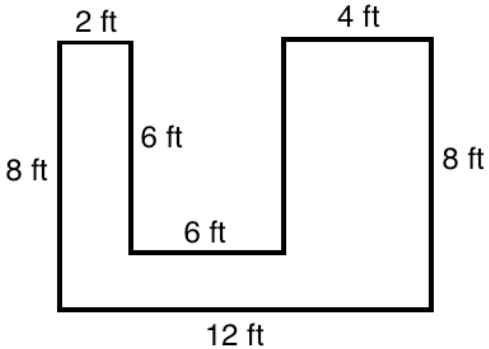
Representational:

Students can model the additive nature of area.
Students can represent whole-number products as rectangular areas in mathematical reasoning.
Students can represent the distributive property in mathematical reasoning.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand the distributive property Students will what is the best way to decompose a shape into rectangles or squares. Students will know multiplication facts. Students will know what area is. **Procedural:** Students can use multiplication facts Students can solve addition problems. Students can apply the distributive property. **Representational:** Students can model decomposing shapes Students can model finding lengths of sides not given. Students can model the distributive property.
Academic Vocabulary and Notation
product, additive, distributive property, rectilinear, decompose

Instructional Strategies Used	Resources Used
Students will need to use tiles to find the area, recognize the similarities to an array, and create the algorithm. Practice dividing a rectangle into two parts and finding the lengths of the sides. Then find the area of the whole rectangle using the distributive property. Decompose nonrectangular rectilinear shapes into rectangles, find the area of each part, then	http://www.mathplayground.com/PartyDesigner/PartyDesigner.html http://pbskids.org/cyberchase/math-games/airlines-builder/ http://pbskids.org/cyberchase/videos/area-alert/ http://math.pppst.com/perimeter.html Burstein, John. “Calculating Area Space Rocket.” *Weekly Reader,* 2003. Arvoy, Marsha. *Area (My Path to Math).* Crabtree, 2010.

add the areas of the various rectangles together.	
Assessment Tasks Used	
Skill-Based Task: Multiply the side lengths to find the rectangle's area.  Multiply the side lengths and use the distributive property to find the rectangle's overall area.  	**Problem Task:** Susan and her friends were asked to design their ideal snow fort. After much thought, they came up with a u-shaped fort. Their fort is represented below. Find the total area of the wall of the fort. Then find the total area of the fort. 

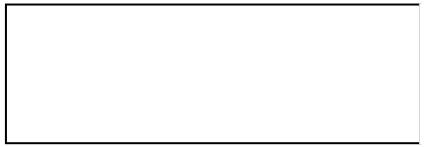
Core Content

Cluster Title: Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.
Standard 8: Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.
MASTERY Patterns of Reasoning:
Conceptual: Students will recognize polygons in real-world situations. Students will be able to recognize the perimeter of polygons in real-world situations. Students will understand that polygons with the same area can have different perimeters and that polygons with the same perimeter can have different areas. **Procedural:** Students know how to find the perimeter of polygons, including finding an unknown side length. Students can solve mathematical problems with polygons of the same perimeter, finding varying areas, and apply them to real-world situations. Students can solve mathematical problems with polygons of the same area, finding varying perimeters, and apply them to real-world situations. Students can find the perimeter of polygons in real-world situations. Students can find an unknown side length in a problem situation. **Representational:** Students can represent, pictorially or with objects, polygons with missing sides and find the length of the missing side. Students can represent polygons with a fixed perimeter and varying areas. Students can represent polygons with a fixed area and varying perimeters.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand the concepts of perimeter and area. Students will be able to analyze a real-world or mathematical problem in order to solve it. Students will know how to find the side length of a polygon. **Procedural:** Students can find the perimeter of a polygon. Students can find the area of a polygon. **Representational:** Students can draw polygons and show the perimeter and the area. Students can draw polygons and identify the sides.
Academic Vocabulary and Notation
polygon, side length, area, perimeter

Instructional Strategies Used	Resources Used
Use a task such as the following to help students understand this concept: Mei wants to build a pen for her dog to be safe and run around without being chained up. Her dad gave her 30 meters of fence to use. What is the perimeter of the pen that will give her the largest area for her dog to run in? Allow the students to use whatever strategies and materials they choose to solve the problem. Have students share strategies. Debrief with the students, making sure that the mathematical concepts are stressed and understood.	http://www.mathgoodies.com/lessons/toc_vol1.html My UEN: “Math All Around Us: Space to Play”

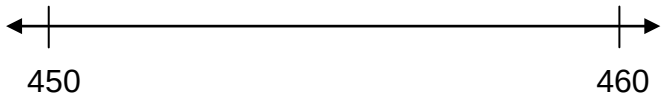
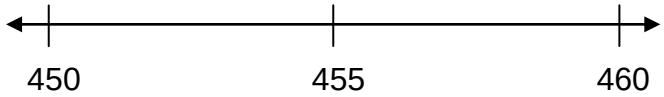
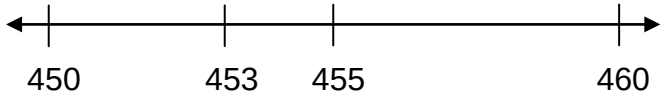
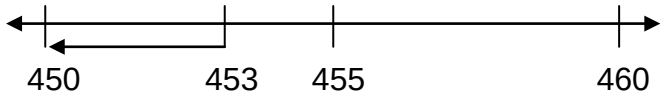
Assessment Tasks Used	
Skill-Based Task: Find two possible perimeters for a rectangle with an area of 60 square feet. Find the length of the missing side of this polygon: 20 cm 8 cm 	**Problem Task:** Jaclyn's stepfather told her she could make a garden in her backyard large enough for 100 square feet of flowers. She wants to make it easy to work on without stepping on flowers or dirt. Create a garden that Jaclyn will like. Find the perimeter of the garden you create.

