Core Content

Cluster Title: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Standard 4: Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand points, lines, line segments, rays, and angles.
Students will understand right, acute, and obtuse angles.
Students will understand perpendicular and parallel lines.
Students will understand how all of these fit into two-dimensional figures.

Procedural:

Students can identify points, lines, line segments, and rays on a two-dimensional plane (paper).
Students can identify angles (right, acute, and obtuse).
Students can identify different types of lines (perpendicular and parallel).

Representational:

Students can use models, manipulatives, and pictures to create points, lines, line segments, rays and angles.
Students can draw and label points, lines, line segments and rays in two-dimensional figures.
Students can draw and label angles (right, acute, obtuse) in two-dimensional figures.
Students can draw and label different types of lines (perpendicular and parallel) in two-dimensional figures.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Students will understand different types of two-dimensional figures (e.g., rhombus, square, rectangle).
Students will understand characteristics and parts of angles.

Procedural:

Students can recognize two-dimensional figures (e.g., rhombus, square, rectangle).
Students can identify number of angles in two-dimensional figure models.
Students can identify number of sides in two-dimensional figure models.

Representational:
Draw two-dimensional figures (e.g., rhombus, square, rectangle).

Academic Vocabulary and Notation

point, line, line segment, ray, angle, obtuse, acute, right, parallel, perpendicular, two-dimensional, figure, ∥, ⊥, <

Instructional Strategies Used	**Resources Used**
Use a version of “Simon Says” with a symbol for each geometric term to play with the students. Start by making a line symbol (both arms extended outward, hands open), point symbol (a fist), and ray symbol (a fist with left hand and right arm extend out, hand open). Practice symbols with students before playing. Call the different terms and students show the symbols. **Teacher:** Simon says *line.* [Students show symbol.] **Teacher:** Simon says *ray.* [Students show symbol.] **Teacher:** *Point.* [Students should stay with the ray symbol.] Continue playing and adding in new symbols for all of the terms. Build geometric shapes using straws and pipe cleaners.	Free geometry software: http://nrich.maths.org/1335 http://nrich.maths.org/public/viewer.php?obj_id=312&part= http://www.geogebra.org/cms/

Assessment Tasks Used

Skill-Based Task: Have students draw and identify each of the geometric terms.	**Problem Task:** Have students locate examples of each geometric term in the classroom, take pictures of the examples outside, or cut out pictures of examples from a magazine. Create a geometric collage using what students found.

Core Content

Cluster Title: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Standard 2: Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand how to classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines.
Students will understand how to classify two-dimensional figures based on the presence or absence of angles of a specified size.
Students will understand how right triangles are different from other triangles.
Students will understand why right triangles fit into their own category.

Procedural:

Students can identify presence or absence of parallel and perpendicular lines.
Students can identify presence or absence of acute and obtuse angles.
Students can identify presence of right angles separate from acute and obtuse angles.
Students can identify and classify right triangles.

Representational:

Students can use models, manipulatives and pictures of two-dimensional figures to identify presence or absence of parallel and perpendicular lines, and acute, obtuse, and right angles.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Students will understand the definition of a two-dimensional figure.
Students will understand the definitions of parallel and perpendicular lines.
Students will understand the difference between parallel and perpendicular lines.

Students will understand the definitions of lines and angles.
Students will understand the definitions of a triangle.
Students will understand the difference between acute, obtuse, and right angles.

Procedural:

Students can identify parallel and perpendicular lines in two-dimensional figures.
Students can identify acute, obtuse, and right angles in two-dimensional figures.
Students can identify and classify two-dimensional figures.
Students can identify right triangles.

Representational:

Students can use models, objects, and pictures to represent geometric terms.

Academic Vocabulary and Notation

classify, right triangle, category, parallel line, perpendicular line, acute angle, obtuse angle, right angle, presence, absence, two-dimensional figure

Instructional Strategies Used	Resources Used
Give students several pictures of various two-dimensional figures. Practice finding figures by giving students criteria for classifications (i.e., parallel lines, perpendicular lines, acute angles, obtuse angles, and right angles). Finally, have the students identify the right triangles and classify these into a separate group.	http://www.uen.org/Lessonplan/preview.cgi?LPid=11235 http://illuminations.nctm.org/LessonDetail.aspx?ID=L270

Assessment Tasks Used

Skill-Based Task:	Problem Task:
Give students different two-dimensional figures and have them classify them and justify their classification.	Create an art project using all of the new geometric classification terms.

Domain: Geometry Grade: 4

Core Content

Cluster Title: Draw and Identify lies and angles, and classify shapes by properties of their lines and angles.
Standard 3: Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.
MASTERY Patterns of Reasoning:
Conceptual: Students will understand the definition of symmetry. Students will understand the definition of a line of symmetry. Students will understand how to find symmetry in two-dimensional figures. Students will understand how to create a line of symmetry. Students will understand that lines of symmetry create congruent figures. **Procedural:** Students can identify two-dimensional figures that have a line of symmetry. Students can draw lines of symmetry in two-dimensional figures. **Representational:** Students can use models and pictures to identify symmetrical figures. Students can use models and pictures to draw lines of symmetry.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will know that two-dimensional figures have length and width. Students will understand what a line is. **Procedural:** Students can identify two-dimensional figures. Students can identify that figures are congruent. Students can draw lines.

Code: 4.G.3

Representational: Students can use models and pictures to identify two-dimensional figures and lines.	
Academic Vocabulary and Notation	
symmetry, two-dimensional, matching parts, symmetrical, line, line of symmetry, congruent	
Instructional Strategies Used	**Resources Used**
Give students a symmetrical two-dimensional figure. Ask students to fold the figure in half and draw a line on the fold. Then ask students to identify whether both parts match. Define where the line of symmetry is on the figure. Define what makes the figure symmetrical. Try the same thing with a two-dimensional figure that is not symmetrical.	http://www.linkslearning.org/Kids/1_Math/2_Illustrated_Lessons/4_Line_Symmetry/index.html http://www.innovationslearning.co.uk/subjects/maths/activities/year3/symmetry/shape_game.asp
Assessment Tasks Used	
Skill-Based Task: Identify symmetrical figures. Draw lines of symmetry on figures given to them.	**Problem Task:** Use pattern blocks, tangrams, or pentaminoes to create a figure that has at least one line of symmetry. Students will draw a two-dimensional replica of their figure showing the lines of symmetry. http://nrich.maths.org/1840

Core Content

Cluster Title: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Standard 1: Know relative sizes of measurement units within one system of units, including km, m, cm; g, kg; lb, oz; L, mL; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1,12), (2,24), (3,36), …

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand the difference between standard and metric units of measurement.
Students will understand relative size within one system of units (e.g., km, m, cm; ton, lb, oz, etc.).
Students will understand equivalent measurements in larger and smaller units within one system.

Procedural:

Students can identify the different measurement systems (standard and metric).
Students can identify the units of measurement within each system (standard and metric).
Students can identify the equivalent units of measurement within one system (e.g., 3 ft = 1 yd).
Students can record measurement equivalents within one system in a two-column table.
Students can convert between units of measurement within one system (e.g., 300 cm = 3 m).

Representational:

Students can use models, manipulatives, and pictures to compare relative sizes within one system.
Students can use models, manipulatives, and pictures to show equivalent measurements within one system.
Students can use models, manipulatives, and pictures to convert between units of measurement within one system.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Students will know and understand the basic units for each system.
Students will understand which measurement unit would be most reasonable to use to measure length, capacity, weight and time (e.g., grams versus kilograms, pounds versus ounces, etc.).
Students will understand which unit to use to measure length, capacity, weight and time (e.g., measure weight with grams or kilograms, not liters).
Students will understand how to use measurement tools (scale, clock, ruler, etc.).
Students will understand part-to-whole and whole-to-part relationships (e.g., 1 inch = 1/12 foot; 1 foot = 12 inches).
Students will understand how to generate and analyze patterns (Refer to 4.OA.5).

Procedural:

Students can measure different items using both systems of measurement (standard and metric).
Students can estimate the measurement using length, capacity, weight and time.
Students can multiply and divide numbers.

Representational:

Students can use pictures to show the measurement of different items.
Students can use measurement tools for length, capacity, weight and time.

Academic Vocabulary and Notation

UNITS OF MEASUREMENT		
Components	Standard	Metric
Length	in., ft, yd, mi.	mm, cm, dm, m, km
Capacity	fl. oz., c, pt, qt, gal.	mL, L
Weight	oz., lb	g, kg
Time	sec., min., hr., day, week, month, year	

Instructional Strategies Used	Resources Used
1. Using different measurement tools provide time for students to measure items using various units within one system (e.g., the door is 2 yards tall, but 6 feet as well). Have students compare and record their measurements on a two-column table. This allows students to explore and discover relative size of measurements. a. Have students use both the standard and customary units of measure for the same items and create charts for both. 2. Have the students measure the length of the room with one-inch tiles, one foot rulers and with yardsticks. Students should notice that it takes fewer yardsticks to measure the room than rulers and tiles. a. Have the students complete the same activity using centimeters and meters.	Measurement tools including, but not limited to, rulers, balances, cups, weights, beakers and clocks: http://www.harcourtschool.com/activity/con_math/g04c24.html Various activities relating to measurement: http://www.jmathpage.com/JIMSMeasurementpage.html

Assessment Tasks Used	
Skill-Based Task: 5000 mL = _____ L _____ cm = 5 m 6 cups = _____ pints _____ quarts = 2 gal	**Problem Task:** Ask students to represent real-life situations (such as the length of an animal, the weight of a rock, etc.) in different units of measurement (e.g., the rock weighs 2 kg or 2,000 g).

