Utah State Office of Education

core standards *for* MATHEMATICS

A REVIEW

Utah STATE OFFICE of Education

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Larry K. Shumway, Ed.D. State Superintendent of Public Instruction

Response to Pioneer Institute Presentation regarding Utah Core Mathematics to Education Interim August 15, 2012

General Myths and Facts about Utah's 2010 Mathematics Core

Myth: The Utah Mathematics Core is not supported by mathematicians.
Fact: The Utah Math Core is supported by a number of Utah's premiere mathematicians, including, but not limited to Dr. Hugo Rossi, Dr. Peter Trapa, and Dr. James Cangelosi. Other mathematicians have been involved in creating and providing professional development for teachers through the Core Academy, including Dr. Brynja Kohler and Dr. Emina Alibegovic. In 2010 the Utah Board of Regents Math Majors committee voted to support the Common Core and to support the integrated pathways for secondary mathematics. USOE has received and continues to receive positive support for the core from educators across the state. (See Appendix A: Statements of Support)

Several prominent mathematicians and mathematics-related organizations have reviewed the core standards and support them:

Dr. Hugo Rossi, University of Utah
"The heart of the core is in the Standards of Practice. The content standards are not very far from the 2007 Utah Standards; in fact those Standards were ranked highly by the committee writing the CCSS, and were among existing documents used in the preparation of the CCSS. The Utah Core Standards understands that role that mathematics plays in the curriculum (besides being essential for people going into STEM professions) is to develop problem solving and critical thinking skills in all students. The compartmentalization of skills and processes of traditional Cores allows for easy creation of assessment materials, and de-emphasis of the development of thinking skills. The Standards of Practice and the Connections among topics that is exposed in the new Utah Core, especially since Utah has adopted the integrated core of the CCSS, prevent this from happening. One sees this in the reactions of teachers in their professional development preparing them for the Core's implementation: I have heard many say, "this is exciting! This is why I entered this profession – to help students develop and embrace their critical thinking skills and their understanding of how things work."

Dr. James Cangelosi, Utah State University

"The *Common Core Standards in Mathematics* as it has been adapted here in Utah is a major step forward to encouraging the following in Utah's school curricula: The development of students' (1) abilities to interrelate various mathematical sub-disciplines (e.g., Euclidean geometry, classical algebra, trigonometry, conceptual ideas of the calculus, probability, and discrete mathematics); (2) practical problem-solving skills; (3) deep understanding of fundamental mathematical concepts and relationships; and (4) computational fluency. Of particular value to mathematics teachers and consequently their

students has been the rich professional development activities throughout the state – professional development activities motivated by the advent of *Common Core Standards*."

Dr. Elmina Alibegovic, University of Utah
"The new Utah Mathematics Core affords the teachers an opportunity to engage in collaborative efforts to improve students' understanding and achievement in mathematics. While the content standards are not that different from the standards that Utah's 2007 Core contained, the progressions of topics allow for development of connections between the mathematical ideas students have traditionally seen as disconnected. This, in turn, improves their understanding as well as the skills necessary for success in college and careers. The exciting part of the new core, which was lacking in the old one, is the practice standards. They outline the ways in which students should engage with mathematics and the habits of mind they should develop whose usefulness extends far past the mathematics classroom. Overall, the Utah Mathematics Core is an important step forward in our quest to improve our students' education."

National Support

Dr. Hung-His Wu, University of California, Berkley (Dr. Wu is often quoted as being opposed to the core standards. In fact, he supports them.)
"Phoenix Rising: Bringing the Common Core State Standards to Life" by Hung-Hsi Wu http://www.aft.org/pdfs/americaneducator/fall2011/Wu.pdf
http://www.corestandards.org/assets/2010_6_10_WU.pdf

Dr. Bill Schmidt, Michigan State University
http://news.msu.edu/story/7929/http://news.msu.edu/story/7929/

A joint public statement of the National Council of Teachers of Mathematics (NCTM), the National Council of Supervisors of Mathematics (NCSM), the Association of State Supervisors of Mathematics (ASSM), and the Association of Mathematics Teacher Educators (AMTE)
http://www.corestandards.org/assets/k12_statements/NCTM-NCSM-ASSM-AMTE-joint-statement.pdf

American Statistical Association
http://www.corestandards.org/assets/CommonCoreStateStandardsforStatisticsProbability.pdf

Fordham Foundation: Grade A-
"Common Core provides admirable focus and explicitly requires standard methods and procedures, enhancements that would benefit Utah's standards."
http://www.edexcellencemedia.net/publications/2010/201007_state_education_standards_common_standards/CommonCore_Math.p

Myth: The Utah Math Core is not research-based.
Fact: The Utah Core is based on the Common Core which cites 102 articles and research papers regarding mathematics education. (CCSS 147-149) These include *Adding it Up: Helping Children Learn Mathematics*, the most comprehensive research of student learning of mathematics from the National Science Foundation, the NAEP Validity Study of the NAEP Mathematics Assessment, standards from high performing states, and works from such notable scholars as D.H. Clements, J. Confrey, J. Van de Walle, B. Reys, and H. Wu (See Appendix E Research).

Myth: The Utah Math Core will not prepare students for admission to Utah's Tier 1 Universities. It will leave students two years behind.
Fact: The Utah Core Standards are aligned with college and career expectations and include rigorous content and application of knowledge through high-order skills. Utah Tier 1 University professors have reviewed the core materials and assured us that achievement of these standards is sufficient preparation for success in college level coursework at their institutions. (See Appendix B, letter from Peter Trapa, University of Utah Mathematics Department Chair)

Myth: The Utah Math Core places Algebra I in 9th grade. Secondary I is a repeat of Algebra.
Fact: The Utah Core Standards no longer include a course titled Algebra I, but have replaced the Algebra-Geometry-Algebra II series with a trajectory similar to that of high performing countries, titled Secondary Mathematics I, Secondary Mathematics II, and Secondary Mathematics III. Algebra concepts begin in elementary school and are fully fleshed out through Secondary III, the rough equivalent of Math 1010 at the university level. In fact, by the end of 8th grade all students will have over 80% of the current algebra class, the major difference being that quadratics have moved to 10th grade to make room for exponential function (formerly in Precalculus and Algebra II) and polynomials have moved to 10th grade to make room for geometry (formerly in Geometry). This reorganization makes it impossible for any state to equate any course in the new core with a particular course in the 2007 core. (For more information see Appendix C *Algebra Topics in the Utah Core*).

Myth: The large majority of students will not reach Calculus in high school as expected by elite colleges.
Fact: The new core is designed to give students greater support through the mathematics curriculum to adequately prepare them for college level mathematics. The standard and honors pathways lead students to College Algebra 1050 or Calculus, the two courses named most desirable by Utah mathematics departments, at our own elite in-state elite universities. The honors sequence will provide a pathway to high school Calculus for those with interest and ability. The regular sequence provides a pathway to College Algebra and Trigonometry in the senior year, which is preparation for College Calculus. Students taking four years of high school mathematics on the regular pathway will be solidly prepared for calculus in college.

Myth: The Utah Math Core forces all students to learn the same mathematics.
Fact: The Utah Mathematics Core is designed to provide access to and support deep understanding of mathematics for all; however, the standards are best supported through differentiated instruction and attention to individual student needs at each grade level and in each course. The curriculum guides

provide information regarding possible intervention strategies and suggest problems that will deepen understanding for high achieving students. The Utah Core coursework has been purposefully designed to allow for increased choice and flexibility as students move to higher grades. Local education agencies have designed additional options for students regarding scheduling that can be supported by their resources.