Core Content

Cluster Title: Use place value understanding and properties of operations to perform multi-digit arithmetic. (Note: A range of algorithms may be used.)
Standard 1: Use place value understanding to round whole numbers to the nearest 10 or 100.
MASTERY Patterns of Reasoning:
Conceptual: Students will understand the basic principles of rounding whole numbers (if the digit is five or greater the digit to the left moves up one number, if the digit is four or less the digit to the left stays the same). **Procedural:** Students can identify the place to which they are rounding. Students can identify the digit that affects how the number is rounded. Students can identify the rounding choices (digit stays the same or rounds higher). Students can round whole numbers to the nearest 10 or 100. **Representational:** Students can represent rounding using number line, place value drawings, base ten blocks, or hundreds charts.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will know place and value of whole numbers less than or equal to 1,000. **Procedural:** Students can read and write multi-digit whole numbers less than or equal to 1,000. **Representational:** Students can model numbers less than or equal to 1,000 (e.g., base-ten numerals, number names, expanded form, number lines, base-ten blocks, hundreds chart).
Academic Vocabulary and Notation
base-ten, benchmark number, compare, digits, expanded form, order, place value, round a whole number, standard form

(number name), whole number, word form	
Instructional Strategies Used	**Resources Used**
Determine the place value to which you are rounding. Step 1: Identify the two benchmark numbers on either side of the target number. Step 2: Find the halfway point between the target numbers. Step 3: Place the target number on the number line. Step 4: Choose the benchmark number that is closer to the target number. What do you do if the number is at the halfway point? Example: Round 453 to the nearest 10. Step One:  Step Two:  Step Three:  Step Four:  Use tasks such as the following to allow students to think deeply about rounding:	Dalton, Julie. *Farmers Market Rounding*. Children's Press, 2007. Arvoy, Marsha. *Rounding My Path to Math.* Crabtree, 2010. http://www.funbrain.com/tens/index.html http://www.ehow.com/way_5182955_math-games-rounding.html http://math.pppst.com/rounding.html http://www.wartgames.com/themes/math/rounding.html http://www.ixl.com/math/grade-3

Ming wants to make a game using blocks. The game will be for 11 people, and each would need 5 blocks. She wants to be sure she has extra blocks in case some get lost. Using rounding determine how many blocks Ming should get to the nearest 10. Your parents are looking for a new apartment. You want to be sure your furniture will fit beneath the windows, so you have measured the width of your table, bed, and sofa. The table is 64 inches, the bed is 79 inches, and the sofa is 93 inches. The landlord told you the space between the windows in the kitchen (for the table) is 70 inches, the space for the bed is 80 inches, and the space for the sofa is 90 inches. She told you she rounded the numbers to the nearest ten. 1. If you round your furniture's measurements, what do you get? Table _____ Bed _____ Sofa _____ 2. Between which benchmark numbers do the measurements in the apartment lie since they have been rounded? Kitchen window space _____ Bedroom window space _____ Living room window space _____ 3. In which spaces will your furniture fit for sure? Where are you not sure your furniture will fit? 4. What area or areas should you ask the landlord to remeasure? (Hint: The rounded amount may seem too small, but it may not be.)	Adapted from: Arrowood, Janet C., *Mathematics for ESL Learners.* Rowman & Littlefield, 2004, p. 46.

Play rounding games (e.g., rounding bingo, rounding war).	
Assessment Tasks Used	
Skill-Based Task: Round 17 to the nearest ten. Round 22 to the nearest ten. Round 234 to the nearest hundred. Round 650 to the nearest hundred. Round 459 to the nearest hundred. Round 987 to the nearest hundred.	**Problem Task:** If you round 250 to the nearest ten, would you still say that Kent has about 300 books on his shelf? Explain why or why not? Use a number line to explain why 450 is the least number that rounds to 500.

Core Content

Cluster Title: Use place value understanding and properties of operations to perform multi-digit arithmetic. (Note: A range of algorithms may be used.)
Standard 2: Fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
MASTERY Patterns of Reasoning:
Conceptual: Students must have a conceptual understanding of the topics listed in the standard (place value, properties of operations, relationship between addition and subtraction) before they can become fluent. Therefore, these understandings are explained in the Background Knowledge section below. **Procedural:** Students can use multiple strategies and algorithms fluently to add and subtract within 1,000. Students can use place value understanding to fluently add and subtract within 1,000. Students can use understanding of the associative, commutative, and identity properties of addition to fluently add within 1,000. Students can understanding of the inverse relationship between addition and subtraction to fluently add and subtract within 1,000. **Representational:** Model addition and subtraction to 1,000 using manipulatives (e.g., place value blocks) to develop fluency. Represent addition and subtraction (e.g., on hundreds charts, number lines, bar models) to develop fluency.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand place value to 1,000. Students will understand vertical and horizontal forms of addition and subtraction. Students will understand order of operations without using parentheses. Students will understand the associative and commutative properties of addition. Students will understand the inverse relationship of addition and subtraction. Students will understand the additive and subtractive identity properties ($n + 0 = n$, $n - 0 = n$). **Procedural:** Students can use understanding of addition fluently to add and subtract within 100. Students can apply understanding of models for addition and subtraction to solve problems within 100. Students can use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation. Students can use understanding of place value and the properties of operations to add and subtract within 100. Students can select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds. **Representational:** Students can use manipulatives such as place value blocks to model addition and subtraction. Students can use hundreds charts and number lines to add and subtract within 100.
Academic Vocabulary and Notation
compare, digits, expanded form, order, place value, standard form, word form, addends, sum, commutative (order) property of addition, identity (zero) property of addition, associative property of addition (grouping), fact family, difference, round, estimate, equation

Instructional Strategies Used	Resources Used
Note: Teachers must recognize that fluency requires using mathematical procedures flexibly, accurately, efficiently, and appropriately and is developed over time through conceptual understanding, using numbers in context, and learning and practicing appropriate procedures. Pose problems such as the following: McKenna had collected 456 pennies. Her friend, Reagan, wanted to start her own collection. McKenna gave her 199 pennies. How many pennies did McKenna have left? Have students solve the problem using whatever strategy they choose, including the traditional algorithm. Have them share their strategies. Strategies may include **adding up** (199 + 1 = 200, 200 + 200 = 400, 400 + 56= 456, therefore 1 + 200 + 56 = 257 pennies), **compensating** (200 + 200 = 400, 400 + 56 = 456, 200 + 56 = 256 plus 1 (from 199 to 200) equals 257), **subtracting to count down** (456 – 56 = 400, 400 - 200 = 200, 200 - 1 = 199 so 56 + 200 + 1 = 257), or **adding by tens or hundreds** (199 + 1 = 200, 200, 300 (100 more), 400 (100 more) 410, 420, 430, 440, 450, 456 (that's 6 more) so 1 + 100 + 100 + 50 (5 tens) + 6 = 257). If students have used other strategies, have them share and explain.	http://nlvm.usu.edu/en/nav/topic_t_1.html - Basic blocks addition and subtraction - Hundreds chart - Number line arithmetic Murphy, Stuart. *Earth Day Hooray! (MathStart 3).* HarperCollins, 2004. Murphy, Stuart. *Shark Swimathon (MathStart 3).* HarperCollins, 2000. Lopresti, Angeline Sparagna. *A Place for Zero: A Math Adventure.* Charlesbridge, 2003. http://www.amblesideprimary.com/ambleweb/mentalmaths/pyramid.html http://www.math.pppst.com/subtraction.html (Scroll down to the PowerPoint "Subtraction by Adding." Be sure to add contexts to the numbers.)