Core Content

Cluster Title: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Standard 2: Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand how to use the four operations to solve word problems using measurement units.
Students will understand how simple fractions and decimals relate to measurement.
Students will understand how to use the simple fractions and decimals to solve word problems using measurement units and money.
Students will understand how measurement quantities fit into a number line.
Students will understand how to convert between larger and smaller units of measurement.
Students will understand how to use measurement diagrams.

Procedural:

Students can identify correct placement of measurement on the number line.
Students can convert between units of measurement within one system.
Students can use the four operations to solve word problems requiring measurement.

Representational:

Students can use models, pictures, manipulatives, number lines and diagrams to solve problems.
Students can use models, pictures, manipulatives, number lines and diagrams to show measurement conversions.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand the different types of measurement (systems and units). Students will understand larger units versus smaller units. Students will understand the use of a number line. Students will understand how decimals and fractions represent measurements. Students will understand when to use each operation. **Procedural:** Students can identify the appropriate units of measure for the given item (e.g., small spider versus tractor) and what is being measured (length, mass, capacity). Students can record data on two-column table. Students can record numbers on a number line. Students can convert between units of measurements within one system. Students can solve equations using the four operations. **Representational:** Students can use models to show conversions. Students can use diagrams to record measurements.
Academic Vocabulary and Notation
distance, time, interval, volume, mass, number line, scale, diagram

Instructional Strategies Used	Resources Used
Have students identify the operation(s) required in word problems by giving students five problems involving measurement. Read the problems with students to determine	http://www.thatquiz.org/tq/previewtest?A/O/Z/P/83711291417153 http://www.helpingwithmath.com/by_subject/word_problems/wor_measurement01_4md2.htm

which ones require conversions. Using the conversion problems, students will identify operation(s) needed to solve each problem.	
Assessment Tasks Used	
Skill-Based Task: Give students word problems to solve. Require students to show work, including any diagrams.	**Problem Task:** Gina decides to figure out how long her class spends actually studying and learning in one day. She arrives at school at 8:30 a.m. The class goes to recess from 9:30 a.m. to 9:45 a.m., and then works in literature circles and writing until 11:30 a.m., when the class goes to lunch. Students are at lunch for 40 minutes. After they return to class, they work on math until their ten-minute afternoon break at 1:30 p.m. After break, they work on science and social studies until school dismisses at 3:10 p.m. How much time are the students in school? How much time are they learning and studying? Give your answer in hours and minutes.

Core Content

Cluster Title: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Standard 3: Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length by viewing the area formula as a multiplication equation with an unknown factor.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand the area model of multiplication in relation to rectangular arrays.
Students will understand the difference between area and perimeter and when to solve for each.

Procedural:

Students can derive formulas for perimeter and area.
Students can identify the use of perimeter and area formulas.
Students can find perimeter and area using addition or multiplication.

Representational:

Students can use models or pictures to show perimeter and area.
Students can use models or pictures to derive the formulas for perimeter and area.
Students can use diagrams or models to solve problems involving perimeter and area.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Students will understand the definition of rectangle.
Students will understand the definition of perimeter.
Students will understand how to use addition to find perimeter.
Students will understand the square unit as the unit of measurement for area.
Students will understand the definition of area.

Students will understand how to use multiplication to find area.
Students will understand that a division problem can be set up using a variable for an unknown factor (e.g., the area of a rectangle is 20 square feet. One side of the rectangle measures 4 feet. What is the length of the adjacent side? 4 ft. × $\square$ = 20 ft.^{2}).

Procedural:

Students can use addition and multiplication facts in relationship to perimeter and area.
Students can add the sides of a rectangle to find the perimeter.
Students can multiply with fluency basic facts 0–9.

Representational:

Students can use manipulatives and pictures to show perimeter.
Students can use manipulatives and array models to find area.

Academic Vocabulary and Notation

rectangle, perimeter, area, array, unit, square units, rectangular perimeter formula (P = 2l + 2w), rectangular area formula (A = l x w)

Instructional Strategies Used	Resources Used
Give students measurement word problems. Read through the problems with students and identify when they need to solve for perimeter or area. Look for word clues. Solve the problems using diagrams.	http://nrich.maths.org/6923 http://nrich.maths.org/2663

Assessment Tasks Used

Skill-Based Task:	Problem Task:
The perimeter of a train ticket is 44 centimeters. The ticket is 14 centimeters long. How tall is it?	A bakery is 13 yards wide and 23 yards long. The baker wants to put a wooden shelf around the inside of the bakery. The wooden shelving costs $4.00 per yard. How much will the baker's shelf cost?

Domain: Measurement and Data Grade: 4th

Core Content

Cluster Title: Represent and interpret data.
Standard 4: Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.
MASTERY Patterns of Reasoning:
Conceptual: Students will understand the construction and interpretation of a line plot. Students will understand how to interpret data to answer a word problem. **Procedural:** Students can construct a line plot using fractions as a reference. Students can decompose a line plot to solve simple fraction problems using addition or subtraction. Students can create a line plot for a given data set that utilizes fractions. **Representational:** Students can use models, illustrations, algorithms, and/or writing to construct and decompose a line plot to calculate the answer.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand how to read a line plot. Students will basic understanding of unit fractions. Students will understand how to add and subtract fractions. Students will understand that fractions represent a number greater than 0 but less than 1 and are located between any two whole numbers on a number line. **Procedural:** Students can place unit fractions on a number line.

Code: 4.MD.4

Students can read data from a line plot.
Students can identify equivalent fractions.

Representational:

Students can represent fractions on a number line.
Students can draw pictures, diagrams, or models to represent unit fractions derived from the same size whole (e.g. $\frac{1}{8}, \frac{1}{6}, \frac{1}{4}, \text{etc.}$).

Academic Vocabulary and Notation

line plot, fraction, measurement, data, data set, unit

Instructional Strategies Used	**Resources Used**
Give students a set of data to place on a line plot and interpret the data. For example, pass out 3 Cuisinaire rods to each student. Set the value for each color of rods—pink is $\frac{1}{2}$, green is $\frac{1}{3}$, and blue is $\frac{1}{4}$. Have students offer their data set while the teacher plots each data point with an "x." When finished, use the classroom graph to discuss how to gather data, plot data, and interpret data. Ask questions that will require students to complete addition and subtraction operations using fractions. How many $\frac{1}{2}s$ are shown? What is the difference in length between the pink rod and the green rod? What is the total length of all the green rods?	Interactive Fraction Games: http://www.uen.org/3-6interactives/math.shtml#fractions
Assessment Tasks Used	
Skill-Based Task: Kwon and Miguel have an insect collection. They have measured the lengths of all their insects. Their data shows that 4 insects are 1/8 inch long, 6 are 1/4 inch long, 8 are	**Problem Task:** Give each student a bag containing a variety of pasta, with multiple examples of each. For the class, give the measurement of each type of past to use for graphing

1/2 inch long, 2 are 1/6 inch long, 1 is 1/12 inch long, and 5 are 1/3 inch long. Create a line plot that shows the data. How much longer is the longest insect from the shortest insect?	purposes (fettuccine is ¼ inch, orzo is $^{1}/_{8}$ inch, spaghetti is $^{1}/_{6}$ inch, etc). Ask the students to create a line plot that displays their set of data. Then ask the students to interpret their data set using a set of questions. What is the difference in length from the longest pasta to the shortest pasta? What is the total length of the shortest pasta?

Core Content

Cluster Title: Geometric measurement: understand concepts of angle and measure angles.

Standard 5: Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

a. An angle is measured with reference to a circle with its center at the common endpoints of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a one-degree angle, and can be used to measure angles.
b. An angle that turns through *n* one degree angles is said to have an angle measure of *n* degrees.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand the definition of an angle.
Students will understand the definition and components of a circle (i.e., point of origin, circular arc, interior, exterior).
Students will understand the fractional relationship of angles to circles.
Students will understand the definition of degree as pertaining to a circle.
Students will understand that degrees are one form of angle measurement.

Procedural:

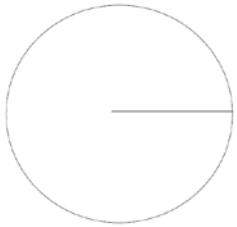
Students can identify the three components of an angle (two rays sharing a common endpoint).
Students can identify a circle as being comprised of 360 one-degree angles.
Students can identify an angle measurement of *n* as being comprised of *n**one-degree angle.

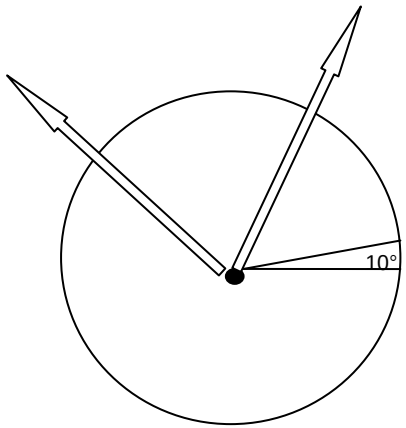
Representational:

Students can use models, manipulatives, and pictures to show various types of angles.
Students can use models, manipulatives, and pictures to show degree as the basic unit of measurement of a circle.
Students can use models, manipulatives, and pictures to show the relationship between an angle and a circle.
Students can use models, manipulatives, and pictures to show how an angle is measured in *n* degrees.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand the definition and components of ray. Students will understand the basic fractional parts of a circle (e.g., ½, ¼, $^3/_4$). Students will understand the four mathematical operations. **Procedural:** Students can identify the construction of a ray. Students can identify the fractional parts of a circle. **Representational:** Students can use models, manipulatives, and pictures to construct a ray. Students can use models, manipulatives, and pictures to represent fractional parts of a circle (e.g., ½, ¼, $^3/_4$).
Academic Vocabulary and Notation
intersection, circular arc, angle, vertex, point of origin, circle, ray, degree, circular interior, circular exterior, end point

Instructional Strategies Used	Resources Used
Students must have practice examining and labeling the components of a circle and recognizing angles formed when rays are drawn from the center of the circle. To make an angle finder: • Cut out two circles using a different color for one of the circles. Fit circles together using the slit. Slide the circle wheel around to form different fractions. 	Video showing angles growing by *n* degrees—good for teacher background ONLY! http://www.teachertube.com/viewVideo.php?title=angles_in_a_circle&video_id=231281 Math quiz using angles and degrees: http://www.ixl.com/math/grade/4 Finding degrees of angles: http://www.mathisfun.com/angles.html

Assessment Tasks Used	
Skill-Based Task: Students use the angle finder created in class. Teacher calls out angles formed by parts of the circle—½, ¼, $^3/_4$—and students create the angles with their finders. Teacher could extend the activity by showing other angles and asking students to form a similar degree angle with the finders.	**Problem Task:** Students will use their angle finder to locate and identify angles in real-life settings (i.e., classroom, playground, home). Students will trace the measurement of the angle finder to represent the measurement of the angle. Students will label the representation with rays, vertex, and interior arc. 