Utah's core standards **are not** used to standardize and inhibit student progress. They are used as standard benchmarks used to help students gauge progress toward fulfilling their individual aspirations. The purpose of Utah's core standards **is not** to drive everyone to achieve the same specific goals for each student or for them to achieve at the same pace. It is not designed to promote sameness. Teachers are to use the standards much like a physician uses developmental standards to understand and plan for each child's needs. The standards are used to help teachers understand in a broad manner what individual children should be able to know and do at each grade level. They are used to benchmark and not judge progress. Our goal is to optimize learning for each student. It is hard to know where an individual student needs assistance or advancement if there are no standards to measure their unique progress.

Myth: The Utah Math Core will disadvantage high achieving students.

Fact: The Utah Math Core is much more rigorous than previous cores. High achieving students in Utah mathematics classes are encouraged to deepen understanding through the pursuit of honors topics, increased rigor, and in some cases, acceleration. The new core provides honors courses beginning in seventh grade and includes a generalized pathway to senior calculus.

In rare circumstances, an LEA may telescope mathematics courses to allow an especially advanced student to take Calculus before the senior year. Extreme care must be taken to properly identify and verify that these students are eligible and ready for such acceleration. "Serious efforts must be made to consider solid evidence of student learning in order to avoid unwittingly disadvantaging the opportunities of particular groups of students" (Common Core, Appendix A, 81). With thoughtful and informed placement and curricular decisions, students can be guided and placed in appropriate classes. Accelerated courses must not skip any content or reduce rigor. Instead, they should move at a faster pace and include multiple assessments to ensure content has been mastered. Districts and schools may use course codes that match the curriculum students are studying, even if that means using multiple course codes within a year. For more information, please see Utah's statement on High Ability Students and the Mathematics Common Core. http://schools.utah.gov/CURR/mathsec/Core/High-Ability-Students-and-the-Mathematics-Common-C.aspx

Myth: The Utah Math Core will require a large dollar investment in new textbooks and materials.

Fact: Utah has a five-to-seven year revision cycle for core standards. Revisions are based on the need to ensure that students learn what they need to know to be successful after graduating from high school. The former core in Mathematics was adopted in 2007. The former Reading/Language Arts was adopted in 2003. The new Common Core standards in these areas were adopted in August 2010 with limited implementation in the 2011-2012 school year. Utah began revising the standards in 2007 after concerns were raised by some (including members of the Legislature) about the rigor of the standards. If Utah had

chosen to reject the Common Core Standards, the core adoption and revision process for both academic areas would have been the same. Both adoptions would have also incurred the same "costs."

The fact is, Utah is not spending more money than usual to implement the new Utah core standards – we are actually spending less. School districts and charter schools have received no additional funds, federal or state, to implement the new core standards, or new instructional materials or curriculum. In fact, funding for professional development, which is used to train teachers in new core standards (including the summer Core Academies) has decreased significantly. Before the current economic downturn school districts and charter schools had $78 million in state funds for professional development through the Quality Educator Block Grant. This has been almost completely eliminated for the past 3 years. For the 2011-12 school year, only $2 million was allocated by the Legislature for professional development, and half of this amount for the following year. Textbook costs remain the same. Typically, when a new core is implemented, school districts and charters phase in new materials. Most have a five to seven year textbook replacement plan.

Since Utah isn't a big textbook market we have never been able to purchase books that are 100% aligned to our core. The lack of materials has incented teachers to find and create lessons and tasks for their students. The Utah network of mathematics teachers is strong and increasing in strength supported by organizations such as the Utah Council of Teacher of Mathematics and the wide availability of technology in schools. Open Education Resources are being widely used by Utah teachers at no cost. Many teachers are now reporting such satisfaction with these resources that they no longer desire the support of a textbook.

Utah is spending money to create new assessments for not just the new math and reading/language arts core, but also the science core. In 2007 former Governor Huntsman convened a Blue Ribbon Panel on Assessment. The panel, stakeholders throughout the state and the Utah State Board of Education, concluded that technology-assisted assessments should be studied and, if successful, should be adopted statewide. Successful pilots were conducted and the State Board concluded that state funds should be sought for technology-assisted assessments to align to state standards. ***The request for a Computer Adaptive Testing system would have been made regardless of which core standards were adopted or in place by the State Board****.*

Besides Computer Adaptive Testing, schools are taking advantage of more and more technology-based instructional resources, and the need for computers and related technologies has increased dramatically over the past several years. To attempt to "blame" the increase in technology costs to the Common Core standards is nonsensical.

Myth: The Utah Math Core (UCS) was adopted without public input from Utah.
Fact: The Utah Core Standards were adopted with unprecedented and extensive public input.
Fact Concerning Mathematics: The State Mathematics Coordinating Committee (SMECC) was first introduced to the draft of the College and Career Readiness Standards, which became the foundation of the Common Core, in October of 2009. Over the course of the next year SMECC was updated at every

meeting and provided with opportunities both for input regarding the documents that were being released by CCSSO and concerning Utah's possible participation. USOE submitted input and responses to drafts from the Common Core developers in February and March of 2010. A Math Advisory Panel was convened which continued the work of the Mathematics Steering Committee. This committee made recommendations to the Board of Education regarding adoption and implementation of the Common Core. Mathematics and education organizations (Curriculum Directors, SMECC, NUCC, Utah Council of Teachers of Mathematics, etc.) were briefed and given an opportunity for input. Every organization supported the adoption of the Common Core for Utah.

Fact in General: The revision and adoption process of a new set of core standards usually lasts for six to twelve months. In this case the process began in October 2007 and ended in August of 2010.
In October of 2007 a USOE led committee began discussions regarding a revision to the mathematics standards. These discussions continued through the summer of 2009. From July through December 2009, USOE held several meetings where participation in development of common core standards was discussed as a part of the agenda. Meetings included conversations with the Education Interim Committee, superintendents, charter directors, curriculum directors, other legislators, PTA members, higher education representatives, and business leaders. As a part of the State Board's **Promises to Keep Initiative**, USOE created *Utah's Comprehensive Reform Plan*. The plan included a process for reviewing, adopting and implementing the common core standards and aligned assessments. In addition, USOE and content advisors around the state continued monitoring the common core standards development process and provided feedback to the developers.

From January through May of 2010, the State Board was briefed on progress toward developing the standards for Mathematics and Language Arts. The State Board agreed that revisions of the two cores and new assessments should be a part of the **Promises to Keep Initiative** and long term improvement plans. USSA was also briefed. USOE continued monitoring the common core standards development process and provided feed back to the developers. USOE continued monitoring the common core standards development process and provided feedback and revisions. The State Board was kept informed of progress at each Board Meeting. On June 4, 2010, the State Board gave preliminary approval for Utah to move ahead in accepting the common core as a framework for setting the state's own standards and curriculum in both English language arts and mathematics. The Board sought public comment during the summer to add to the input they had received for the past years.

During the summer of 2010, USOE held several meetings where common core standards were discussed as a part of the agenda. Feedback was solicited on our website. Meetings included conversations with superintendents, charter directors, curriculum directors, legislators, PTA members, higher education representatives, and business leaders. The core standards for Mathematics and English Language Arts were approved during the Board's August 6 meeting.

Mathematical Myths and Facts

Myth: Geometry teachers will be required to teach with an experimental method not proven to work anywhere in the world.
Fact: The Common Core does not require any standardized method or pedagogy; however, this statement is most likely in reference to the addition of transformational geometry to the standards. This has not come at the expense of Euclidean Geometry, but is an additional way of making sense of our spatial world. It is true that many teachers are unfamiliar with this approach to geometry; therefore USOE has provided and will continue to provide professional development in this area. Universities have already begun plans to adjust pre-service programs to adequately prepare future teachers.

Myth: The Utah Core de-emphasizes algebraic manipulation.
Fact: The examples in the Utah Curriculum Guides make clear that algebraic manipulation is still an important part of mathematical skill. Algebraic procedures are best taught on a foundation of understanding where fluency can develop.