Strategies like those above use place value, properties of operations, and the relationship between addition and subtraction. Repeat with many other problems. Have students practice using problems in context (word problems). It is not necessary for students to use the traditional algorithm to become fluent in addition and subtraction, though that algorithm should be taught at some point.	
Assessment Tasks Used	
Skill-Based Task: 236 + 147 236 +147 236-147 236 - 147	**Problem Task:** There are 236 pieces of candy and my mom bought 147 more. How many do I have in all? I had 236 pieces of candy and I gave away 147 to my friend. How many do I have left? Have students share their reasoning and strategies.

Core Content

Cluster Title: Use place value understanding and properties of operations to perform multi-digit arithmetic. (Note: A range of algorithms may be used.)
Standard 3: Multiply one-digit whole numbers by multiples of 10 in the range 10 - 90 (e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations.
MASTERY Patterns of Reasoning:
Conceptual: Students will understand how multiplication of one-digit factors of multiples of 10 connects with multiplication of two one-digit whole numbers. Students will understand multiplication as finding an unknown factor. **Procedural:** Students can multiply one-digit factors by multiples of 10 up to 90. Students can use strategies involving properties of operations to calculate products relating to this standard. Students can use strategies involving place value to calculate products relating to this standard. **Representational:** Students can use manipulatives to demonstrate understanding of multiplication using multiples of 10. Students can use number lines to demonstrate understanding of multiplication using multiples of 10. Students can use hundreds charts to demonstrate understanding of multiplication using multiples of 10.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand place value. Students will understand that multiplication is repeated addition. **Procedural:** Students can count within 1,000; skip-count by 10s or 100s. Students can read and write numbers to 1,000 using base-ten numerals and number names. Students can fluently multiply within 100 (3.OA.7).

Representational:

Students can demonstrate that 100 can be thought of as a bundle of ten tens — called a “hundred.”
Students can use place value blocks to model multiplication.
Students can multiply on a 100’s chart and number line.
Students can manipulate objects to demonstrate properties of operations**.**

Academic Vocabulary and Notation

multiplication, factors, product, array, row, columns, commutative (order) property of multiplication, multiples

Instructional Strategies Used	Resources Used
Help students to connect their understanding of place value to multiples of ten. Help them understand that since 30 is 3 tens and 80 is 8 tens, they are multiples of ten, since a multiple of ten is 10 multiplied by another number. In this standard we work with the multiples of 10 from 10 to 90. Have students use base-ten blocks, number lines, diagrams, or hundreds charts to multiply one-digit numbers by multiples of tens. Have the students share their strategies with each other and explain their reasoning. Discuss the strategies together as a class. For example, students might remember from 3.OA.1 that 3 x 2 can be interpreted as 3 groups of 2 objects each. Through their understanding of place value, students reason that 3 x 20 can be interpreted as 3 groups of 20 objects each or 6 tens and 6 tens is 60. Have students use the same factor and all multiples of ten and look for a pattern in the multiplication. Then try another factor.	http://www.oswego.org/ocsd-web/games/Ghostblasters1/gbcd.html http://www.ictgames.com/fairyfog10s_v2.html (can help low-level students)

Assessment Tasks Used	
Skill-Based Task: 10 x 6 =	**Problem Task:** There are 10 monkeys and each has 6 bananas. How many bananas are there in all? Tami wanted to prove to her little brother Timothy that his 240 toy cars could be grouped in more than one way. How many groupings could she find using multiples of ten up to 90? What are they? Were Tami's groupings accurate? How do you know? What properties of operations did she use?

Core Content

Cluster Title: Develop understanding of fractions as numbers.
Standard 1: Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.
MASTERY Patterns of Reasoning:
Conceptual: Students will understand that fractional parts must be equal-sized pieces of the same whole. Students will understand how many equal parts make a whole. Students will understand that as the number of equal pieces in the whole increases, the size of the fractional pieces decreases. Students will understand that the numerator of a fraction is the number of equal parts being considered, e.g., 3/5 is three 1/5 units. Students will understand that the denominator of a fraction is the number of equal parts that make up the whole. Students will know the characteristics of a unit fraction (a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts.) **Procedural:** Students can identify the numerator as the number of equal parts being considered. Students can identify the denominator as the number of equal parts that make up the whole. Students can read and write a fraction. Students can divide a region or set of objects into fractional parts. Students can explain fractions verbally and/or in writing. **Representational:** Students can represent fractions using circles, squares, rectangles, fraction bars, number lines, and sets of objects. Students can represent fractions as fair sharing and parts of a whole.

Code: 3NF1

(Note: Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand equal and fair shares. Students will understand that a region is a geometric figure that can be divided into equal parts. Students will understand that sets of objects can be divided into equal parts. Students will recognize that equal shares of identical wholes do not have to have the same shape or size. **Procedural:** Students can make equal parts. Students can use the additive nature of area. Students can partition a set of objects or a region into equal groups/pieces **Representational:** Students can model equal groups using manipulatives.
Academic Vocabulary and Notation
halves (1/2), thirds (1/3), fourths (1⁄4), sixths (1/6) , eighths (1⁄8), fraction, numerator, denominator, equal parts

Instructional Strategies Used	**Resources Used**
Use models, manipulatives, and fraction sets to represent numerator and denominator.	McMIllian, Bruce. *Eating Fractions.* Scholastic Press, 1991.
Develop understanding of fair shares by sharing objects where each person gets the same amount.	Murphy, Stuart. *Give Me Half (MathStart 2).* HarperCollins, 1996.
Partition circles, squares, or rectangles into equal parts.	Adler, David. *Fraction Fun.* Holiday House, 1997.
Partition a set of objects into equal groups.	Pallota, Jerry. *Hershey's Milk Chocolate Bar Fractions Book.* Cartwheel Books, 1999.

Code: 3NF1

(Note: Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)

Fold paper to create a variety of fractional models. Use time as a reference for real-world application of common fractions. Use pattern blocks, geoboards, fraction sets in various shapes, analog clocks, and various objects (e.g., cars, people, shells, buttons, etc.).	Penner, Lucille R. *Clean Sweep Campers.* Live Oak Media, 2008. http://www.amathsdictionaryforkids.com Fraction bar, fraction pieces, naming fractions at: http://www.nlvm.usu.edu/en/nav/grade_g_2.html
Assessment Tasks Used	
Skill-Based Task:   Write the fraction for a visual model.  What fraction of the shapes are rectangles? Make a visual model to show a fraction.	**Problem Task:** Paul made a garden in the shape of a rectangle and divided it into 5 equal parts. He planted carrots on 3/5 of the garden. Draw a picture of what Paul's garden might have looked like. Explain your thinking. Pam had 30 cookies. She needed to take 1/3 of them to a party. How many cookies did she take to the party? Show how you know in numbers, pictures, and words.

Code: 3NF1

(Note: Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)

Core Content

Cluster Title: Develop understanding of fractions as numbers.

Standard 2: Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.
b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand the properties of a unit whole.
Students will understand that fraction is a part of a whole.
Students will understand that a fraction is divided into equal parts.
Students will understand that a fraction as a number on the number line.