Core Content

Cluster Title: Geometric measurement: understand concepts of angle and measure angles.

Standard 6: Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand angles are measured in degrees.
Students will understand how to use a protractor to measure angles.
Students will understand that a protractor is an appropriate tool to measure angles.
Students will understand the benchmark angles (i.e., right = 90^0, straight = 180^0, 0^0 < acute < 90^0, 90^0 < obtuse <180^0).
Students will understand how comparing benchmark angles to other angles determines reasonable angle measurements.

Procedural:

Students can measure a variety of angles in whole number degrees using a protractor.

Representational:

Students can represent degrees with the following symbol (^{0}).
Students can draw a variety of angles with specified measures including benchmark angles.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Students will understand that an angle is comprised of two rays with a common endpoint.
Students will understand that angles are measured with reference to a circle at its center at the common endpoint.

Procedural:

Students can identify the parts of the circle and the angle formed by two rays.

Representational:

Students can draw rays and angles.

Academic Vocabulary and Notation

angle, degree, ray, degree symbol (^{0}), protractor

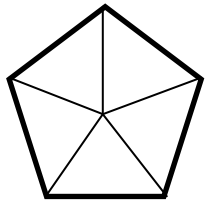
Instructional Strategies Used	Resources Used
Measure angles with a variety of degrees.	http://www.mathopenref.com/

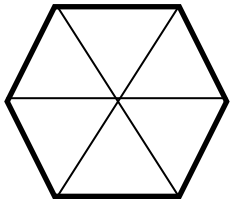
Assessment Tasks Used

Skill-Based Task:

Working with partners, students draw several angles and have partners measure them, and vice versa.

Measure the angles of a regular polygon—students understand that angles are equal.





Problem Task:

Students will locate angles in the classroom or outside. After writing down the angle measurement and item, the students will draw each of the angles found using a protractor.

Core Content

Cluster Title: Geometric Measurement: understand concepts of angle and measure angles.
Standard 7: Recognize angle measures as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.
MASTERY Patterns of Reasoning:
Conceptual: Students will understand the total angle measurement is the sum of its parts. Students will understand two non-overlapping angles can be added together to find the sum of both angles. **Procedural:** Students can identify and justify the operation required to find unknown angles from a diagram, real-life problem or a mathematical equation. Students can accurately measure angles with a protractor. **Representational:** Students can use models, manipulatives, diagrams and equations to demonstrate an understanding of additive angle measurement. Students can using models, manipulatives, and diagrams formulate equations with an unknown value to determine the total measure of the angle.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand benchmark angles: right angle = 90°, straight angle = 180°, 0° < acute < 90°, 90°< obtuse <180°. Students will understand part-to-whole relationships and how this relates to addition and subtraction (e.g., missing addend).

Procedural:

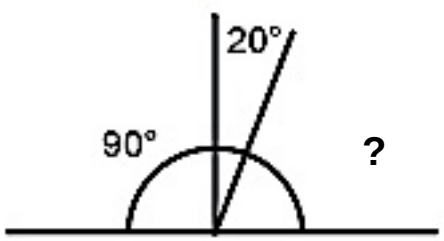
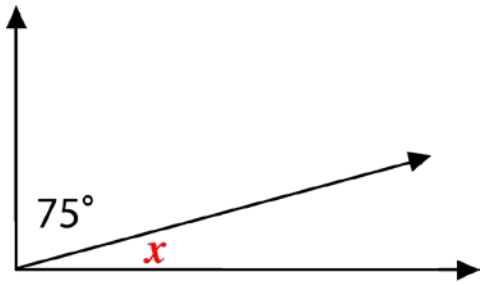
Students can identify benchmark angles.
Students can compare benchmark angles to angle sums for reference purposes.
Students can solve equations with an unknown value.

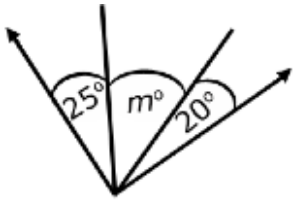
Representational:

Students can represent angles pictorially and express their measure in whole number degrees.

Academic Vocabulary and Notation

angle, protractor, sum, degree (and symbol °) acute angle, obtuse angle, straight angle, right angle, angle measure, perpendicular

Instructional Strategies Used	Resources
Using previous knowledge of angle measure, students will be able to view the diagrams at the right, and determine the missing angles. Using protractors, students will draw angles with missing measure and trade drawings with a partner. The partner will then solve for the missing angle.  180 - (90 + 20) = ? 180 - 110 = 70°	http://www.mathsisfun.com/geometry/complementary-angles.html http://www.mathsisfun.com/geometry/supplementary-angles.html http://www.khanacademy.org/video/complementary-and-supplementary-angles?playlist=Geometry Book: *Sir Cumference and the Great Knight of Angleland*, by Cindy Neuschwander 

Assessment Tasks Used	
Skill-Based Task: Ruby is standing on first base. Jasmine is standing on second base. What is the angle of measure from home plate between the two girls? What is the angle between third base and second base? Answer: 45°	**Problem Task:** Bella and Edward's teacher told them that the two outside rays in this drawing are perpendicular. She asked them to find the missing measure. What is it? Answer: 45° (again!) 

Core Content

Cluster Title: Generalize place value understanding for multi-digit whole numbers.

Standard 1: Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand the places of numbers and the value of each place.
Students will understand that each place increases by ten times the prior place moving to the left.
Students will understand that moving to the right is decreasing the value by tens using an inverse operation.
Students will understand why the procedures work based on place value and properties of operations.

Procedural:

Students can multiply and divide numbers by multiples of tens, hundreds, thousands, etc. to one million (e.g., 70 x 100 = 7,000; 5,000 x 10 = 50,000 and 700 ÷ 70 = 10; 50,000 ÷ 50 = 1,000).
Students can explain why multiplication and division work based on place value and properties of operations.

Representational:

Students can model place and value relationships showing how a digit in one place represents ten times what it represents in the place to its right using manipulatives (e.g., place value blocks, mats, discs, etc.).

Supports for Teachers

Critical Background Knowledge

Conceptual:

Students will understand the place of each digit and the value of its place.
Students will understand that multiplication and division are inverse operations.

Procedural:

Students can multiply and divide within 100.
Students can multiply by multiples of 10.

Representational:

Students can model place value using manipulatives (e.g., place value blocks, mats, discs, etc.).
Students can represent multiplication and division using arrays, objects, drawings, etc.
Students can show how the multiplication and division facts are related using models.

Academic Vocabulary and Notation

inverse operation, base ten numeral (instead of standard form), value, place, and place value, digit

Instructional Strategies Used	**Resources Used**
Have students investigate the patterns associated with the answers obtained with calculators to problems such as the following. (They should relate their findings to the patterns on the place value chart.) 7 x 10 7 x 100 70 ÷ 10 7 x 1,000 700 ÷ 10 7 x 10,000 7,000 ÷ 10	Calculators, papers, pencils, place value chart

Assessment Tasks Used

Skill-Based Task:	**Problem Task:**
800 ÷ 80 = 3,000 ÷ 100 = 500,000 ÷ 5,000 = 500 equals how many tens? 6,000 equals how many hundreds?	Given two numbers (e.g., 2,000 and 20) explain the relationship between the numbers, digits, places, and values.

Core Content

Cluster Title: Generalize place value understanding for multi-digit whole numbers.

Standard 2: Read and write multi-digit whole numbers using base-ten numerals, number names and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand how to write numbers in base-ten numerals, number names, and expanded form less than or equal to one million.
Students will understand how to compare numbers less than or equal to one million based on place and value.
Students will understand the comparison symbols.

Procedural:

Students can write numbers in base-ten numerals, number names, and expanded form using numbers less than or equal to one million.
Students can read numbers in base-ten numerals, number names, and expanded form using numbers less than or equal to one million.
Students can use symbols (>, <, =) to compare multi-digit numbers less than or equal to one million.