Myth: The algebra in the common core is at a very low level.
Fact: What has been described as an emphasis on "functional algebra" is misrepresented to mean basic algebra. In fact, this is a criticism of the basis of algebra understanding rooted in the concept of function. Conceptual understanding of functions leads to procedural fluency and greater understanding of more complex mathematics.

Myth: The Common Core fails to teach prime factorization and consequently does not include teaching about least common denominators or greatest common factors.
Fact: The Utah Core begins laying the groundwork in fourth grade (4.OA.4) where students work with factor pairs. This standard also develops the understanding of prime and composite numbers.
In grade 6, the Utah Core states, "Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. (6.NS.4)
The Utah curriculum guide illustrates that one such task for this standard would be "Find the greatest common factor of 24 and 60." The curriculum guide suggests that teachers use factor towers, Venn diagrams, and factor trees (the preferred method) for finding prime factors.
Evidence that this is understood by teachers can be found in the Jordan District illustration of 6NS4 found
at. http://departments.jordandistrict.org/curriculum/mathematics/elementary/CCSSM6/cf/cfcomputationalfluency.pdf

Myth: The Common Core fails to include conversions among fractions, decimals, and percents, identified as a key skill by the National Research Council, the National Council of Teachers of Mathematics, and the presidential National Advisory Mathematics Panel.
Fact: In sixth grade, students convert fractions to decimals (6NS2d)
In 7th grade students are expected to "develop a unified understanding of number, recognizing fraction, decimals…and percents as different representations of rational numbers." and explicitly 7EE3 says: Solve

multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Myth: The Common Core fails to teach in K-8 about key geometrical concepts such as the area of a triangle, sum of angles in a triangle, isosceles and equilateral triangles, or constructions with a straightedge and compass that good state standards include.
Fact: Area of triangles is found in 6th grade (6.G.1). Triangle angle measurement relationships are found in grade 8 (8.G.5). Formal constructions which are now in Grade 9 (G.CO.12) were formerly found in 9th and 10th grade geometry courses. Specific examples of these can be seen in the curriculum guides.

Myth: Common Core barely touches on logarithms, of great importance for chemistry, physics, and STEM in general.
Fact: Logarithms are important and are in the core. They are studied in the context of inverse relationships and are included in Secondary III honors as described in F.BF.5. This is a more explicit treatment than in our current core.

Statement: The Common Core fails to address mathematical induction.
Fact: The 2007 Utah Core did not address mathematical induction. The input from Utah university mathematicians at that time was that they preferred to keep it in college coursework.

Myth: The core fails to address parametric equations.
Fact: Parametric equations have been identified by Utah teachers as necessary for Calculus and therefore are included in the Secondary III Honors course (under development). They are also included in Precalculus (Standard III, Objective 1) which is part of the Utah Core. Therefore, the Utah Core addresses parametric equations for all students who take four years of mathematics.

Myth: The core fails to address infinite geometric series (progressions with common ratio).
Fact: Geometric series has been and remains a Precalculus topic (Standard I, Objective 2). It is also included in Secondary III Honors. All students who take four years of mathematics will study infinite geometric series.

Myth: The core incompletely addresses conic sections
Fact: Although this claim doesn't explain what might be expected in a "complete" treatment, conic sections are included in Secondary Math II (G.GPE.3) where a complete treatment is described in the curriculum guide.

Myth: The core omits in trigonometry the phase of periodic functions, half-angle formulas, and polar forms and functions.

Fact: Trigonometry formulas are included in Secondary Math II Honors (F.TF.9) where the curriculum guides expand to include half angle formulas. Treatment of trigonometric functions has been and remains in Precalculus.

Response to various emails sent to Legislators

Myth: USOE is promoting or advocating for a constructivist approach to mathematics.
FACT: USOE is not promoting or advocating for a constructivist approach to mathematics. USOE is also not in favor of a strict traditional approach either. It is important to remember that standards **are not** practices!!!! Standards define what students need to know and be able to do. However, using standards and highly structured objectives to develop curriculum or inform practice is generally considered to be traditional in nature.

Traditional mathematics instruction focuses on accuracy and efficiency. It is usually accomplished through direct instruction. In traditional instruction, understanding is sometimes sacrificed as students learn to efficiently solve problems with accuracy. Constructivist mathematics instruction focuses on building deep understanding and building connections between mathematical concepts. Standard algorithms are sacrificed as children "build their own understanding." In many cases, strict constructivists fail to close the circle leaving students confused about how to actually do the math. Too much strict constructivism fails to leverage understanding to build efficiency.

Utah does not promote the exclusive use of either philosophy. **Good teachers use elements of both, as recommended by the National Math Panel**. They give individual attention and assign facts to be learned. They give multiple choice tests and performance based problem-solving tests. They have children practice math facts and justify how they solve a problem. Using only one practice is bad practice. Good teachers build deep understanding and help students make meaningful connections, while building student skills in accuracy and efficiency.

USOE's focus is on what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

Myth: USOE used the summer professional development called core academy to push teachers into using Investigations or adopting a constructivist approach.
Fact: False. The Core Academy this summer was based on a balanced approach to mathematics, including the kind of mathematics instruction that supports fluency (memorization and algorithmic procedures) along with instruction that supports understanding through developing deeper levels of understanding the lead to student abilities to apply mathematics to real situations. In every class, at

every level this summer, teachers were supported in designing lessons that support this kind of learning that leads to legitimate uses of mathematics. We asked all of the academy facilitators if they heard anyone state that "memorizing mathematics facts or learning standard algorithms is not important." They were all appalled and reassured us that this never happened in any academy. We are supporting instructional approaches that have proven to be effective, but we are not prescribing anything.

MYTH: The Mathematics core doesn't include basic computational skills
FACT: The standards clearly include the items basic computational skills. The following website lists the Mathematics core standards. http://schools.utah.gov/core/DOCS/Utah-State-Mathematics-Standards.aspx Furthermore, a review of the Utah mathematics Curriculum Guides will demonstrate that computational and algorithmic skills are expectations in the new core.

MYTH: USOE's integrated approach to High School Math isn't being used anywhere else. It will harm our students.
FACT: USOE has adopted an integrated approach to high school mathematics. An integrated approach means that students learn about number systems, algebra, functions, modeling, geometry, statistics and probability in three one year courses instead of separate courses for each concept. High performing countries such as Singapore use this method (See Singapore Document). Mathematics in grades K-8 has always been taught in an integrated fashion.

Utah decided to adopt an integrated approach after several years of study. In the summer of 2005 The Fordham Foundation, a conservative think-tank and education watch-dog, evaluated state standards in Mathematics. The report gave our Mathematics standards a D. Utah and other states were encouraged to re-think and strengthen standards. In addition, international comparisons showed that students lagged behind their world-wide peers. Prominent mathematicians and higher education leaders asked for stronger standards. Utah responded with a new set of math standards. Fordham gave those standards an A-. Our new standards still did not please many legislators and mathematicians such as Dr. Milgrim. In a letter to the Utah State Board, the co-chairs of the Education Interim Committee of the Utah Legislature asked the State Board to again review Utah's mathematics standards. Specifically they wanted the standards to be "world-class" and more competitive and similar to those of high performing countries and states. USOE with the assistance of an advisory panel began reviewing Utah's mathematics standards, instruction and pre-service teacher training. The end result was a decision to adopt new standards and integrated high school courses. Fordham has given the new Utah common core math standards an A- and recommended that Utah stay with the new standards rather than use the old.

The new core is designed so that MORE students will be calculus ready either after their junior (after taking the honors series) or senior (after the regular series and a Precalculus or 1050/1060 course) year. The new core ensures that students have the depth of knowledge necessary to succeed in higher level mathematics classes, as well as procedural knowledge. It is our goal that more students attain higher levels of mathematics by following the new standards than has ever been possible before. This pathway is supported by the state university mathematics departments that have let us know that many

students, including those who have already taken calculus, are not, in fact, college ready. By solidifying both conceptual and procedural knowledge, we hope to better prepare students for calculus, whenever they are ready.