Procedural:

Students can identify fractions on a number line.
Students can place fractions on a number line.
Students can show a fraction on a number line by marking off equal lengths from 0 to 1.
Students can divide a number line between 0 and 1 into equal parts and define the unit fraction.

Representational:

Students can represent a fraction on a number line diagram by marking off lengths from 0.

Code: 3NF2

(Note: Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)

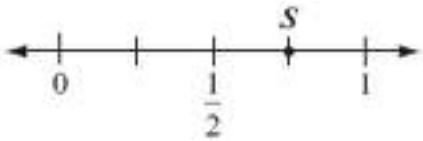
Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand the words *halves*, *thirds*, *half of*, *a third of* (for the denominators identified in grade three domain), and describe the whole as two halves, three thirds, four fourths, etc. Students will show understanding of placement of numbers on a number line. **Procedural:** Students can partition circles and rectangles into two, three, four, six or eight equal shares, and describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, or four fourths. Students can place a number on a number line. Students can divide a region or set of objects into fractions. **Representational:** Students can model fractions with area or sets of objects.
Academic Vocabulary and Notation
halves (1/2), thirds (1/3), fourths (1/4), sixths (1/6), eighths (1/8), fraction, numerator, denominator, equivalent fractions, equal parts

Instructional Strategies Used	Resources Used
Use a strip of paper as a number line. Label 0 on the left and 1 on the right. Then fold the paper number lines into various fractional pieces. Fold one strip in half, the next in thirds, the next in fourths, etc. Label the fractional parts on each strip. Give the student a number line labeled 0-1 and have them draw various fractional parts.	Murphy, Stuart. *Jump Kangaroo Jump (MathStart 3).* HarperCollins, 1999. http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/maths/fractions/level4.htm

Code: 3NF2

(Note: Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)

Use adding machine tape to measure classroom objects and divide into fractional pieces (e.g., show 1/2 of the length of my desk, filing cabinet, door, white board, etc.). Label 0-1 as a number line on the adding machine tape. Use rulers.	
Assessment Tasks Used	
Skill-Based Task: Show 1/4 on a number line. Show 3/8 on a number line. Name the fraction shown on the number line.  	**Problem Task:** Solve the following problems: You have a piece of licorice. You need to cut it into three equal parts to share with your friends. On a number line, show where you would cut it and label to show what fraction of the licorice each person would get. An inchworm reached its full size of an inch by growing a 1/4 of an inch each month. On the number line, draw the inchworm after three months.

Code: 3NF2

(Note: Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)

Core Content

Cluster Title: Develop understanding of fractions as numbers.

Standard 3: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
b. Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.
c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.
d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand that a whole number can be expressed as a fraction.
Students will understand the definition of equivalence.
Students will understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
Students will understand simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3).
Students will understand that two fractions with the same numerator or the same denominator can be compared.
Students will understand that comparisons are valid only when the two fractions refer to the same whole.

Procedural:

Students can recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3).
Students can explain why the fractions are equivalent.
Students can compare fractions by reasoning about their size.
Students can express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.
Students can compare two fractions with the same numerator or the same denominator.
Students can express the comparison of fractional models by using <, >, or =.
Students can recognize that comparisons are valid only when the two fractions refer to the same whole.
Students can explain verbally and in writing all of the procedures for this standard.

Code: 3NF3

(Note: Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)

Representational:

Students can model comparisons of fractions with manipulatives.
Students can model equivalent fractions by using a visual fraction model.

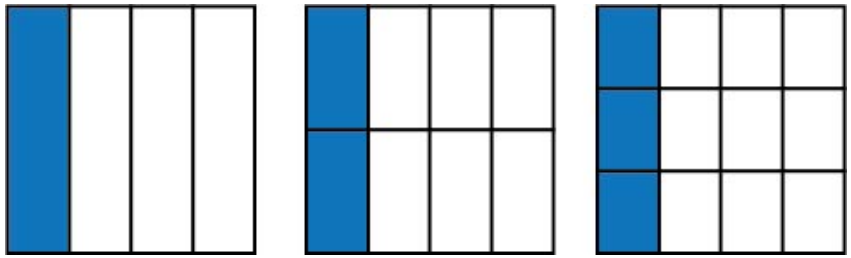
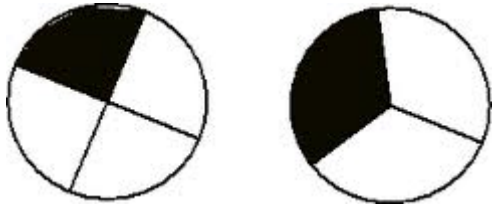
Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand the words *halves*, *thirds*, *half of*, *a third of*, etc. Students will understand numbers using >, =, and < symbols to record the results of comparisons. **Procedural:** Students can partition regions into two, three, or four equal shares; describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc.; and describe the whole as two halves, three thirds, four fourths. Students can compare numbers using >, =, and < symbols to record the results of comparisons. Students can describe the whole as two halves, three thirds, four fourths, etc. **Representational:** Students can model the division of regions and sets of objects into fractional parts.
Academic Vocabulary and Notation
halves (1/2), thirds (1/3), fourths (1/4), sixths (1/6), eighths (1/8), fraction, numerator, denominator, equivalent, equivalent fractions, equal parts, compare

Instructional Strategies Used	Resources Used
Use fraction manipulatives to compare the size of different fractions of the same whole. Find equivalent fractions and label them. Use manipulatives to show that it takes 6/6, 4/4, 3/3, etc. to	Pallota, Jerry. *Hershey's Milk Chocolate Bar Fractions Book.* Cartwheel Books, 1999. Pallotta, Jerry. *Apple Fractions.* Cartwheel Books, 2003.

Code: 3NF3

(Note: Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)

make a whole. Use manipulatives such as rulers and pattern blocks with the same numerator or same denominator to compare the size of various fractions.	Murphy, Stuart. *Jump, Kangaroo, Jump (MathStart 3).* HarperCollins, 1999. Adler, David. *Fraction Fun.* Holiday House, 1997. http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/maths/fractions/level4.htm http://www.gamequarium.com/fractions.html http://nlvm.usu.edu/en/nav/topic_t_1.html
Assessment Tasks Used	
Skill-Based Task:  Write the equivalent fractions.  Write <, >, or =.	**Problem Task:** Jamie jumped 2/4 of the length of the sidewalk. Find an equivalent fraction. Use a visual model to solve the problem. Explain your model. My friend and I each ordered a medium pizza. I ate 1/4 of my pizza. My friend ate 1/3 of his pizza. Who ate more? How do you know?