Representational:

Students can use manipulatives (e.g., place value blocks, mats, charts, etc.) to model the different forms of numbers.
Students can use manipulatives (e.g., place value blocks, mats, charts, etc.) to compare different numbers.
Students can use manipulatives (e.g., place value blocks, mats, charts, etc.) and symbols (>,<, =) to compare numbers.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand how to read, write, and compare numbers less than or equal to 1,000. **Procedural:** Students can read and write numbers in base-ten numerals, number names, and expanded form using numbers less than or equal to 1,000. Students can use place and value of digits to compare numbers less than 1,000. Students can use comparison symbols (>, <, =). **Representational:** Students can use manipulatives (e.g., place value blocks, mats, charts, etc.) to show the different forms of numbers. Students can use manipulatives (e.g., place value blocks, mats, charts, etc.) to compare different numbers. Students can use manipulatives (e.g., place value blocks, mats, charts, etc.) and symbols (>, <, =) to compare numbers.
Academic Vocabulary and Notation
base-ten numeral (formally known as standard form), number names (formally known as word form), expanded form, greater than (>), less than (<), equal to (=)

Instructional Strategies Used	Resources Used
Students build a number on their place value chart (mat, blocks, etc.). They practice reading the number to a partner, then write the number in different forms. Students build a number on their place value chart (mat, blocks, etc.). They compare their number with their partner's number using the symbols. They need to read and say the comparison correctly. Students will be given examples of numbers in base-ten numeral form. They must match it to the correct example in	Read and write numbers: http://www.uen.org/Lessonplan/preview?LPid=18917 Comparing numbers: http://www.aaastudy.com/cmp.htm#topic3 http://nrich.maths.org/public/search.php?search=compare

number name and expanded form. Students will read and write the number in both forms. For example: 1,234 = one thousand two hundred thirty-four = 1,000 + 200 + 30 + 4 In pairs, students use dice or number cards to generate their number. Students write both numbers down and use the correct symbol (>,<,=) to show the comparison.	
Assessment Tasks Used	
Skill-Based Task: Using ten-sided dice, roll and place digits on a place value mat to create a six digit number. Compare that number to a partner's using vocabulary such as *greater than*, *less than*, or *equal to*. Then record the results in many different forms.	**Problem Task:** Johnny read 5,076 pages in a year. His friend read 5,126 pages in the same year. Who read more, how do you know? Meg has a collection of 23,566 marbles. What two other ways can you write to show how many marbles she has?

Core Content

Cluster Title: Generalize place value understanding for multi-digit whole numbers.
Standard 3: Use place value understanding to round multi-digit whole numbers to any place.
MASTERY Patterns of Reasoning:
Conceptual: Students will understand how to round whole numbers less than or equal to one million. Students will understand that rounding can be applied to any place within a number. **Procedural:** Students can identify the place to which they are rounding. Students can identify the digit that affects how they round the number. Students can identify the rounding choices (the digit stays the same or rounds higher). Students can correctly write the rounded number in base-ten numeral form. **Representational:** Students can use visual models to illustrate place value in rounding e.g., number line, place value drawings, base ten blocks.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will know the place and value of whole numbers less than or equal to one million. Students will know how to round numbers to the nearest 10 and 100. **Procedural:** Students can read and write multi-digit whole numbers less than or equal to one million. Students can do the following for numbers less than 1,000: Identify the place to which they are rounding. Identify the digit that affects how they round the number. Identify the rounding choices (the digit stays the same or rounds higher).

Correctly write the rounded number in base-ten numeral form.

Representational:

Students can model numbers less than or equal to 1,000.

Academic Vocabulary and Notation

estimate, round, about, close to, almost, exact, benchmark, place value, base-ten

Instructional Strategies Used	**Resources Used**
Locate a target number on the number line. Determine the place value to which you are rounding. Identify which two benchmark numbers are on either side of the target number. Choose the benchmark number that is closer to the target number. Show populations of different cities. Ask students to estimate how many people are in each city.	Number lines: http://www.321know.com/grade4.htm#topic49

Assessment Tasks Used

Skill-Based Task:	**Problem Task:**
Round 345,782 to the nearest: 10 100 1,000 10,000 100,000	On Saturday, 45,672 people visited Hogle Zoo. Sam and Dee both estimated how many people were there. Sam said that about 45,000 people visited the zoo on Saturday, and Dee said that about 46,000 people visited the zoo on Saturday. Who rounded the number of visitors correctly? Explain your choice.

Core Content

Cluster Title: Use place value understanding and properties of operations to perform multi-digit arithmetic.
Standard 4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.
MASTERY Patterns of Reasoning:
Conceptual: Students will understand addition and subtraction of whole numbers less than or equal to one million. Students will understand what the standard algorithm for addition and subtraction is and why it is important to know how to use it fluently. **Procedural:** Students can use the standard algorithm to fluently add and subtract. **Representational:** Students can model the standard algorithm with place value representations. e.g., place value mat, place value drawing and base ten blocks.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand meanings of addition and subtraction and their inverse relationship. Students will understand place value of whole digit numbers. Students will recognize and understand how to use the standard algorithm for numbers less than or equal to 1,000. *Note: It is incorrect to state that, in a problem such as 367 – 289, you cannot subtract 9 from 7, or 89 from 69. It is mathematically possible to subtract a larger number from a smaller number. However, the result is a negative number. Students should be taught at this stage of development that, in the standard algorithm, you regroup if the subtrahend is smaller than the minuend.*

Procedural:

Students can add and subtract facts and whole numbers less than or equal to 1,000.

Representational:

Students can model addition and subtraction with manipulatives.

Academic Vocabulary and Notation

algorithm, sum, difference, total, addend

Instructional Strategies Used	**Resources Used**
• Have students use virtual manipulatives to add and subtract multi-digit whole numbers. • Use a place value mat to model addition and subtraction of multi-digit whole numbers.	http://nlvm.usu.edu/en/nav/frames_asid_154_g_2_t_1.html?from=category_g_2_t_1.html http://nlvm.usu.edu/en/nav/frames_asid_155_g_2_t_1.html?from=category_g_2_t_1.html

Assessment Tasks Used

Skill-Based Task:	**Problem Task:**
93,486 + 17,049 = 45,001 – 13,808 =	1. According to the U.S. Census Bureau, the population of Cache County in 2009 was 115,269. In 1990 it was 70,183. How many more people were there in Cache County in 2009? Justify your answer. 2. According to the U.S. Census Bureau, the population of Cache County in 2009 was 115,269 and the population of Washington County was 137,473. How many people were there in both counties in 2009? How do you know?

Core Content

Cluster Title: Use place value understanding and properties of operations to perform multi-digit arithmetic.
Standard 5: Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
MASTERY Patterns of Reasoning:
Conceptual: Students will understand the properties of operations. Students will understand how place value affects multiplication. Students will understand how place value affects decomposing numbers (breaking numbers apart) to multiply. **Procedural:** Students can use multiplication equations to solve a given problem. Students can use basic facts to solve equations. Students can decompose (break apart) a number to multiply. Students can multiply four-digit numbers by one-digit numbers. Students can multiply two-digit numbers by two-digit numbers. **Representational:** Students can model multiplication with manipulatives (e.g., place value blocks, mats, discs, etc.). Students can model multiplication with rectangular arrays and/or area models. Students can illustrate multiplication using graph paper, arrays and/or area models.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand the meaning of multiplication. Students will understand multiplying by tens. Students will understand the components in a multiplication equation (e.g., factors, products)

Procedural:

Students can understand and use basic multiplication facts fluently.

Representational:

Students can represent multiplication with different models such as arrays, equal size groups, area models and combinations.

Academic Vocabulary and Notation	
product, rectangular array, equation, area model, factors, properties of multiplication, rows, columns, partial products	
Instructional Strategies Used	**Resources Used**
Show the problem of 5 x 34. Put out 5 baskets for the 5 groups. Each group needs 34 objects (e.g., blocks, discs) to show that number. Then students can add or multiply the 5 groups of 4 ones, then the 5 groups of 3 tens. Add the partial products to solve 5 x 34. Use area model for multiplication of two digit times two digit numbers. (See Unit 4 under "Resources Used.")	baskets, discs, blocks, etc. http://www.prongo.com/math/multiplication.html Use Unit 3 at: http://eduplace.com/math/mthexp/g4/mathbkg/ Use Unit 4 at: http://eduplace.com/math/mthexp/g5/mathbkg/
Assessment Tasks Used	
Skill-Based Task: 8 x 8,256 87 x 36	**Problem Task:** Students will measure the length and width of the classroom and then multiply the dimensions to find the area. Repeat in the hall, the cafeteria, and the playground. Determine the proper unit of measurement to use in each case.

Core Content

Cluster Title: Use place value understanding and properties of operations to perform multi-digit arithmetic.

Standard 6: Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand that the remainder is separate from the quotient.
Students will understand the relationship between multiplication and division.
Students will understand how place value affects division.
Students will understand properties of operations.
Students will understand that division is either finding the number of items in a group when the number of groups is known (quotative or measurement model) or how many groups there are when the number in each group is known (partitive or sharing model). Note: It is not necessary for students to know the names of the models.

Example:

Quotative Model: Juanita has twelve apples. She wants to put 3 apples in a bag. How many bags will she need? 12 apples ÷ 3 apples per bag = 4 bags
Partitive Model: Juanita has 12 apples. She wants to put them in 3 bags. How many apples will go in each bag? 12 apples ÷ 3 bags = 4 apples per bag

Procedural:

Students can use multiplication and division facts fluently.
Students can decompose (break apart) numbers to divide.
Students can use repeated subtraction and sharing as division strategies.
Students can solve equations involving division.
Students can divide up to four-digit dividends and one-digit divisors.
Students can use multiple strategies to divide these numbers. Multiple algorithms are encouraged at this step.

Representational:

Students can model division with rectangular arrays, area models and manipulatives (e.g., place value blocks, mats, money, etc.).