Response to July 11 Email from Oak Norton:
"From: oak <oak@oaknorton.com>
Sent: Wed, Jul 11, 2012 8:01 am
Subject: [UACC Petition] Report and Quick Action Needed"

> "Among the points brought up was this salient tidbit. There were 3 people who drafted the original math standards for Common Core before they were reviewed. Only one of the 3 was a mathematician and he said the standards were not very high, certainly not on par with Asian countries who finish algebra 1 by 8th grade. Common Core sets algebra 1 completion in 9th grade. Dr. Jim Milgrim, the only mathematician on the review panel refused to sign off on the standards for this same reason that we were falling behind our Asian counterparts by design. The CC geometry standards also contain a method of teaching congruent angles that has never been successfully used anywhere in the world instead of standard methods."

1. Dr. Milgrim was not the only mathematician that worked on the standards. The three people referred to in the email are Dr. Phil Daro, an education professor at Berkeley with a career focused on mathematics education, Dr. Jason Zimba, with a Master's in Mathematics and PhD in Physics, and Dr. William McCallum, a PhD in Mathematics at the University of Arizona.
http://math.arizona.edu/~wmc/Research/2012_07_12_ICME_McCallum.pdf

2. The mathematician who is quoted in the email as saying that "the standards were not very high," is Dr. McCallum. His actual statement was imbedded in this article from January 2010, six months before the standards were completed and released;

"McCallum, a math professor at the University of Arizona, took the criticism in stride. He reminded the forum panelists that they were looking at draft language that had not yet been made public, and he warned against taking individual standards out of context. While acknowledging the concerns about front-loading demands in early grades, he said that the overall standards would not be too high, certainly not in comparison other nations, including East Asia, where math education excels." http://toped.svefoundation.org/2010/01/17/common-core-standards-under-fire/

McCallum did not say they wouldn't be high. He said they wouldn't be too high. McCallum's comments were made in 2010 before the standards were completed. Many changes were made to the standards between January and June of 2010. Dr. McCallum has been entirely supportive of the core since they were completed in June of 2010.
Dr. William McCallum, University of Arizona
http://math.arizona.edu/~wmc/Research/2012_07_12_ICME_McCallum.pdf

Appendix A

Statements of Support

Higher Education Statements

Utah STATE OFFICE of Education

Leadership...Service...Accountability

August 22, 2012

Dear Utah State Board of Education and Utah State Board of Regents:

This letter is to acknowledge that the implementation of College and Career Readiness Standards based on the Common Core will support student readiness for participation in higher education mathematics. Professors of mathematics and mathematics education have been working in partnership with the Utah State Office of Education to assure alignment by articulating standards and courses. The Utah State Office of Education is actively involved in USHE committees, such as those involved in TICE and Tuning to ensure continued alignment. There are no changes necessary to the Utah Core Standards for Mathematics necessary for alignment.

Sincerely,

Diana Suddreth
STEM Coordinator
Utah State Office of Education

Hugo Rossi
Director, Center for Science & Math Education
University of Utah

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Larry K. Shumway, Ed.D., State Superintendent of Public Instruction

UtahState
UNIVERSITY

August 17, 2012

Persons Interested in the *Utah's Mathematics Common Core Standards*:

Several times a week for the past two years, people have questioned me about these "new" Common Core Mathematics sstandards (e.g., "Jim, what do you think of this 'new' way of teaching math?"). Such questions suggesting that *Utah's Mathematics Common Core Standards* are based on any *new* ideas regarding mathematical pedagogy would be amusing to me except that I heard those same questions about the (a) 2010 Mathematical Association of America's (*MAA*) discussion papers on the *Committee on the Undergraduate Program in Mathematics* (*CUPM*); (b) *2007 Utah Core Mathematics Standards*; (c) MAA's 2004 *CUPM* report, (d) *Teaching Mathematics in Secondary and Middle School: An Interactive Approach, 3rd edition* (Cangelosi, Merrill/Prentice-Hall, 2003); (e) *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics (*NCTM*), 2000; (f) the National Science Foundations sponsored *Calculus Reform Movement* (National Science Foundation (NSF), 1987); (g) *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989); (h) *Educating Americans for the 21st Century: A Plan for Improving the Mathematics, Science, and Technology Education for All American Elementary and Secondary Students so that Their Achievement Is the Best in the World by 1995* (NSF, 1983); and (i) Curricula of *the School Mathematics Study Group* (NSF, 1958–1977).

It is frustrating to hear, "Why do we have to teach this 'new' way? What's wrong with the 'old' tried-and-true way?" The historical records with respect to theories of the intellect, pedagogy, and mathematics education demonstrates that this "new" way of teaching mathematics is ancient dating back to Socrates (BC 469–300), refined by the works of scholars such as Johann Pestalozzi (1746–1827), and proven effective by contemporary research in cognitive science and mathematics education (see, e.g., the mathematical learning models proposed by Richard Skemp's *the Psychology of Learning Mathematics* (1971) and the myriad of empirical studies reported in journals such as *Journal of Research in Mathematics Education*).

So people ask, "So, if these research-based mathematical teaching methods that are consistent with *Utah Common Core Standards* have been around so long, why do our children seem to 'underperform' on mathematics tests?" Here is the sad answer: These standards and principles for teaching and learning mathematics espoused by my aforementioned sample list, a–i, of reports have only been implemented by too small of a portion of mathematics teachers throughout the U.S. – teachers who enjoy both a strong mathematical and pedagogical preparation. These are the exceptional teachers whom my university students reference when I ask, "Why did you decide to major in mathematics (or mathematics teaching or physics or engineering)?" "I hated math until I had Rob Hoggan at Sky View (or Liz Mott at Logan High or Daren Lentz at Mount Logan Middle or Stephanie Swainston at North Cache Middle, ...); he (she) led me to discover the theorems underlying algorithms. We didn't just memorize rules without understanding them. It's the first time it made sense - like why the product of a pair of real numbers is positive." My own enlightenment about the beauty of mathematics did not occur until rather late in my undergraduate days at Louisiana State University. My preparation coming out of high school was less than adequate and my family enjoyed no tradition of attending college. But because of an athletic scholarship I enrolled at LSU totally naive about university life or academics. I

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chose mathematics as a major because it was the only item on the majors list I recognized ("What's a 'major,' I wondered?"). Through my first two years, LSU's version of "schoolmath" was neither challenging nor interesting. But finally, James Brandon (a visiting assistant professor from the University of Texas) engaged me in my first course in advanced calculus. Wow! Within weeks I was formulating conjectures and proving theorems that brought beauty and understanding to all of the miserable, boring, inapplicable algorithms I had memorized and forgotten during my prior experiences with what I previously thought of as mathematics. Professor Brandon introduced me to the principles underlying Utah's Mathematics Common Core Standards. Thank you, Dr. Brandon.

Of course, what my students or I used to hate wasn't mathematics at all. It was this oxymoron that D. Fowler in his 1997 article in *MAA's Focus* refers to as "schoolmath" – which employs an epistemology based on appeal to authority (which is appropriate in some other disciplines (e.g., religion) but is contrary to the epistemology of mathematics which bases its truths (e.g., the product of a pair of negative real numbers is positive) on conjectures built on inductive reasoning and proofs built on deductive reasoning.

So how did "schoolmath" come to dominate mathematics curricula throughout the U.S.? As suggested earlier, the more familiar "schoolmath" is newer than the less familiar research-based mathematical pedagogy consistent with the *Utah's Mathematics Common Core Standards*. "Schoolmath" is a product of the ill-fated faculty psychology movement of the late 19th and early 20th century in Europe and the U.S. Faculty psychology, rooted in the craniology of Franz Gall (1758–1828) and the commercially-successful practice of phrenology promoted by Johan Spurzheim (1776–1832), was proven invalid in the 1920's but its impact – with an emphasis on training the mind by working "the brain's memory faculties" – devastated mathematics curricula in the U.S. With the endorsement of "authorities such as the Committee of Ten of 1892 and the commercial success of the "schoolmath" textbook industry that was born in the heyday of faculty psychology, "schoolmath continues to dominate today.