Code: 3NF3

(Note: Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)

|-----|-----|-----|-----|-----|-----|-----|-----| 0 1 Place the following fractions on number line above. $\frac{1}{2}, \frac{2}{2}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}, \frac{8}{8}$ Use your number line to answer these questions: What are two fractions equivalent to 1? What are two fractions equivalent to 1/2?	

Code: 3NF3

(Note: Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)

Core Content

Cluster Title: Represent and solve problems involving multiplication and division.

Standard 1: Interpret products of whole numbers (e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each). For example, describe a context (story situation) in which a total number of objects can be expressed as 5 x 7.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand that multiplication is combining equal groups of objects.
Students will understand that multiplication is repeated addition.
Students will understand that skip counting can be used to solve multiplication.
Students will understand that in a multiplication equation, the first factor equals the number of groups and the second factor equals the number in each group.

Procedural:

Students can find the total number of objects within equal groups (e.g., 5 x 7 = 35).
Students can use repeated addition to find the product of equal groups.
Students can use skip counting to find the product of equal groups.

Representational:

Students can model equal groups of objects.
Students can draw equal groups of objects (e.g., an array).
Students can write repeated addition expressions and multiplication expressions that represent their pictures.
Students can draw pictures that represent the multiplication expression.
Students can model skip counting on a number line.

Supports for Teachers

Critical Background Knowledge	
Conceptual: Students will understand repeated addition can find the total number of items in equal groups. Students will understand grouping of objects. Students understand the meaning of skip counting. **Procedural:** Students can solve repeated addition problems. **Representational:** Students can connect equal groups to the operation of repeated addition.	
Academic Vocabulary and Notation	
factor, array, product, multiple, equation, multiplication, equal groups, x (multiplication symbol), row, column	
Instructional Strategies Used	**Resources Used**
Read a book to help students understand skip counting. Have students skip count on a number line to show adding of equal groups. List real-life situations where items come in equal groups. Model equal groups by putting the same number of objects in each group. Link number of objects in each group to repeated addition. Link repeated addition to multiplication equation. Associate that the first factor is the number of equal groups and the second factor is the number of objects in each group.	Aker, Suzanne. *What Comes in 2's, 3's, and 4's?* Aladdin, 1992. Fosnot and Uittenbogaard. *Minilessons for Early Multiplication and Division.* Heinemann, 2008. Hamm and Palmer. *How Many Feet in the Bed?* Alladin/Simon and Schuster, 1994. Neuschwander, Cindy. *Amanda Bean's Amazing Dream.* Scholastic Press, 1998. Warburton, Tom, Director. *Schoolhouse Rock: Multiplication Classroom Edition Interactive DVD.* Disney Educational Products, 2008. Math Vocabulary Word Wall Cards that show

	vocabulary, picture, and definition: http://www.graniteschools.org/depart/teachinglearning/curriculuminstruction/math/Pages/MathematicsVocabulary.aspx
Assessment Tasks Used	
Skill-Based Task:  Write an equation that can help you find the total number of points on the above stars. Teacher note: Make sure students can show the number of groups vs. the number of items in the group (3 x 5 = 15).	**Problem Task:** Frank bought twelve boxes of crayons. Each box of crayons had 8 crayons in it. How many boxes of crayons does he have? • Solve the problem. • Write a repeated addition equation. • Write the multiplication equation. • Draw a visual representation of the equation.

Core Content

Cluster Title: Represent and solve problems involving multiplication and division.

Standard 2: Interpret whole-number quotients of whole numbers (e.g., interpret 56 ÷ 8 as the number of objects in each group when 56 objects are partitioned equally into 8 groups). For example, describe a context in which a number of groups can be expressed as 56 ÷ 8.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand that division represents two different situations.

- Partitive (Equal groups): determining how many objects are in each group.
- Quotative (Measurement): determining how many groups can be made from a specific amount of equal objects.

Students will understand that division is repeated subtraction.

Procedural:

Students can find how many equal groups can be made out of a certain number of objects.
Students can find how many objects can be shared equally among a certain number of groups.
Students can use repeated subtraction to find the number of equal groups.
Students can solve division problems.

Representational:

Students can model a division equation using pictures, objects, or numbers.
Students can demonstrate equal groups of objects.
Students can draw equal groups of objects.
Students can write repeated subtraction expressions and division expressions that represent their pictures.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand that division is sharing equally among groups. Students will know, when given a certain number of groups, how many items will be in each group. Students will understand that measurement (quotative) division is when students are given a certain number of objects, they need to find out how many groups will have an equal number of items. **Procedural:** Students will be able to use repeated subtraction. **Representational:** Students can model problems involving equal group.
Academic Vocabulary and Notation
quotient, dividend, divisor, divide, equal groups, whole numbers, partitive, quotative, ÷, $\overline{)}$

Instructional Strategies Used	Resources Used
While reading *The Doorbell Rang*, give counter manipulatives in baggies. so students can move the objects around. Give students tiles and have them model both types of division problems. Partitive (sharing): There were 42 jellybeans. I put them into 6 cups. How many jellybeans would be in each cup? Quotative (measurement): Brendon had 42 jellybeans. He put 7 into each cup. How many cups of jellybeans did he make? Read division books and model problems based on the content. Relate repeated subtraction to division.	Murphy, Stuart. *Divide and Ride.* HarperCollins, 1997. Pinczes, Elinor J. *A Remainder of One.* Houghton Mifflin, 1995. Hutchins, Pat. *The Doorbell Rang.* Greenwillow Books, 1989. Pinczes, Elinor J. *One Hundred Hungry Ants.* Houghton Mifflin, 1993. 3-5 Numbers and Operations: http://nlvm.usu.edu/ Number line arithmetic Rectangle division

Assessment Tasks Used	
Skill-Based Task: 56 ÷ 7 = 48 - □= 8 36 - □□= 9	**Problem Task:** Quotative (measurement): Damion has 27 cups of cereal. He eats 3 cups every morning. How many days will he have cereal? (Use pictures, words, and numbers to show how you solved this problem.) Partitive (equal groups): Anna had 27 beads. She made 9 necklaces. How many beads were on each necklace? (Use pictures, words, and numbers to show how you solved this problem.)

Core Content

Cluster Title: Represent and solve problems involving multiplication and division.

Standard 3: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand that word problems can be represented in multiple ways (e.g., equation, array, equal groups, repeated addition, repeated subtraction, number line, table).
Students will understand what a symbol represents in an equation (e.g., in 4 x △ = 16, △ = 4).
Students will understand that the symbol can represent a different component of the equation.

Procedural:

Students can create and solve a multiplication or division word problem.
Students can create and solve a word problem using a symbol to represent the unknown number.

Representational:

Students can model objects in an array.
Students can model objects in groups.
Students can model using equal jumps on a number line.
Students can model using repeated addition (multiplication) or subtraction (division).
Students can write an equation that represents the word problem.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Students will understand the meaning of multiplication.
Students will understand the meaning of division.
Students will understand how to write an equation for multiplication and division.
Students will understand how to solve a word problem.