Code: 4.NBT.6

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand meaning of multiplication and division. Students will understand the relationship between multiplication and division (e.g., inverse operations). Students will understand place value. Students will understand the properties of operations. Students will know how to use basic multiplication and division facts fluently. Students will know and be fluent in basic subtraction facts. **Procedural:** Students can fluently multiply and divide within 100. **Representational:** Students can model simple division with arrays.
Academic Vocabulary and Notation
divisor, dividend, quotient, remainder, array, area model, inverse operations, properties of operations, product, factor

Instructional Strategies Used	Resources Used
Model division by sharing money equally (partitive). For example, four students have earned a total of $56 in good behavior bucks. The teacher awards the group of students the money in five $10 bills and six $1 bills. The division problem is: 4) $ 56 Place the five $10 bills on the table and divide them evenly into 4 piles, resulting in one $10 in each pile with one $10 bill left over. The teacher is left with one $10 bill and six $1 bills for a total of $16. The $10 can be exchanged at the “bank” for ten $1 bills. The teacher now has sixteen $1 bills that can be divided evenly between the 4 students, with $4 for each. Each student receives a total of $14.	Place value manipulatives (e.g. money, base ten blocks, discs, etc.) and graph paper http://nrich.maths.org/6402 http://www.kidsnumbers.com/long-division.php Use Unit 4 at: http://eduplace.com/math/mthexp/g4/mathbkg/ http://nrich.maths.org/6402

Code: 4.NBT.6

Model division by subtracting groups of the divisor from the dividend (measurement or quotitive).

```
6) 158
   - 60     10
     98
   - 60     10
     38
   - 36     + 6
      2      26
```

Remainder Quotient

How many groups of 6 are in 158? (At least 10)
Use 10 as the first partial quotient. 10 x 6 = 60
Subtract 158 – 60 = 98

How many groups of 6 are in 98? (At least 10)
Use 10 as the second partial quotient. 10 x 6 = 60
Subtract 98 – 60 = 38

How many groups of 6 are in 38? (At least 6)
Use 6 as the third partial quotient. 6 x 6 = 36
Subtract 38 – 36 = 2

Add the partial quotients and record any remainders.

Assessment Tasks Used

Skill-Based Task:
487 ÷ 6 =
2,426 ÷ 2 =
342 ÷ 3 =

Problem Task:
Each year our school has a field day where students rotate between 7 different activities. If the physical education teacher divides the 434 students evenly between the activities, how many students will there be at each activity? Will there be any students left out? Justify your answer.

Core Content

Cluster Title: Extend understanding of fraction equivalence and ordering.

Standard 1: Explain why a fraction *a/b* is equivalent to a fraction *(n* x *a)/(n* x *b)* by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

MASTERY Patterns of Reasoning:

Note: Expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Conceptual:

Students will understand a fractional quantity can be subdivided into an infinite number of equal pieces while maintaining the original fractional quantity, e.g., 1/2 can be subdivided into 2/4, 4/8 and so on. Those subdivisions are called equivalent fractions.

Students will understand the identity property of multiplication and its relationship to fractions (1/1, 2/2, 3/3, 4/4,… n/n = 1)

Students will understand how the identity property of multiplication is employed to create equivalent fractions [(n*a)/(n*b) = na/nb]

Procedural:

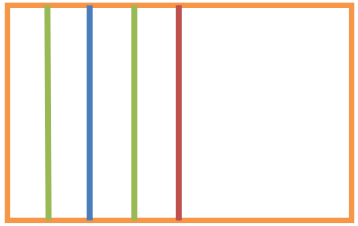
Students can identify differences in two equivalent fractions, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size.

Students can identify how the identity property of multiplication transforms a fraction into its equivalent fraction.

Representational:

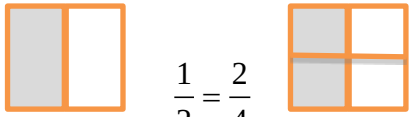
Students can represent equivalency of fractions pictorially (*a/b* is equivalent to *(n x a)/(n x b)*).

Students can construct models of equivalent fractions using manipulatives such as paper, color tiles, fraction bars, and fraction circles. The visual model, with subdivisions, might look like this:



The red line represents 1/2 of the box. When 1/2 is subdivided into two pieces with the blue line, the fraction becomes 2/4 because each piece represents 1/4 of the box. When each fourth is subdivided into two pieces by the green lines, the fraction becomes 4/8 because each piece represents 1/8 of the box.

Using a fraction model, students can subdivide both numerator and denominator by multiplying with the same factor (*a/b* is equivalent to (*n x a)/(n x b),* because both a and b were changed by the same factor *n)*, e.g. 1/2 = 2/4 because (2 x 1)/(2 x 2) = 2/4, or, pictorially:



$$\frac{1}{2} = \frac{2}{4}$$

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand fractions as equal parts of a whole. Students will understand that the larger the denominator, the smaller the size of the unit. Students will understand that a numerator tells the number of pieces being considered out of the whole. Students will understand that two fractions are equivalent if they represent the same amount or quantity. Students will understand the identity property of multiplication (*n* x 1 = *n*). **Procedural:** Students can visually identify equivalent fractions that are created from the same whole. **Representational:** Students can model equivalent fractions using area models, set models, picture models and number lines.
Academic Vocabulary and Notation
fraction, equivalent fraction, numerator, denominator, equivalent, number line model, area model, identity property of multiplication, *n* x 1 = *n*

Instructional Strategies Used	**Resources Used**
Using a sheet of paper, fold end to end and color one side. Continue folding paper and document changes in the fraction of the colored side (1/2, 2/4, 4/8, 8/16) as you proceed to fold	

the paper. Using the area model, change a given fraction into an equivalent fraction by drawing additional vertical and/or horizontal lines that result in equal parts of the whole. Using paper strips (3” x 18” construction paper), subdivide by successive folding to create several equivalent fractions. Label the fractions. Use fraction tiles and circles to create equivalent fractions. Use number line fraction bars to compare fractions.	*Elementary and Middle School Mathematics: Teaching Developmentally* by John A. Van de Walle, p. 304 Virtual manipulatives (National Library of Virtual Manipulatives, grades 3-5; Number and operations; Number line bar): http://nlvm.usu.edu/en/nav/frames_asid_265_g_2_t_1.html?open=activities&from=category_g_2_t_1.html
Assessment Tasks Used	
Skill-Based Task: Look at the model. Name three equivalent fractions for the part that is shaded. 	**Problem Task:** My mom left ½ of a cake on the counter. The doorbell rang and one of my friends came over. If we cut what’s left into equal parts, what fraction of the whole cake did we each eat? If 3 of my friends came over and we cut ½ cake that’s left into equal parts, what fraction of the whole cake did we each eat? (Ask the child to extend this reasoning as far as he/she is able.)

Core Content

Cluster Title: Extend understanding of fraction equivalence and ordering.

Standard 2: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

MASTERY Patterns of Reasoning:

Note: Expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Conceptual:

Students will understand that fractions represent a single quantity that can be compared.
Students will understand benchmark fractions (1/4, 1/3, 1/2, 2/3, 3/4).
Students will understand that fractions can be compared by attending to either numerators, denominators, or benchmark fractions.

Procedural:

Students can identify how fractional pairs can be changed to have equivalent denominators to determine >, =, <.
Students can identify how fractional pairs can be changed to have equivalent numerators to determine >, =, <.
Students can use benchmark fractions to determine >, =, < of various fraction pairs.

Representational:

Students can change fractional pairs using area model, number lines, set models, pattern blocks and other manipulatives or pictorial representations in order to compare two fractions.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Students will understand fractions as equal parts of a whole.
Students will understand that the larger the denominator, the smaller the size of the unit.
Students will understand that numerators name the portion out of the whole being considered.

Students will understand comparisons of fractions are only valid when they refer to the same size whole.
Students will understand equivalence in fractions.
Students will understand the symbols >, =, <.

Procedural:

Students can place benchmark fractions on a number line.
Students can identify simple equivalent fractions.

Representational:

Students can use models, manipulative and pictures to construct simple fractions and their equivalents (e.g.,1/2 = 2/4, 4/6 = 2/3).

Academic Vocabulary and Notation

benchmark fractions (thirds, halves, fourths), numerator, denominator, >, =, <, equivalent fractions

Instructional Strategies Used	Resources Used
Use pattern blocks to solve the following problems: 1. If a red trapezoid is one whole, which block shows 1/3? 2. If the blue rhombus is 1/3, which block shows one whole? 3. If the red trapezoid is one whole, which block shows 2/3? Use a variety of models such as fraction strips, Cuisenaire rods, number lines, etc. to represent and compare fractions of a common whole. Use individual shapes or objects to create sets that can be used to match fractional parts. Emphasize that comparisons cannot happen unless the fractions are part of the same whole.	National Library of Virtual Manipulatives—comparing fractions: http://nlvm.usu.edu/en/nav/frames_asid_159_g_2_t_1.html?from=category_g_2_t_1.html NCTM illuminations—comparing fractions, eggsactly with fractions lessons 4, 5, and 6. http://illuminations.nctm.org/LessonDetail.aspx?id=U112

Demonstrate finding common numerators using small fractions such as $\frac{3}{4}$ and $\frac{2}{3}$. Multiply the first fraction by $\frac{2}{2}$ and the second fraction by $\frac{3}{3}$ to get two new fractions $\frac{6}{8}$ and $\frac{6}{9}$ that can be compared. Ask the students to tell you why this procedure works.	
Assessment Tasks Used	
Skill-Based Task: Compare two fractions with different denominators and numerators to determine whether one is >, =, < to the other and explain how you arrived at your answer. Mario has 3/5 of a candy bar. Tisha has 2/3 of the same kind of candy bar. Who has more? Why?	**Problem Task:** I made a beaded necklace that was 2/6 blue, 3/5 green and the rest was white. Did the necklace have more blue or green beads? Explain your answer two ways (using a benchmark number, common numerators, or common denominators). Justify your answer.