Utah's Mathematics Common Core is another in our string of efforts to supplant "schoolmath" with research-based mathematical pedagogy. To date it has provided a major step forward to encouraging the following in Utah's school curricula: The development of students' (1) abilities to interrelate various mathematical sub-disciplines (e.g., Euclidean geometry, classical algebra, trigonometry, conceptual ideas of the calculus, probability, and discrete mathematics); (2) practical problem-solving skills; (3) deep understanding of fundamental mathematical concepts and relationships; and (4) computational fluency.

Of particular value to mathematics teachers and consequently their students has been the rich professional development activities throughout the state – professional development activities motivated by the advent of *Common Core Standards*.

Sincerely,

James S. Cangelosi, Ph.D.
Professor of Mathematics
Utah State University

THE UNIVERSITY OF UTAH

The Center for Science and Mathematics Education
155 South 1400 East, Rm. 233 Salt Lake City, Utah 84112-0090 801-585-1985 Fax 801-581-4148

August 20, 2012

Utah State Office of Education
250 East 500 South
PO Box 144200
Salt Lake City, Utah, 84114-4200

Chair Deborah Roberts and Superintendent Larry Shumway,

I am writing to express my conviction that at this time – and it is a very short time that we have – the adoption of the integrated model of the Common Core standards by the state of Utah, subsequently adapted as the Utah Core Standards (UCS) to meet specific state objectives in education and economic development, provides us with a unique and important opportunity that we should not squander. This conviction is based on the quality of thinking that went into the creation of this core, the strength of its intellectual structure, and the collaborative effort in its implementation.

At this moment the UCS is being questioned: the claim is that it is a top-down instrument forced on us from outside Utah. To support their claim, the opposition bring in expert testimony from outside Utah. This logic defies ... well, logic. What is now going on in Mathematics education in Utah is the development of a curriculum based on the Utah Core Standards, relying on materials already developed by Utah teachers in their respective districts. This is a unique and potentially highly successful approach that develops a home-grown model for education that will challenge the status of international models that at present command emulation.

Because those apposing the implementation of the Core in Mathematics bring in outside 'experts,' I feel a necessity to describe my own credentials. I was raised during the second world war in a lower-class neighborhood in the Bronx. My parents relocated from a bucolic village in New Jersey, because the New York system of education, from K to 16 was – at that time - the best in the world and free. When I graduated from the College of the City of New York I went to graduate school at MIT on a full scholarship. After earning my PhD I taught at Princeton and Brandeis. I joined the faculty of the University of Utah in 1974, with the express purpose of building a world-class mathematical research department. In 1981, the national survey of mathematics departments cited the University of Utah as the most improved program in the nation, and then (and still today) ranked us in the top 35 mathematics departments in the United States. I served as Chair of the

department and Dean of the College of Science, bringing to the University several mathematics and science education programs that still thrive. In 1997 I took leave from Utah and joined the administration of the Mathematical Sciences Research Institute at UC Berkeley. I served five of the next nine years at MSRI; my main functions were oversight of the postdoctoral program and community outreach. In the latter capacity I participated in the Math Wars for which California was a central battlefield. I had a singular opportunity to see that both sides had much to offer the educational system and benefit children immensely.

When I returned to Utah in 2006 I brought with me that thinking. I was asked to serve on the task force to rewrite the Utah Mathematics Core Standards. We finished our work in the summer of 2007. That core was attacked as lacking in rigor and precision. Then, as now, those opposed brought in the same outside 'experts' to testify on their behalf. I find it interesting that they are now requesting that we return to the 2007 Core, seeming to have forgotten that they fervently opposed it at the time. What has changed their minds? I challenge them to respond: what have they learned in the past 5 years about the 2007 core that convinces them that it was, after all, optimal. Was it the Fordham grade of A- (replacing a C-) a factor?

It is my belief that their objection has little to do with the Core Curriculum but has to do with change itself. The opposition could benefit from study of the UCS, particularly the emphasis it places on the role mathematics plays in the understanding of change. The 2007 Utah Core was a strong document; as is the CCSS that received the same grade. But the new Utah Core Standards are even better. They incorporate the idea of integration of content, and reliance on student active involvement in context for understanding. Many fewer processes are introduced, as the emphasis has become that on conceptual understanding and analysis of problems, rather than routine application of operations. Perhaps the opposition feel that the 2007 Standards are good enough, something they did not express when opposing them. So the question is this: Is "good enough" sufficient for our children's education? It wasn't for my parents.

The authors of the CCSS nationally, and the Utah Core Standards locally, promote an understanding of the problems with our mathematics education and propose a remedy: a set of standards based on the mathematical concepts and processes that are fundamental to success in the contemporary world, developing an understanding of the use of these tools and a flexibility in using them, The UCS gives us an opportunity to change the way mathematics is taught by creating a new curriculum that are based on those standards. That work has already begun, and that beginning holds great promise.

I have attached a set of "talking points" on the UCS. Sincerely,

Hugo Rossi

Hugo Rossi, Professor Emeritus of Mathematics, University of Utah

The Utah Core Standards are genuinely Utah Standards, and not a relabeling of the CCSS.

Utah has adopted the integrated model, on which the CCSS has little detail. We are one of three states to adopt that model, and the first to implement it.

The Standards, both of Content and Practice, develop objectives for K12 education in mathematics; they provide neither curriculum nor ways of achieving those objectives.

The fact that Utah is at this time alone in implementing the integrated model, and has withdrawn from the Greater Balance Assessment project, leaves us with the role of developing that curriculum and the instruments, texts and tests, and to deliver it on multiple platforms. The Utah State Office of Education, many in Higher Education and in the districts have embraced that role. It is an opportunity to develop mathematics education without publisher pressure, and the freedom to think in terms of what our students should understand about mathematics, not what questions will be on the CRT.

In this way we can work together to develop a comprehensive approach to education, taking full advantage of technology, online delivery and problem- based learning .

Utah has already begun this process: the summer core academy was dedicated to development of the basic ideas of the standards that will underlie a new curriculum, districts are developing materials for teachers to use in the classroom; and, with support from the legislature, a broad-based group has begun work on texts for 7th and 8th grades that will be available online,

The Utah Core Standards can be viewed as an extension of the Utah 2007 Core with a stronger, better defined, and consistent perception of the goal.

The Core is propelled forward throughout by concentration on five strands:
Number Sense, Functions, Algebra, Geometry, Statistics.

Weaved into these topics is a development of understanding of mathematics as an instrument to model, simulate and analyze processes.

Focus is moved away from at one extreme, "having fun with math" to, at the other extreme,"never-ending drill on algorithms." Mathematics is presented as a serious enterprise, essential for all people to function in the modern world, and in particular, as fundamental to much of the work that will occupy us in the 21st century.

Tools are developed to understand and analyze arguments and presentations of data in the news, on the job and in human relations is a central concern of the UCS.

The Common Core State Standards are the product of many of the best education minds in the country. This nationwide group of experts included experts from Utah, so that our local expertise was combined with expertise from 49 other states to create the best possible set of standards. Why would we suppose that by subtracting 98% of the expertise in the CCSS group (the experts from the other 49 states) and developing our own proprietary standards using only 2% of the expertise in the original group, we could possibly create something better?