Procedural:

Students can find out what operation the problem is asking them to perform.
Students can solve multiplication problems.
Students can solve division problems.

Representational:

Students can write an equation that matches the word problem.
Students can model a variety of strategies.

Academic Vocabulary and Notation

array, area model, equal groups, multiply, divide, product, factor, quotient, divisor, dividend, row, column, symbol

Instructional Strategies Used	Resources Used
Use trade books to present real-world problems and have students model, write, and solve. The students will solve their own story problems and solve other students' problems. Find the array that matches given expressions. Analyze another student's word problem for viability.	Giganti, Paul. *Each Orange Had 8 Slices.* Greenwillow, 1999. Pinczes, Elinor J. *One Hundred Hungry Ants.* Houghton Mifflin, 1993. Tang, Greg. *Best of Times.* Scholastic Press, 2002. http://nlvm.usu.edu/en/nav/category_g_2_t_1.html http://www.ixl.com/math/grade-3/division-word-problems-facts-to-10

Assessment Tasks Used	
Skill-Based Task: Maya had 4 bunnies. Each bunny had 8 babies. How many babies were there in all? Solve. Maya had 40 carrots. She gave 5 bunnies the same number of carrots. How many carrots did each bunny get?	**Problem Task:** 72 ÷ 9 = ___ Write a word problem that represents this equation, then solve. Show your thinking in pictures, words, and numbers. 7 x ____ = 21 Write a word problem that represents this equation, then solve. Show your thinking in pictures, words, and numbers.

Core Content

Cluster Title: Represent and solve problems involving multiplication and division.

Standard 4: Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the following equations: 8 x ? = 48, 5 = □ ÷ 3, 6 x 6 = ?

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand that there can be a result unknown (don't know the answer), change unknown (know the first number of the equation, but not the second), or a start unknown (first number in equation not known) within an equation.

Students will understand the use of a symbol to represent an unknown number.

Procedural:

Students can apply multiplication or division to solve for an unknown in an equation.

Representational:

Students can use a model to solve for the unknown whole number in an equation.

Students can represent an equation by putting the numbers in a real-world problem.

Students can write a word problem that represents an equation with an unknown.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Students will understand that both sides of an equation equal the same amount.

Students will understand that a symbol can be used for an unknown within an equation.

Procedural:

Students will know how to write an equation.

Students can use a symbol to represent an unknown number.

Students can solve a multiplication or division equation.

Representational:

Students can model multiplication and division using a variety of strategies.

Academic Vocabulary and Notation	
symbol, equal, =, x, ÷	
Instructional Strategies Used	**Resources Used**
Use the multiplication table to help find a missing factor. Provide problems with an unknown to be found. Teach students to use a variety of symbols to represent the unknown. Use an input/output strategy where either the input or output is unknown. Use a number line to model the missing number.	Murphy, Stuart. *Safari Park.* HarperCollins, 2001. The Product Game http://illuminations.nctm.org/LessonDetail.aspx?ID=L272 Complete the Division Sentence Facts to 10 http://www.ixl.com/ Interactive Chart http://www.mathsisfun.com/tables.html

Assessment Tasks Used	
Skill-Based Task: 24= 3 x ____ 56 ÷△ = 7	**Problem Task:** This is a skill-based standard. No problem is provided.

Core Content

Cluster Title: Understand properties of multiplication and the relationship between multiplication and division.

Standard 5: Apply properties of operations as strategies to multiply and divide. Examples: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known (commutative property of multiplication). 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or by 5 x 2 = 10, then 3 x 10 = 30 (associative property of multiplication). Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16 = 56 (distributive property).

Note: Students need not use formal terms for these properties at this grade level.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand that multiplication is commutative and division is not commutative.
Students will understand the distributive, associative, and commutative properties of Multiplication.
Students will understand the multiplicative identity property (i.e., multiplying by 1).
Students will understand the zero property of multiplication (i.e., multiplying by 0).
Students will understand that division by 0 is undefined (the zero property).

Procedural:

Students can multiply two factors in any order.
Students can multiply by grouping using parentheses and three factors in various ways.
Students can simplify a multiplication expression into smaller problems to make solving easier
Students can find the product when multiplying by 1.
Students can find the product when multiplying by 0.
Students can find the quotient when dividing by whole numbers.

Representational:

Students can use an array or grouping to model the commutative property.
Students can model the associative property using three factors.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand the commutative and associative properties of addition. Students will understand that numbers can be broken apart into smaller numbers. Students will understand the identity property of addition. Students will understand that three numbers can be multiplied together. **Procedural:** Students can find the product of a multiplication equation. Students can break addition equations into smaller addition problems to solve. **Representational:** Students can make an array. Students can show equal groups. Students can model a division equation using pictures, objects, or numbers. Students can draw equal groups of objects. Students can represent repeated subtraction expressions and division expressions pictorially (and write the expressions). Students can represent repeated addition expressions and multiplication expressions pictorially (and write the expressions).
Academic Vocabulary and Notation
array, product, factor, (), x, ÷, =, commutative property, associative property, distributive property, zero property, dividend, divisor, quotient

Instructional Strategies Used	Resources Used
Associative Property: • Have student lay out 18 counters in 3 groups, which have 2 rows of 3 counters each. • Write an expression using parenthesis to represent the model (e.g., (2 x 3) x 3). • Ask students to use 18 more counters and model 2 x(3 x 3). • Have them compare the models.	Giganti, Paul. *Each Orange Had 8 Slices.* Greenwillow Books, 1999. Multiplication chart http://illuminations.nctm.org/LessonDetail.aspx?ID=U110

• Explain that the concept of changing the grouping of factor does not change the product. Commutative property: • Have the students make 2 groups of 3. Then have them make 3 groups of 2. Compare and discuss the similarities and differences of the two models. • Connect to the concept that changing the order of the factors does not change the product. Note: arrays are also good models to teach this concept. Distributive property: • Make an array of 6 rows of 8 counters (6 x 8). • Break the array into 5 rows of 8 and 1 row of 8(5 x 8) + (1 x 8). • Add together the products of both arrays (40 + 8 = 48). • Connect the total area of the larger array to the area of the two smaller arrays. • Have the students draw an array of 4 rows of 9 (4 x 9) on graph paper. • Cut into 4 rows of 4 (4 x 4) and 4 rows of 5 (4 x 5). • Add together the product of both arrays (16 + 20) • Connect the total area of the larger array to the area of the two smaller arrays. Identity Property: • Bring up 9 students and give them each one object. How many total objects do they have? • Add 3 + 1 and then multiply 3 x 1 and compare what happens between the two. Zero Property: • Add 3 + 0 and then multiply 3 x 0 and compare what happens between the two. • Have 9 students come up. Tell them they will each get zero cookies. How many cookies will they have? (9 x 0 = 0)	Interactive Multiplication Chart http://www.mathsisfun.com/tables.html http://www.aaamath.com/pro74b-propertiesmult.html

Assessment Tasks Used	
Skill-Based Task: 5 x 8 = 40; so 8 x 5 = (8 x 3) x 2 = ___ x (2 x 3) 8 x 6 = (8 x ___) + (8 x ___) 6 x 0 = _____ 8 x 1 = _____	**Problem Task:** Robin says I can find 8 x 7 by multiplying 4 x 7 and then doubling it. Is she correct? How do you know? Use pictures words, and numbers to explain your reasoning. Paul put his toy cars into 8 rows of 2. What is another way he could group his cars? Show Paul's group and then show your grouping. Emily and Sam multiply 5 x 4 x 3. Emily multiplies 5 x 4 first and then multiplies by 3. Sam multiplies 4 x 3 first and then multiplies by 5. Which student is correct? Use pictures, words and numbers to explain your thinking.