Core Content

Cluster Title: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

Standard 3: Understand a fraction *a/b* with *a* > 1 as a sum of fractions 1/*b*.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

MASTERY Patterns of Reasoning:

Note: Expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Conceptual:

Students will understand that addition and subtraction of fractions is joining and separating parts referring to the same whole.
Students will understand that a fraction can be decomposed into a sum of fractions with the same denominator.
Students will understand that a mixed number is a whole number and a fraction, e.g., 2¼.
Students will understand the definition of unit fraction.

Procedural:

Students can use varied strategies, including algorithms, for adding and subtracting fractions with like denominators.
Students can describe various ways to decompose (break apart) a fraction.
Students can decompose a fraction and whole numbers into unit fractions.
Students can add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
Students can solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Representational:

Students can model the joining and separating of fractions with like denominators referring to the same whole.
Students can illustrate decompositions of fractions using various models (set, area, measurement, etc.).
Students can represent unit fractions with models.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Students will understand that addition and subtraction are the joining and separating of parts.
Students will understand that 1 = *n/n*.
Students will understand the definition of a unit (a unit is one of what we are counting).
Students will understand how to add and subtract fractions with like denominators.
Students will understand relationship of equivalent forms.

Procedural:

Students can place fractions on a number line.
Students can compare fractions.
Students can identify parts of the whole, numerator, and denominator.
Students can determine equivalent fractions.

Representational:

Students can understand that whole numbers can be expressed in a variety of ways (e.g., 8 = 5 + 3 = 4 + 4 = 2 + 6).
Students can represent fractions using area, set, number line, and measurement models.
Students can model addition and subtraction of fractions using various models (set, area, measurement, etc.).

Academic Vocabulary and Notation

decompose (decomposition), mixed numbers, fractional form, unit fraction
(Note: We have intentionally excluded the term “improper fraction” and instead use the term “fractional form.”)

Domain: Number and Operations - Fractions

Grade: 4th

Instructional Strategies Used	Resources Used
Illustrate adding and subtracting of fractions and mixed numbers using number lines, fraction strips, area models, set models, rulers, etc. Illustrate decomposing of fractions and mixed numbers with number lines, fraction strips, area models, set models, rulers, etc.	http://studyjams.scholastic.com/studyjams/jams/math/fractions/add-sub-common-denom.htm http://www.ncpublicschools.org/docs/acre/standards/support-tools/unpacking/math/4th.pdf

Assessment Tasks Used

Skill-Based Task:

Missing addend:

1/6 + ___ = 5/6

5/5 + ___ + 2/5 = 2 2/5

2/3 - 1/3 = ___

3/10 + 2/10 + 4/10 = ____

4/4 - 3/4 = ____

A cake recipe calls for 3/4 cup of milk, 1/4 cup of oil, and 2/4 cup of water. How much liquid was needed to make the cake?

3 2/5 + 4/5 = ____

5 1/4 - 2 3/4 = ____

Problem Task:

- Draw a picture to show why these equations are true, and explain your reasoning:

  7/8 = 4/8 + 3/8

  $2\frac{1}{4} = \frac{3}{4} + \frac{6}{4}$

- Sally said that 1/10 + 7/10 + 4/10 is the same as $1\frac{2}{10}$. Is she correct? Explain and use a model to illustrate your explanation. (This is for Standard 4.c.)

- Draw a model of the garden plot according to the data table below. The plot is divided into 15 sections. What fraction of the plot will be potatoes?

Crop	Number of sections
Corn	4
Peas	2
Strawberries	2
Tomatoes	3
Potatoes	The rest

Code: 4.NF.3

Core Content

Cluster Title: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

Standard 4: Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation $\frac{5}{4} = 5 \times \left(\frac{1}{4}\right)$.

b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times \left(\frac{2}{5}\right)$ as $6 \times \left(\frac{1}{5}\right)$, recognizing this product as $\frac{6}{5}$. (In general, $n \times \left(\frac{a}{b}\right) = \frac{(n \times a)}{b}$.)

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be five people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

MASTERY Patterns of Reasoning:

Note: Expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Conceptual:

Students will understand that a fraction (1/*b)* multiplied by a whole number (n) is *n/b* ($\frac{1}{7} \times 3 = \frac{3}{7}$).

Students will understand that $3 \times \frac{1}{7} = \frac{1}{7} + \frac{1}{7} + \frac{1}{7}$.

Students will understand that $a \times \left(\frac{b}{c}\right) = a \times b \times \left(\frac{1}{c}\right)$ and $a \times b \times \left(\frac{1}{c}\right) = \frac{a \times b}{c}$.

Procedural:

Students can form equivalent expressions by writing fractions as a multiple of a unit fraction (e.g., 4/3 = 4 x (1/3)).

Students can use the associative property of multiplication needed in the problem $a \times \left(\frac{b}{c}\right) = a \times b \times \left(\frac{1}{c}\right)$.

Students can solve word problems involving multiplication of a fraction by a whole number.

Representational:

Students can represent visually 5/4 = 5 x (1/4) using drawings, number lines, and other forms of modeling.

Students can represent visually problems involving $a \times \left(\frac{b}{c}\right) = a \times b \times \left(\frac{1}{c}\right)$.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Students will understand multiples and factors of whole numbers
Students will understand equivalent fractions
Students will understand the relationship between repeated addition and multiplication
Students will understand that n/n = 1
Students will understand the associative property of multiplication with whole numbers.

Procedural:

Students can add fractions referring to the same whole.
Students can decompose a whole number or a fraction into unit fractions.

Representational:

Students can model addition of fractions using various models (set, area, measurement, etc.).

Academic Vocabulary and Notation

unit fractions, multiple, variable, fractional form (5/4), mixed number, associative property

Instructional Strategies Used	Resources Used

Use number lines, fraction strips, area models, set models, rulers, etc. to show multiple addition of unit fractions. Connect this to multiplication of a fraction by a whole number. Emphasize through the use of concrete representations the idea expressed by $n \times \left(\frac{a}{b}\right) = \frac{(n \times a)}{b}$. The algebraic notation is not what students will be responsible for. Be aware that the notation from the standard skips a step: $n \times \left(\frac{a}{b}\right) = (n \times b) \times \frac{1}{b}$ and $(n \times a) \times \frac{1}{b} = \frac{(n \times a)}{b}$.	The web site http://www.ixl.com allows visitors up to 20 problems without purchasing. http://nlvm.usu.edu/en/nav/frames_asid_194_g_2_t_1.html?from=category_g_2_t_1.html http://www.homeschoolmath.net/teaching/f/multiplying_fractions_1.php
Assessment Tasks Used	
Skill-Based Task: Solve. Simplify your answer. 5 x 1/4 = $a \times \frac{1}{6} = \frac{7}{6}$ $6 \times \frac{2}{3} =$	**Problem Task:** Draw a picture to explain why $\frac{8}{5} = 8 \times \frac{1}{5}$. Kathy is having a party. She wants $\frac{2}{3}$ cups of trail mix per guest. She expects 6 guests. How much trail mix should Kathy prepare? Write an equation and justify your solution with a visual model.

Core Content

Cluster Title: Understand decimal notations for fractions, and compare decimal fractions.
Standard 5: Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.
MASTERY Patterns of Reasoning:
Note: Expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. However, addition and subtraction with unlike denominators is general is not a requirement at this grade level. **Conceptual:** Students will understand why a common denominator must be used in order to add fractions. Students will understand that when fractions are added they must refer to the same whole. **Procedural:** Students can add fractions with denominators of 10 and 100. **Representational:** Students can model addition of fractions with base-ten denominators (10, 100)

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand that a fraction refers to equal parts of a whole. Students will understand equivalent fractions. Students will understand that 100 is 10 times larger than 10 (e.g., 4 x 1 = 4; 4 x 10 = 40; 4 x 100 = 400). Students will understand a unit whole.

Procedural:

Students can add fractions with like denominators.

Representational:

Students can use a variety of models to represent addition of fractions with like denominators.

Academic Vocabulary and Notation

base-ten fractions, common denominator, equivalent fraction

Instructional Strategies Used	**Resources Used**
Model tenths and hundredths using dimes (1/10) and pennies (1/100), 10 x 10 grid, meter stick, etc. to illustrate the use of common denominators. Use a 10 x 10 grid to demonstrate addition of 7/10 + 6/100. Explain why the fractions can be added as they are when using the grid, but when writing out the equation they must have a common denominator.	*Elementary and Middle School Mathematics: Teaching Developmentally* by John A. Van de Walle, pp. 315-317

Assessment Tasks Used

Skill-Based Task:	**Problem Task:**
5/10 = *a*/100 70/100 = *b*/10 2/10 + 40/100 = *c*	Develop a model to describe the addition of 7/10 + 3/100.

Core Content

Cluster Title: Understand decimal notation for fractions, and compare decimal fractions.

Standard 6: Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand that decimals are special types of fractions that can be written with a denominator that is equal to 10 or 100 (at this grade level).
Students will understand that decimals represent parts of a whole.
Students will understand that decimals can be decomposed using expanded form (e.g., 0.32 = 0.3 + 0.02 = $\frac{3}{10} + \frac{2}{100}$).
Students will understand that the number of digits to the right of the decimal point indicates the number of zeros in the denominator, so that 0.70 = $\frac{70}{100}$.

Procedural:

Students can read decimals in word form and write decimals in word form.
Students can write decimals using digits and write the equivalent fraction.
Students can place decimals on a number line using background knowledge of fractions as a guide for placement.