Please remember the time before Utah adopted the Common Core, when every state had its own isolated and proprietary education standards.
Entrepreneurs, researchers, and other innovators only had financial motivations to pay attention to Texas, California, and Florida. Utah was ignored and left to adopt curriculum designed specifically to support these other states' standards. By adopting the Common Core, Utah's children and teachers have gained access to a nationwide marketplace of curriculum and innovations written directly to Utah standards. The decision to adopt the Common Core State Standards gives our students access to the very best educational materials and experiences available. Rejecting the Common Core State Standards would sentence Utah's children to another decade of being ignored by the creators of educational materials and educational technologies. I'm sure that's not what we want to do.

David Wiley, PhD
Brigham Young University
Associate Professor, Instructional Psychology and Technology Director of Research, Center for the Improvement of Teacher Education and Schooling Senior Fellow, National Center for Research in Advanced Information and Digital Technologies (Digital Promise) CAREER Grant Recipient, National Science Foundation

Straight Up Conversation: Math Scholar Hung-Hsi Wu on the Common Core

By **Rick Hess** on October 5, 2011 8:01 AM
Retrieved August 16, 2012 **http://blogs.edweek.org/edweek/rick_hess_straight_up/2011/10/straight_up_conversation_berkeley_math_professor_emeritus_hung-hsi_wu_on_the_common_core.html**

A few weeks back, **I penned a post** about the lack of response we'd received regarding our in-the-works *Education Next* forum on the Common Core math standards. I heard from a number of individuals who offered to defend the standards. One was Hung-Hsi Wu, professor emeritus in mathematics from UC-Berkeley, who has just penned the **cover story** on this topic for AFT's magazine *American Educator*. Dr. Wu, who started teaching at Berkeley in 1973, has been actively involved in math education for the past two decades, helping write California's 1999 Mathematics Framework and California's Standards Tests. He was also a member of NAEP's Mathematics Steering Committee, 2000-2001, that contributed to the revision of the NAEP Framework.

I appreciated Dr. Wu's offer to share his take and was impressed by his willingness to talk frankly about the Common Core effort, as he sees it. Here's our (e-mail) conversation.

Rick Hess: In layman's terms, what do you see as the big differences between the Common Core math standards and those in most existing state standards?

Hung-Hsi Wu: The Common Core math standards place great emphasis on mathematical integrity, [in other words] the statements of the standards are mathematically correct and the progression from topic to topic is logical. In this regard, it is at least comparable to the best state standards, such as those of California and Massachusetts. However, the Common Core math standards are unique in being sensitive to the multiple defects in the existing de facto national curriculum that is already embedded in existing textbooks (see **my article** for further discussion) and address these defects directly. For example, there is a profound common misunderstanding about something as basic as what it means to solve an equation. ...The Common Core math standards, however, ask that students "understand solving equations as a process of reasoning" and say explicitly what needs to be taught about this process (see Standard A-REI 1 in High School Algebra). As another example, when state standards ask that the concept of congruence be taught in middle school, they do not realize that what students will end up getting is that *congruence means same size and same shape*. As a *mathematical* definition, the latter is completely unacceptable. By contrast, the Common Core standards explain that congruence means what one gets by a sequence of rotations, reflections, and translations (grade 8, Standard 8.G 2). Such sensitivity to the existing defects is absolutely essential to any meaningful improvement in our math education; in this regard, the Common Core standards leave all rivals far behind.

RH: What do you make of the concerns some have raised that the thematic focus of the 9-12 is an awkward fit for the familiar organization of courses like algebra, geometry, and calculus?

HW: One would feel this way only if one is already wedded to the traditional offering of one

year each of Algebra I, Geometry, and Algebra II in high school. There are mathematical reasons why this sequence is not an optimal way to organize high school mathematics. For example, mathematics is best taught without being handicapped by such rigidity. On the other hand, those who are bent on following the so-called American Integrated Curriculum also find fault with the high school set-up of the Common Core, but there are also valid reasons to argue that such an integrated curriculum, by not being sufficiently attentive to mathematical integrity, is not an optimal way to organize high school mathematics either. In any case, the 9-12 standards of the Common Core are what they are because the Common Core made a conscientious decision to stay neutral in this debate by describing only the mathematical content of the various strands in high school and allow[ing] each state to make its own decision. This flexibility makes it possible to formulate a high school program that conforms to neither the traditional nor the integrated format; see **here** for example.

RH: What's your response to the concerns raised by UPenn dean Andy Porter, who has suggested that, in practice, the standards "do not represent a meaningful improvement over existing state standards" and that they have "a greater focus than certain state standards and a lesser focus than others?"

HW: These conclusions are based on data that are demonstrably wrong: for example, the claim that state standards in grades 3-6 spend 14.47% of instruction time on "Advanced algebra" and 0% on "Measurement" (compared with 0% and 17.79% in Common Core standards, respectively)...His claim that Finland puts "far less emphasis on higher order thinking skills, and far more on basic skills" than do the Common Core standards is also not consistent with the data of Finnish students' performance on their own internal exams. It may be more profitable to wait for Porter to clarify his dissatisfaction with the Common Core using valid data before we discuss this issue further. In the meantime, I would like to make a general statement about Porter's methodology. He did not mention having looked at the mathematical quality of the Common Core standard but relied solely on the findings of a content-analysis procedure (the Surveys of Enacted Curriculum) for his conclusion. There is no denying that such a procedure, when used properly in conjunction with other data, could be a valuable research tool. But when it is used all by itself, it is a crude instrument. This explains why, for example, Porter missed the essential mathematical information about the Common Core described in the answers to questions 1 and 2 above.

RH: What do you think of the concerns raised by critics who argue that the math standards have never been benchmarked against international competitors by independent analysts?

HW: Usually such benchmarking is done by asking whether topic X is taught by a certain grade, and whether each grade teaches too many topics. If topic X is fixed, then the usual criterion of excellence seems to be that the earlier X is taught, the better the curriculum. The Common Core math standards do not play this game, but are nevertheless fully consistent with the research findings of the National Mathematics Advisory Panel on curriculum from an international perspective (see Chapter 3 of the *Report of the Tasks Groups*). People who are worried that the Common Core math standards have not been benchmarked against international competitors may be those who have bought into some myths, e.g., all high-achieving nations finish Algebra I in grade 8. A rational discussion of this issue would show that there is no intrinsic merit in finishing Algebra I by grade 8. When it comes to school algebra, it is not how early you teach it but, rather, how *well* you teach it. The standards of those states in

the U.S. that mandate the completion of Algebra I in grade 8 manage to do so *only* by stinting on the necessary background material that students need in order to learn linear equations and their graphs. Furthermore, the math standards of both China and Japan postpone the teaching of quadratic equations and functions to grade 9, and these are two of the highest-achieving nations in the world in math education.

RH: What do you say to teachers concerned that moving objectives, units, and skills across grade levels may not seem like a big deal in theory, but that it will pose big headaches for today's teachers?

HW: I presume the "moving objectives and skills across grades" refers to, for example, spreading the teaching of fraction addition over three grades: grades 3 to 5. Contrary to what the question implies, this *is* a big deal because it is part of Common Core math standards' design to optimize mathematics learning by giving students enough time, whenever feasible, to absorb the material as well as time for teachers to teach the material. For children, the addition of fractions is so conceptually complicated that they need the time to internalize the whole process. This particular treatment of fraction addition is one of the outstanding features of the Common Core standards. A forthcoming document from CCSSO, "Progressions on Fractions," will elaborate on this process; in the meantime, teachers can look at a somewhat discursive discussion **here**. Ultimately, what is at issue is that all teachers owe it to every child to give [him or her] the best chance to learn. If the student takes more than one grade to do it, then that is what it takes. If it takes the Common Core standards to wake us up to our basic obligation to children, then we should applaud these standards.

RH: Okay, softball. What would you argue are a couple of really good things about the Common Core math standards that people generally do not yet know?