Core Content

Cluster Title: Understand properties of multiplication and the relationship between multiplication and division.

Standard 6: Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand that multiplication and division are related. Therefore, one operation can help solve the other.
Students will understand unknown-factor problems and how to solve them.

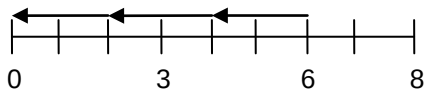
Procedural:

Students can use fact families and/or number bonds to help solve division equations.
Students can use related multiplication facts to solve for a missing factor in a division equation.
Students can find an unknown factor in a division problem.

Representational:

Students can use an array model to show related multiplication and division equations (e.g., 3 x 2 = 6; 2 x 3 = 6; 6 ÷ 2 = 3; 6 ÷ 3 = 2).
Students can model a division problem to show an unknown factor.
Students use a number line to represent missing factor problems.

6 ÷ △ = 2



Supports for Teachers

Critical Background Knowledge

Conceptual:

Students will understand that multiplication and division are inverse operations.
Students will understand commutative property for multiplication.
Students will understand the meaning of the equal sign.
Students will understand basic multiplication.
Students will understand that division is reducing a number (repeated subtraction).

Procedural:

Students can solve a multiplication equation.
Students can solve a division equation.

Representational:

Students can model multiplication and division in a variety of ways (arrays, groups/sets).

Academic Vocabulary and Notation

fact family/related facts, multiplication, division, inverse, ÷, /, commutative property, factor

Instructional Strategies Used	Resources Used
Have the students create an array to represent the related multiplication and division equations. Write the related fact families from the arrays. Have students use equal sharing to represent division (groups and sets). Have students create and solve story problems.	Long, Lynette. *Dazzling Division*. Wiley, 2000. Greenberg, Dan. *Mega-Funny Division Stories*. Teaching Resources, 2002. http://www.ixl.com/math/grade-3/relate-multiplication-and-division http://www.amblesideprimary.com/ambleweb/mentalmaths/dividermachine.html http://math.pppst.com/division.html http://www.wartgames.com/themes/math/division.html http://www.aaamath.com/div39j-divall.html (scroll down to practice)

Assessment Tasks Used	
Skill-Based Task: Create arrays with related multiplication and division facts. Find the missing number. Use the related multiplication fact to help you. 16 ÷ 2 = ____ 8 x 2 = 16 28 ÷ 7 = ____ 7 x 4 = 28 $15 \div 3 =$_____ ____ $\times 3 = 15$	**Problem Task:** Ana separates 42 kids into 7 teams for kickball. How many kids are on each team? Write the multiplication fact you use to help you divide. Explain your thinking.

Core Content

Cluster Title: Multiply and divide within 100.

Standard 7: Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one also knows 40 ÷ 5 = 8) or properties of operations. By the end of grade 3, know from memory all products of two one-digit numbers.

MASTERY Patterns of Reasoning:

Conceptual:

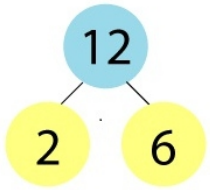
Students will understand the inverse relationship of multiplication and division.
Students will know from memory all products of two one-digit numbers.
Students will understand commutative and distributive properties.

Procedural:

Students can apply a strategy to solve multiplication and division equations.
Students can solve multiplication and division problems fluently (i.e., flexibly, accurately, efficiently, and appropriately).
Students can show how a problem was solved using commutative/distributive properties.

Representational:

Students can illustrate multiplication number bonds as a means of developing fluency.



Supports for Teachers

Critical Background Knowledge

Conceptual:

Students will know multiplication fact families.
Students will understand the commutative, associative, and distributive properties of multiplication.
Students will understand the identity and zero properties of multiplication.

Procedural:

Students can solve multiplication and division equations using a variety of strategies.

Representational:

Students can show how a problem was solved using commutative/distributive properties.

Academic Vocabulary and Notation	
product, factor, dividend, divisor, quotient, fact family, related facts, x, ÷, commutative property, distributive property	
Instructional Strategies Used	**Resources Used**
Note: Fluency in mathematics means solving problems flexibly, accurately, efficiently, and appropriately. Be certain that students have conceptual understanding of multiplication before they begin to commit the products of two one-digit numbers to memory. • Doubles (2 x 2 = 2 + 2) • Double and double again (4 x 2 = (2 x 2) x 2) • Halve, then double (6 x 8 = (3 x 8) + (3 x 8)) • Doubles plus one more set (3 x 7 = (2 x 7) + 7) • Add one more set (6 x 7 = (5 x 7) + 7) • Decomposing into known facts (i.e., use facts you know to solve the ones you don't) • Halves (12 ÷ 2 = 6) • Doubles and halving (36 ÷ 4 = 72 ÷ 2 = 144 ÷ 1)—you double the dividend and halve the divisor to make a simpler problem • Multiplying by zero and one • Patterns in 9's • Fact families • Number bonds	Tang, Gregory. *Best of Times.* Scholastic Press, 2002. Pinczes, Elinor J. *My Full Moon Is Square.* Scholastic Press, 2002. Literature resources: http://www.graniteschools.org/depart/teachinglearning/curriculuminstruction/math/elementarymathematics/Pages/default.aspx Multiplication chart

Domain: Operations and Algebraic Thinking Grade: 3

Assessment Tasks Used	
Skill-Based Task: 4 x 6 = ____ 7 x 9 = ____ 9 x 10= ____ 56 ÷ 8 = _____ 72 ÷ 9 = _____	**Problem Task:** There are 5 tables in the lunchroom. Six students sit at each table. How many students are in the lunchroom? Mari has 48 crayons. She knows that 8 crayons can fit in a box. How many boxes will she need?