Representational:

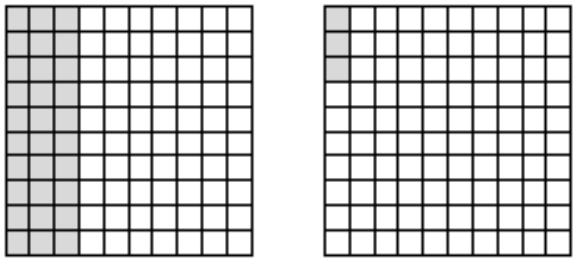
Students can use models such as 10 x 10 grids or base ten blocks to represent decimals.
Students can use place value charts to display decimals.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand that the value of each place is ten times the value of the place to its immediate right. Students will understand that fractions represent parts of a whole. Students will understand that fractions have many names in the form of equivalent fractions (e.g., ¾ = $^{6}/_{8}$ = $^{75}/_{100}$ or $\frac{3}{10} = \frac{30}{100}$). **Procedural:** Students can place unit fractions on a number line. Students can form equivalent fractions. Students can write whole numbers using digits, words, and expanded form. **Representational:** Students can use place value charts, illustrations or manipulatives to represent whole numbers.
Academic Vocabulary and Notation
tenths, hundredths, expanded from, decimal, equivalent fraction, decompose

Instructional Strategies Used	**Resources Used**
1. Link to students' background knowledge of place value where the value of each place is ten times the value of the place to its immediate right in order to extend the pattern to decimal place value. 2. When comparing two decimals, remind students that, as in comparing two fractions, the decimals need to refer to the same whole. Allow students to use visual models to compare two decimals. They can shade in a representation of each decimal on a 10 x 10 grid. The 10 x 10 grid is defined as one whole. The decimal must relate to the whole.	

For example:



0.3 0.03

3. Call out decimals to the class. Have students write the decimal using digits and words. Then have them convert the decimal to its equivalent fraction.
4. Create a number line using a length of rope or string. Make sure the length spans 3 to 5 feet. The line begins at zero and ends at 1. Write unit fractions on index cards and pass out to students. Ask students to place the unit fractions on the length of rope and explain how they know each is in the correct place. Discuss how decimals relate to fractions. Supply students with index cards with decimals or equivalent decimal fractions on them. Follow the same procedure, placing the cards on the length of rope with an explanation.

Assessment Tasks Used

Skill-Based Task:	**Problem Task:**
Give a length of adding machine tape to each student. The tape covers only the span of 0-1. Offer a variety of decimals and decimal fraction to place on their individual number line.	Supply the students with a decimal fraction such as 67/100. Have them write a letter using pictures, numbers, and words to a student from a different class, explaining how to convert to a decimal.

Core Content

Cluster Title: Understand decimal notation for fractions, and compare decimal fractions.

Standard 7: Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand that decimal values can only be compared when they refer to the same whole.
Students will understand that two decimal values can be written differently but still be equivalent (0.1 = 0.10).

Procedural:

Students can compare two decimal values with the symbols >, =, or <.

Representational:

Students can record the comparison of two decimal values by using a visual model, e.g., grid drawing, base ten blocks, pictures, tiles.
Students can justify the comparison of two decimal values by using a visual model, e.g., grid drawing, base ten blocks, pictures, tiles, number line model or meter stick.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Students will understand that decimals represent part of a whole and can be represented as fractions with denominators of 10 or 100 (see 4.NF.6.).
Students will understand comparison of whole numbers based on place value.

Procedural:

Students can write a decimal by looking at a visual model.

Students can use the symbols >, =, < to compare whole numbers and fractions.
Students can locate decimal values on a number line (see 4.NF.6).

Representational:

Students can use a model to visually represent fractions.
Students can place decimal values on a number line (see 4.NF.6).

Academic Vocabulary and Notation

decimal, tenth, hundredth, fraction, equivalent, >, =,<, 0.00

Instructional Strategies Used	**Resources Used**
1. Use grids or base ten blocks to match fraction representations to decimal representations, e.g. 1/10 = 0.1. Students benefit from hands-on practice more than from just observing the teacher work examples. 2. Emphasize the use of 0 when there are no whole units, e.g., 0.1. 3. Use a meter stick and number line to show decimal placement and relationships.	The following link could be used to compare, even though it is intended for addition and subtraction: http://nlvm.usu.edu/en/nav/frames_asid_264_g_2_t_1.html?from=category_g_2_t_1.html

Assessment Tasks Used

Skill-Based Task:	**Problem Task:**
Compare 1. 0.1 and 0.7 2. 1.2 and 2.1 3. 0.3 and 0.30 4. 0.5 and 0.05 5. 0.4 and 0.17	Ron says 0.17 is greater than 0.4. Kym says Ron is wrong. Who is right? Justify your answer with written explanation and a visual model.

Core Content

Cluster Title: Use the four operations with whole numbers to solve problems.
Standard 1: Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
MASTERY Patterns of Reasoning:
Conceptual: Students will recognize that any two factors and their product can be read as a comparison (e.g., 8 is the same as 4 sets of 2 or 2 sets of 4; 8 is 4 times as many as 2, or 2 times as many as 4). Students will recognize that multiplication represents groupings of numbers, and identify that the first factor in the equation represents the number of groups and the second factor represents how many within each group. Students should be able to make a comparison that 5 groups of 7 is the same as 7 groups of 5. Both products are 35. Students will understand that this representation illustrates the Commutative Property of Multiplication. **Procedural:** Students can interpret a multiplication equation. Solve for the product. **Representational:** Students can represent and solve multiplication equations through the use of models (e.g., arrays), illustrations, and writing. Students should be able to illustrate that 5 groups of 7 is the same product as 7 groups of 5.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will possess an understanding of number sense, such as decomposing numbers (35 is the same as 10 + 10 + 10 + 5 or 30 + 5 or …) and the reasonableness of answers. Students will be able to identify place value, including writing numbers in expanded form to recognize grouping by

place value.
Students will understand the base ten number operations of addition.

Procedural:
Students can skip count to compute.
Students can solve problems using repeated addition.
Students can fluently use basic multiplication facts 0-9.

Representational:
Students can model skip counting, repeated addition and basic multiplication facts using manipulatives, drawings, algorithms, and journaling.

Academic Vocabulary and Notation
multiply, compare, equation, represent, base ten, array, operation, factor, product, Commutative Property of Multiplication, expanded form, place value, decompose

Instructional Strategies Used	**Resources Used**
1. Concrete—Pictorial—Abstract Representation a. Have students use manipulatives to show what *x* groups of *y* looks like (e.g., 3 groups of 5; 4 groups of 7). Solve for the product. b. Build arrays for equations using the Commutative Property of Multiplication (e.g., with 5 groups of 7 and 7 groups of 5, 5 rows with 7 chairs in each row looks different from 7 rows with 5 chairs in each row, but there are 35 chairs in both sets). c. In math journals, have the students create a representation (e.g., array or picture illustrating the grouping) of the process that they completed using manipulatives. Write the two equations that are equivalent to the given representations, and solve for the product.	Number and Operations (Grades 3-5) Rectangle Multiplication—This virtual manipulative tool allows students to create arrays displaying different grouping (e.g., 3 groups of 6 and 6 groups of 3): **http://Nlvm.usu.edu/en/nav/topic_t_1.html** This resource includes a short lesson and a game for students to practice building arrays, writing equations, and solving for a product: **http://Mathstory.com/mathlessons/arrayrace.htm** Locate the activity “Groups of Dogs” at the following site for students to look at arrays using objects: **http://www.internet4classrooms.com/grade_level_help/solve_problems_math_fourth_4th_grade.htm**

2. In cooperative learning groups, have each group create either an equation or a model. Then have the students rotate through each group and construct an equivalent form of what is observed. 3. Fact families—reinforce the principles of related facts using triangle flash cards or Memory-type games.	
Assessment Tasks Used	
Skill-Based Task: Evaluate 8 x 6, 5 x 9, 7 x 3, etc. Write the equations for each multiplication problem using the Commutative Property of Multiplication (e.g., 8 x 6 = 6 x 8). Write and then solve the given equation using another method. Use a verbal statement to explain the chosen method.	**Problem Task:** Pedro has invited 8 of his friends to a summer party. He asked each of them to bring 7 pieces of candy. Create a representation of the total number of candy pieces the friends will share. Write the equation that represents the illustration you created. Solve for the answer.

Domain: Operations and Algebraic Thinking Grade: 4th

Core Content

Cluster Title: Use the four operations with whole numbers to solve problems.

Standard 2: Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand that a symbol represents an unknown variable in an equation.

Students will recognize division as the inverse of multiplication.

Students will distinguish when a word problem is asking to solve using a multiplicative or additive comparison.

Procedural:

Students can use concrete, pictorial, and abstract methods to demonstrate the relationship between multiplication and division.

Students can distinguish the difference between multiplicative comparison (e.g., Maria has 9 stickers. Joe has 3 times as many stickers as Maria. How many stickers do they have altogether?) and additive comparison (e.g., Sara has 5 picture books and 3 chapter books. How many more picture books than chapter books does she have?).

Solve a word problem by creating an equation using a variable or symbol to represent the unknown number.

For example, a pink rod is 4 inches long. A green rod is 3 times as long as the pink rod. How long is the green rod? Let *y* represent the green rod.

3 *x* (length of the pink rod) = *y*

3 *x* (4 inches) = *y*

y = 12 inches

Division example: The green rod is 12 inches long. It is 3 times as long as the pink rod. How long is the pink rod? Let n represent the pink rod.