HW: The Common Core math standards provide guidance to the teaching [of] fractions in a way that is pedagogically sensible and mathematically correct. Since the fear of fractions has almost become a national pastime, these standards---if properly implemented--- will bring relief to many parents and students. The same can be said about these standards on negative numbers. In addition, the teaching of geometry in middle and high schools is so defective at present that it cries out for a new approach; essentially nothing can make things worse in most cases. The Common Core math standards outline a new approach that makes mathematical sense and, for the first time, provide a seamless transition from middle school geometry to algebra and high school geometry. So finally, there is at least some hope of changing the culture of failure in the teaching of school geometry.

RH: What gives you confidence that teacher preparation and professional development are going to rapidly and effectively make the necessary changes? What have you seen on this score that's worrisome or reassuring?

HW: Nothing, and nobody, has ever given me such confidence. But for the record, let me say in no uncertain terms that, the state of school mathematics education being what it is, we need better teacher preparation and improved professional development in order to stay educationally afloat *no matter what the standards may be*. If we cannot get better teacher preparation or improved professional development, then we would be better off with a set of standards that is at least mathematically sound. In the meantime, the Common Core people are striving to provide teachers with as much help as possible. There will be a set of *Progressions documents* that highlight the main ideas of each major strand in the standards. There is also the

Illustrative Mathematics Project that will provide problems to illustrate the standards. Various individuals are also pitching in to help teachers. My homepage already has a long document explaining how fractions can be taught according to the Common Core standards; by the end of the year, I will have some documents on the teaching of geometry. So there are resources to make the situation more tolerable. What I find most worrisome is the fact that many educators and administrators believe that the status quo (of doing nothing) is plenty good enough. It is not. We need effective professional development, period.

RH: How aware is the professional mathematics community of the Common Core effort? As a policy observer, it seems like there's been relatively little activity on behalf of the standards by math professors or interested professionals. Is that fair? If so, why is that?

HW: What I have observed among most mathematicians in major research universities is a longstanding apathy towards all things related to schools in general, and the reason for that is complex but partly understandable. So long as school math education continues to be long on politics but short on intellectual substance, the apathy will remain. It has to be said, too, that the reward system in a research university does not favor work done about school mathematics; the reason in this case is perhaps self-explanatory. Nevertheless, there are very knowledgeable mathematicians like Richard Askey and Roger Howe who are making an effort to improve math education, and there are responsible organizations such as the American Mathematical Society that are trying to make the math community aware that, for a change, the extraordinary quality of the Common Core Standards merits extraordinary action. So we should not lose hope yet.

RH: What's your take on the state of the Common Core math assessments? How concerned are you about potential problems, delays, or fears that they'll give insufficient attention to "hard" math skills?

HW: I am not as well-informed about the math assessment efforts as I should be, but in general terms, I want to make sure that students will not be in any way over-assessed, and that the mathematical quality of the test items be above reproach (which has not always been the case; see Chapter 8 of the National Mathematics Advisory Panel *Report of the Tasks Groups*). Students should be assessed, but there is such a thing as too much of a good thing. On the other hand, I do not believe that a good mathematics education should pursue "hard" skills per se. But by maintaining the high mathematical quality of test items, one will automatically give proper attention to such "hard" math skills. In order to maintain such high mathematical quality, however, very competent mathematicians will have to be involved in the assessment process every step of the way.

RH: Last question: Big picture, what does "success" for the Common Core math standards look like in 2015? If things go well, how different will teacher preparation, math instruction, and assessment look?

HW: Nobody can pass judgment on the success or failure within a year of the kind of profound change promulgated by the Common Core math Standards unless the standards are an immediate disaster (which I hope they are not). I think a more reasonable date to make such a judgment is 2017. If things go well, teacher preparation will begin to concentrate on the most urgent need of the moment: better content knowledge. Math instruction in classrooms will be long on reasoning and short on giving out orders, and textbooks will at least be free of ghastly errors. Assessment will pay equal attention to one-step questions as well as those that require

multi-step reasoning. For anyone who is aware of what mathematics education is like at present, such seemingly modest goals, if achieved, would already be cause for celebration.

Public Education Statements

Utah Council of Teachers of Mathematics
45 East State Street
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(801) 402-5303

April 27, 2012

Utah State School Board
250 East 500 South
Salt Lake City, UT 84114-4200

Members of the Utah State Board of Education,

The Utah Council of Teachers of Mathematics (UCTM) is the largest professional organization of math educators in the state. Our organization includes hundreds of teachers, professors, math curriculum specialists, and administrators from Utah's school districts, charter schools, colleges, and universities. This letter represents the opinions, consensus, and recommendations of the UCTM executive board.

Some within our community have called for the abandonment of Utah's Common Core State Standards (CCSS). Their claims are based on misconceptions and are clearly laced with political ideology and rhetoric. Jockeying for state rights and ideological arguments about federal overreach are issues that belong in a caucus meeting, political convention, or in the capitol building – not in conversations about educational standards. Appropriate conversations about the CCSS should focus on the quality of the standards themselves and how to implement the standards so students will have increased opportunities for college and career readiness.

In order to provide a backdrop for the appropriate CCSS conversation, we offer the following observations:

- The CCSS are of the utmost quality and rigor (better than any other available standards).
- CCSS standards are vertically aligned (through grade levels) and have an appropriate breadth while focusing on depth of knowledge. This attribute is a rarity in education standards.
- The standards for mathematical practice will lead to better pedagogical strategies (with mathematical skills and aptitudes being taught alongside mathematical knowledge). Improved pedagogy will lead to increased student interest and success in science, technology, engineering, math, and other critical fields of study.
- The CCSS are based on the best research in developmental science and math education.
- Having been developed by a consortium of states, the standards were authored by the some of the sharpest minds in the country.
- The common nature of the CCSS allows for smoother transitions as students transfer between schools, districts, and states.

We raise our voices in strong support for continued implementation of the CCSS. We applaud your attention to this critical issue, and thank you for your efforts in advocating for the highest quality education standards in Utah.

Sincerely,

Utah Council of Teachers of Mathematics – Executive Board

Logan Toone – President (Davis)
Christine Walker (UVU)
April Leder (Alpine)
Pam Dallon (Alpine)
Dawn Barson (Alpine)
Daniel Carroll (A.L.A. Charter)
Travis Lemon (Alpine)
Joyce Smart (Logan)
Carrie Ziegler (Salt Lake)
Cynthia Price (Davis)
Andrew Glaze (Davis)
Amy Jeppsen (BYU)
John Hanks (Alpine)
Joy Coates (Iron)
Jerry Schaffer (Granite)
Jennifer Boyer-Thurgood (Weber)

Voices of Utah Educators

Kelly Thayer, Diana Karren, Kathy Price, Joshua Frampton
"We are the kindergarten team at Daybreak Elementary School and have been informed that the legislature wants to change the CORE standards again. We must put a stop to the madness. Our team has spent over $5,000 in getting grants to help us buy things to teach the new science and now the new math. We have spent over $500 or more each out of our own pockets to get the things necessary to teach the new core. We are planning on getting $8,000 in Donorschoose funds in September to accommodate the new math program. If the state keeps changing the CORE, we'll be at a loss and will have to start over again. This is simply not fair to the teachers that truly care about teaching children correctly; which means not work sheeting them to death, but making learning fun and enjoyable! We take our teaching the CORE very seriously and put our whole hearts into it. Please keep it in place for at least 5-10 years."