Code: 3OA7

Core Content

Cluster Title: Solve problems involving the four operations, and identify and explain patterns in arithmetic.
Standard 8: Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. Note: This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (i.e., order of operations).
MASTERY Patterns of Reasoning:
Conceptual: Students will understand that a symbol/letter can be used to represent an unknown number. Students will understand how to use mental math, rounding, and estimation Students will understand the definition of reasonableness. Students will know the basic order of operations (i.e., multiplication and division come before addition and subtraction). **Procedural:** Students can solve two-step word problems using addition, subtraction, multiplication and division. Students can solve problems where a symbol/letter represents an unknown number. Students can solve problems using mental math, rounding, and estimation. Students can justify the reasonableness of their answer. Students can write and solve a two-step word problem. **Representational:** Students can draw pictures to represent two-step word problems. Students can use manipulatives involving problems where a symbol/letter represents an unknown number.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will know what whole numbers are. Students will know how to solve one-step word problems using addition, subtraction, multiplication and division. Students will know how to fluently add and subtract within 1,000. Students will know how to fluently multiply and divide within 100. Students will know how to add up to four two-digit numbers. Students will know how to relate the word problem to an equation. **Procedural:** Students can solve addition and subtraction equations within 1,000 flexibly, accurately, efficiently, and appropriately. Students can solve multiplication and division problems within 100 flexibly, accurately, efficiently, and appropriately. Students can solve one-step word problems using operations. **Representational:** Students can model using concrete models or drawings to add and subtract within 1,000, and relate the strategy to a written method. Students can model how addition and subtraction strategies work, using place value and the properties of operations. Students can model how multiplication and division strategies work, using place value and the properties of operations.
Academic Vocabulary and Notation
Addends, sum, difference, round, estimate, equation, difference, multiplication, factors, product, array, multiples, division, divisor, dividend, quotient, reasonableness, symbol, $\times$, $\div$, $/$,interchangeable

Instructional Strategies Used	Resources Used
Have the students write their own word problems that include more then one step and/or operation. Use the *Alexander, Who Used to Be Rich…* book to set up for two-step problems such as adding each amount spent to get a total and then subtracting that from the dollar. Use the *Pigs Will Be Pigs* book to set up for two-step problems. Practice estimation strategies: • Using benchmark numbers • Front-end estimation • Rounding • Talk about whether the answer is reasonable. How do you know? Multiple problem solving strategies • Draw diagrams • Table • Use physical objects • Pictures • Number lines • Models • Use the IXL link for instruction with whole group participation.	Giganti, Paul. *Each Orange Had 8 Slices.* Greenwillow Books, 1999. Viorst, Judith. *Alexander, Who Used to Be Rich Last Monday.* Atheneum Books, 1987. Axelrod, Amy. *Pigs Will Be Pigs.* Aladdin, 1997. http://www.mathplayground.com/gsmbegin.html http://teacher.scholastic.com/maven/zoo/index.htm http://teacher.scholastic.com/maven/adder/index.htm (Good for addition and subtraction review) http://www.mathplayground.com/NewThinkingBlocks/thinking_blocks_multiplication_division.html http://www.mathplayground.com/NewThinkingBlocks/thinking_blocks_addition_subtraction.html http://math.pppst.com/wordproblems.html (the first PowerPoint teaches how to read word problems, and is rewriteable) http://www.ixl.com/math/grade-3/multi-step-word-problems

Assessment Tasks Used	
Skill-Based Task: Solve: 2 + 3 x 4 Solve: 9 ÷ 3 + 6 Solve: 15 – 5 × q = 0 Solve: 35 ÷ 5 + r = 7	**Problem Task:** Joe had 4 packages of bubblegum. Each package had 8 sticks. He shared them with his friends. At the end of the day, he had 10 pieces left. How many sticks were chewed? Model the problem and explain how you know the answer is reasonable. Tami blew up 24 red balloons and 15 blue balloons for her party. Seven balloons popped. How many balloons were left? Juanita did 16 math problems on Monday and 3 pages of math problems on Tuesday. Each page had an equal number of problems. She did 40 problems in all. Write an equation representing this problem using a letter for the unknown quantity. How many problems were on each page? How do you know your answer is reasonable?

Core Content

Cluster Title: Solve problems involving the four operations, and identify and explain patterns in arithmetic.
Standard 9: Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.
MASTERY Patterns of Reasoning:
Conceptual: Students will recognize arithmetic patterns that can be found on a hundreds chart, a number line, an addition and a multiplication table. Students will recognize multiplication patterns that can be found on a hundreds chart and a multiplication table. Students will know that multiplication by an even number results in an even number. Students will know that multiplication of an odd number by another odd number results in an odd number. Students will know that multiplication of an odd number by an even number results in an odd number. Students will explain arithmetic patterns using properties of operations. **Procedural:** Find the skip counting patterns on a hundreds chart 2-12 Find the products of the commutative property on the multiplication chart. Find patterns on the multiplication chart for 0-12. **Representational**: Model addition and multiplication patterns with a number line. Model addition and multiplication patterns with the hundreds chart and the multiplication chart.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will know the difference between an even number and odd number. Students will know that an even number plus another even number equals an even number. Students will know that an even number plus another odd number equals an odd number.

Students will know that an odd number plus another odd number equals an even number.
Students will understand what a pattern is.
Students will know multiplication facts.

Procedural:

Students will be able to skip count.
Students can use a number line, hundred chart, addition chart, and multiplication chart.

Representational:

Students can highlight numbers on number lines, hundred chart, addition chart, and multiplication chart.

Academic Vocabulary and Notation

compare, digits, addends, sum, multiplication, multiples, factors, product, sequence, pattern, row, column, input-output table, commutative property

Instructional Strategies Used	Resources Used
Use input and output tables. Color the pattern on a multiplication and addition table. Start by giving students base ten blocks or cubes and have them build groups of tens. Discuss findings about what happens as they multiply a factor by ten. Then explore other skip counting patterns.	http://www.321know.com/patra10.htm#section2 http://pbskids.org/cyberchase/math-games/crack-hackers-safe/ http://www.funbrain.com/cracker/index.html http://math.pppst.com/multiplication.html (use the fourth PowerPoint) http://multiplication.com/classroom_games.htm http://www.mathplayground.com/functionmachine.html http://teams.lacoe.edu/documentation/classrooms/amy/algebra/3-4/activities/functionmachine/functionmachine3_4.html http://www.mathwire.com/games/algebragames.html Hutchins, Pat. *The Doorbell Rang.* Greenwillow, 1989. *School House Rock: Multiplication* [DVD]. Disney, 2007.

Assessment Tasks Used

Skill-Based Task:

Input	Output
5	12
9	16
13	20
15	?

What is the missing number?

Input	Output
5	30
7	42
8	48
9	?

What is the missing number?

Problem Task:

If Bailey receives $5.00 a week for mowing a lawn, how much money does he have after 7 weeks? Show the pattern. Explain your thinking using pictures, words, or numbers.