3 x *n* = 12 inches

12 inches ÷ 3 = 4 inches

Code: 4.OA.2

Representational:

Students can represent multiplicative comparisons in word problems using models, illustrations, and/or writing.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Students will recognize that any two factors and their product can be read as a comparison (e.g., 8 is the same as 4 sets of 2 or 2 sets of 4; 8 is 4 times as many as 2, or 2 times as many as 4).
Students will recognize that multiplication represents grouping of quantities, and identify that the first factor in the equation represents the number of groups and the second factor represents how many within each group.
Students should be able to make a comparison that 5 groups of 7 is the same as 7 groups of 5. Both products are 35.
Students will understand how to solve a word problem using multiplication and division (e.g., I have 4 packs of bubblegum. Each pack contains 8 pieces of gum. How many pieces of bubblegum do I have in all? There are 24 students to ride in vans. Each van has 6 seats. How many vans are needed?)

Procedural:

Students can fluently use basic multiplication and division facts 0-9.

Representational:

Students can model how to solve basic multiplication and division word problems using manipulatives, drawings, algorithms, and journaling.

Academic Vocabulary and Notation

variable, inverse operations, multiplicative comparison, additive comparison, symbol

Instructional Strategies Used	Resources Used
1. Use multiplication and division fact families to show the relationship between division and multiplication.	http://www.helpingwithmath.com Look at Multiplication and Division Word Problems for

Explain how one is the inverse of the other. 2. Give the students story problems with multiplicative comparisons. Have them draw a picture or create a model of the problem, write an equation with a symbol for the unknown variable, and solve.	examples to use with students of multiplicative comparison. http://www.mathplayground.com/wordproblems.html Challenging examples of word problems using multiplicative comparison. http://www.mathscore.com/math/standards/Common%Core/4th%20Grade/ Scroll down to the correct domain and standard to find a listing of online problems displaying multiplicative comparison.
Assessment Tasks Used	
Skill-Based Task: Create and solve an equation from a given word problem.	**Problem Task:** Over the summer, Raul read 8 books. Natalia read 4 times as many books. How many books did Natalia read? Draw a picture or create a model of the problem, write an equation with a symbol for the unknown variable, and solve.

Domain: Operations and Algebraic Thinking Grade: 4th

Core Content

Cluster Title: Use the four operations with whole numbers to solve problems.

Standard 3: Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.

MASTERY Patterns of Reasoning:

Conceptual:

Students will decode and understand multistep word problems that may or may not include remainders.

Students will decode and understand multistep word problems and create an equation with a variable representing the unknown number.

Students will determine the reasonableness of the calculated answer using mental computation and estimation strategies.

Procedural:

Students can interpret a multistep equation that includes the four basic operations.

Students can create and solve an equation using a variable or symbol to represent an unknown number in a multistep problem, interpreting remainders when needed.

Students can evaluate the reasonableness of the answer through mental math, estimation, and rounding.

Representational:

Students can represent and solve multistep word problems that include the four basic operations and which may or may not include remainders through the use of models, illustrations, and/or writing.

Code: 4.OA.3

Domain: Operations and Algebraic Thinking Grade: 4th

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will possess an understanding of number sense. Students will be proficient in the four basic operations. Students will identify the relationship between the four basic operations. Students will understand how to solve a word problem. Students will understand the use of variables for unknown numbers. Students will understand the meaning of remainders. **Procedural:** Students achieve mastery of basic math facts using the four operations. Students can write equations that include unknown variables. Students can solve division problems with remainders. Students can use mental math and estimation to determine the reasonableness of an answer. **Representational:** Students can use manipulatives, drawings, algorithms, and/or journaling to solve multistep word problems.
Academic Vocabulary and Notation
multistep word problem, mental math, estimation, rounding, remainder, variable, operations, equation, reasonableness

Instructional Strategies Used	Resources Used
1. Polya's Problem-Solving Method: a. Understand the problem. b. Devise a plan. c. Carry out the plan. d. Look back and evaluate the answer.	**http://www.mathplayground.com** Click on the "Word Problems" tab and select "Word Problems with Katie" for different types of multistep problems. **http://www.mathscore.com/math/practice/Word%Problems%20With%20Remainders/**

Code: 4.OA.3

	The site has multiple problems listed that require students to solve different operations in steps to determine the answer. **http://www.ixl.com/math/grade-4/multi-step-word-problems** The site offers additional examples of word problems. **http://www.internet4classrooms.com/grade_level_help/solve_problems_math_fourth_4th_grade.htm** Look for the activity "Two-step Computation" to play a game with multi-step operations.
Assessment Tasks Used	
Skill-Based Task: Solve an equation from a given multistep word problem. Then check the reasonableness of the answer using mental math or estimation.	**Problem Task:** A 17-inch long piece of rope is cut into 2-inch pieces. How many 2-inch pieces are there? How much of the rope is left? Draw a picture or diagram that illustrates the problem. Write an equation using a symbol for the unknown variable. Solve the equation. Use mental math or estimation to determine the reasonableness of your answer. Write an explanation of how you know you are right.

Domain: Operations and Algebraic Thinking Grade: 4th

Core Content

Cluster Title: Gain familiarity with factors and multiples.
Standard 4: Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
MASTERY Patterns of Reasoning:
Conceptual: Students will understand factor pairs as two whole numbers that multiply together to get one product. Students will understand that prime numbers have exactly one factor pair. Students will understand that composite numbers have more than one factor pair. Students will understand multiples as a product of two given whole numbers. **Procedural:** Students can list the multiples of the numbers 2 through 9 up to 100. Students can create a list or chart of factor pairs of whole numbers 1-100. Students can identify, from a list or chart, which whole numbers are prime or composite. **Representational:** Students can use tools such as number lines, hundreds charts, arrays, or cubes to model relationships between factors and multiples.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand whole numbers as the counting numbers plus 0. Students will understand the Commutative Property of Multiplication. **Procedural:** Students have fluency in basic multiplication and division facts 0-9

Code: 4.OA.4

Domain: Operations and Algebraic Thinking Grade: 4th

Representational: Students are familiar with number line structure Students have the ability to build arrays for basic facts
Academic Vocabulary and Notation
factor, multiple, prime, composite, whole number

Instructional Strategies Used	Resources Used
Using a hundreds chart, color-code the multiples for each of the numbers 2 through 9. Identify patterns for multiples of single digit numbers. Use the Sieve of Eratosthenes to find the prime numbers from 1 to 100. **Note: The video is for teacher information only. Do NOT simply show it to the students.** Students should work through the 100 chart using the sieve method described in the video. Take several days for this activity.	**http://illuminations.nctm.org/LessonDetail.aspx?ID=L620** Students distinguish between numbers with several factors and those with only a few factors. **http://illuminations.nctm.org/lessons/FactorGame/FactorGame-AS-Problems.pdf** This link offers a worksheet that assesses students' knowledge after playing the Factor Game. **http://www.xpmath.com/forums/arcade.php?do=play&gameid=60** Play King Kong by whacking him if he's holding a prime number. **http://www.aaamath.com/fra63ax2.htm** The computer lists a number and the student identifies it as prime or composite. **http://www.mathplayground.com/howto_primenumbers.html** Watch a video that defines the terms from the standard, including factor, prime, and composite.
Assessment Tasks Used	

Code: 4.OA.4

Domain: Operations and Algebraic Thinking Grade: 4th

Skill-Based Task:	**Problem Task:** Students at Creek Elementary are going to an assembly. Each class arranges its chairs in a rectangular form. What are all the possible arrangements for the following classes? Miss Franklin 30 students Mr. Clark 27 students Ms. Rodriguez 31 students Mrs. Smith 13 students Help the students as they work through the task by asking questions to help them use multiple strategies. Then debrief, talking about the characteristics of the above numbers. Which ones had equal rows, which did not and why? Have the students write in their math journals how they solved the problem and why their answer is correct.

Code: 4.OA.4

Domain: Operations and Algebraic Thinking Grade: 4th

Core Content

Cluster Title: Generate and analyze patterns.
Standard 5: Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.
MASTERY Patterns of Reasoning:
Conceptual: Students will understand that number and shape patterns follow a given rule. Students will understand that there are sometimes features of the pattern that are not stated in the rule. **Procedural:** Students can complete a given number or shape pattern (e.g., 3, 6, 9, ___ , ___ , 18). Students can determine the rule of a given pattern (e.g., 3 , 6, 9, 12, 15, 18, … The rule is to skip count by 3 or multiples of 3). Students can generate a number pattern that follows a rule and state the rule. Students can generate a shape pattern that follows a rule and state the rule. Students can identify and state any alternate features of the pattern that are not stated in the rule. **Representational:** Students can demonstrate the ability to complete a given number or shape pattern using manipulatives, skip counting, pictures, journals, etc.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will possess an understanding of number sense. Students will know whether a number is odd or even. Students will understand the relationship between the four basic operations. Students have familiarity with number and shape patterns.

Code: 4.OA.5

Domain: Operations and Algebraic Thinking Grade: 4th

Procedural: Students have concrete understanding of basic math facts. Students can solve math operations and sequences that include unknowns. Students can use mental math strategies. **Representational:** Students can use manipulatives, drawings, algorithms, and/or journaling to complete various patterns.
Academic Vocabulary and Notation
number pattern, shape pattern, pattern rule, sequence, alternate

Instructional Strategies Used	**Resources Used**
Play "Count around the Room." Give the counting rule (e.g., count by 7s) as well as the starting place (e.g., begin at 3). Students continue with a given pattern, each student giving a number of the pattern as you move around the room.	**http://www.uen.org/3-6interactives/math.shtml#patterns** A listing of online pattern games that students can play
Assessment Tasks Used	
Skill-based Task: Create 5 to 10 number and/or shape patterns. Students will write a rule that corresponds with each pattern**.**	**Problem Task:** A chicken laid 3 eggs on Wednesday, 6 eggs on Thursday, 12 eggs on Friday, 24 eggs on Saturday, and 48 eggs on Sunday. If this pattern continues, how many eggs will the chicken lay on Monday and on Tuesday? What is the rule demonstrated by this pattern?

Code: 4.OA.5