John Thomas, Principal, Teacher Spring City Elementary
"I am writing to support the new Utah Mathematics and Language Arts standards, referred to as the Common Core State Standards. I have been in the education community for many years and during that time, many changes have occurred in the Utah curriculum. The previous math standards, for example, have been effective but were criticized as not being rigorous enough. The new standards are much more rigorous and I believe they will better prepare students for a more effective mathematics experience in college or in a career. I have had the opportunity to study and teach mathematics using a problem solving approach for many years following the teaching standards developed by the National Council of Teachers of Mathematics. I am excited the new Utah State Mathematics Standards emphasize helping students learn to think deeply about mathematics and not focus exclusively on facts or arithmetic. This is a challenging transition, but it is critical that we do not give up these more rigorous standards because of fear. Student performance can and will improve only if we are willing to change our thinking and believe that they can learn at a deeper level. Of course, certain challenges exist such as comparing our state with other states, but these standards will actually, for the first time, provide a comparable measure. There are those who fear this comparison and others who seem to believe that these common standards are somehow the result of a federal initiative. I hope we do not let fear drive what we do in our Utah Education System, but rather focus on the purpose of these standards which is to help all students be better prepared to enter university scholarship or enter an increasingly competitive work force.

Becky Ball, Centerville Junior High

"I teach math at Centerville Junior High and am so excited to teach this new curriculum this year. Over the past two years I have slowly started adapting lesson plans to move toward this improved method of instruction. I have enjoyed teaching more than I ever have before. Please don't reverse this course! How fun it has been to teach this way, with carefully structured peer interaction and problem-centered

learning based on progressions of problems. The students do not have to ask "When are we ever going to use this?" because they ARE using it to solve real-world problems. I had so many positive comments from parents last year who said that math used to be their children's least favorite class and that it was now their favorite class. Many of the students who I taught last year in 8th grade requested to be in my class again for 9th grade, not because I am a fabulous teacher, but because they enjoyed this type of learning environment that teaching the common core standards involve.Those who are encouraging us to go back to our old ways must have never taught both methods or did not teach problem-centered strategies correctly. Give it a chance! Thanks for your concern."

Cathleen Gilbertson, Sixth Grade, South Clearfield Elementary

"I am a teacher at South Clearfield Elementary in Davis District. My first experience with the common core was last year in math in my sixth grade class. My opinion of the common core from two angles. I am a parent of three children and a retired military member as well as a teacher. The benefits that I foresee to use the common core as a teacher is that it allows us, teachers, to go further in-depth with each concept. Reducing the number of concepts that need to be taught makes it possible for the students to really understand the concept in a way that time did not allow before. It creates an environment that promotes more critical thinking skills by allocating time to question and discover the math concepts. It is a way to improve on the students' understanding on how they discovered the answer instead of just knowing the answer. Furthermore it creates an education system that is standardized nationwide. When we compare the states to each other the data from the standardized testing is being used. If the tests are not created from the same curriculum for each grade it is not an accurate assessment on the value of the education system. When each state uses their own curriculum one grade may be tested on more difficult or less difficult concepts. If every state was basing their testing on the same concepts at the same grade level it would make the data from the standardized testing more accurate for comparison. As a parent that moved from one state to the next, it was often my concern that my children would be ahead of or worse behind the concepts that were being taught each time we moved. This is a big concern for military members when they are transferred. As a teacher it is sometimes frustrating to welcome a new student into the class, and to realize that they came from a state in which they are not taught the same concepts. Math is very much building on skills. When a student misses a concept because it was not taught in the last school, it makes it difficult and frustrating on the student as well as the teacher trying to get them caught up. My school has many such cases because we are located near Hill Air Force Base. "

LOGAN CITY SCHOOL DISTRICT

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August 19, 2012

Dear Education Interim Committee Members:

I have been an educator for over 30 years. I have been a Math Teacher Specialist for the past 5 years. During my time as an educator in the State of Utah I have witnessed a lot of change in regards to Math Standards.

I was elated when I first heard about the Common Core Standards and how they came to be. I have spent the last three years educating and preparing teachers in the Logan School District to be prepared to teach them. WE ARE PREPARED AND READY. We began early implementation last year and the teachers have told me it was difficult for the students but that they were up for the challenge. They saw gains in students' ability to perservere and problem solve. Those practices are embedded in the mathematical practice standards of the Common Core. These standards work and they will help students understand math better. DON'T CHANGE THE COURSE!!!.

Never in my carreer have we had a time when our standards remotely matched the rest of the states, until now. It has been extremely difficult to match assessments with textbooks and the Utah Core. It has always been a "huge disconnect". I don't understand why we would want to dismiss the Common Core now.

I presented at a National Math Conference in Las Vegas this past July. It was so refreshing to be able to present materials with a scope that matches across the fifty states. I was able to have discussions with educators all across the U.S. and for the first time, we were all on the same page.

So much time and energy has gone into researching "best practices" for these Standards and I think they are truly amazing. Are they perfect? No. Nothing ever is. But please don't pull the plug before we've even had a chance to try them out.

Utah has a poor track record for exhibiting a very short attenion span in regards to education curriculums and programs. We rarely give something long enough to determine it's validity before we're changing the course.

Please consider all of the hours of preparation that have gone into the Common Core Standards, thus far. and give it a FAIR CHANCE TO SUCCEED.

Sincerely,
Barbara Child
Math Teacher Specialist
Logan City School District

GOOD
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ACADEMY

Building Knowledge and Character

August 20, 2012

Thoughts on Education in Utah and the Common Core

According to *Education Week* August 2012, Utah ranks 40th out of 50 states in its ability to educate students (Edweek). *Huffington Post* ranked states recently on their teaching of math and science using national scores to rank. Utah falls at 21, in the average range. The Science and Engineering Readiness Index (SERI) measures how high school students are performing in physics and calculus -- based on publicly available data, including Advanced Placement scores, National Assessment of Educational Progress reports, teacher certification requirements by state and physics class enrollment data (Huff Post).

This data and many other easily accessible rankings prove that obviously the current educational structure in Utah is found lacking, and in order to gain ground for students who are currently behind, and those entering the system, Utah needs to make some radical changes.

The common core standards are such that they simply state an "acceptable and expected" level of education that should be achieved by each student before moving onto the next level. Now that Utah has put aside the No Child Left Behind Act, educators adopting the core can measure student knowledge as they progress, and actually keep children back that have not achieved the core standard for the grade. Students who are then retained have another year to grasp and master material knowing that their peers in almost all states in the country have mastered at least the basics outlined in the core.

The Common Core is *not* a curriculum, simply a guide used to determine the base minimum for students to master. The fact that it is not a curriculum allows schools like ours to not only meet the core, but exceed it widely by choosing a rigorous curriculum.

As a teacher in a public charter school, who has transplanted in Utah from New York, I can honestly state that students in Utah need the common core skills as a minimum. As a nominee for the Presidential Award for Teaching of Math and Science, currently finalist in the Math Hero award from Raytheon, a math coach for the 5/6 grade team at my charter school, and Utah's 2011 Charter School Educator of the Year, I can say with all due respect that Utah desperately needs a well articulated and attainable standard in order to compete nationally. I have attended the Utah Common Core Academy for two years in a row, once for math and once for science. I spent weeks with educators that were not prepared to explain simple elementary concepts necessary for a foundation in both of those domains. Utah teachers need to take the core seriously and drive themselves forward to learn and master the material before teaching it. Rigor and valuable learning experiences will allow for better education and the ability to compete, but only if administrators and educators are willing to do the work and see the outcome. Personally I totally support the adoption of the Common Core for Utah schools. I see it as a way to promote Utah to move up on the national rankings.

Respectfully submitted,

Susan Goers

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FOUNDATIONS
ACADEMY

Building Knowledge and Character

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Gunnison Valley Elementary School
Grant Hansen, Principal
PO Box 369
550 South 300 East
Gunnison, UT 84634
435-528-7880

August 16, 2012

To Whom It May Concern:

I am writing this letter in support of the Utah Common Core Standards in Mathematics. The new standards are more clearly defined. They provide better scope and sequence between grade levels. They are more closely related to the maturity level of the students. The new standards are more rigorous. The pacing of instruction will be increased and the mastery of concepts will be emphasized to a greater extent than what is expected in the current standards. The focus will be on greater understanding of the basic math concepts rather than a cursory understanding of many, many concepts.

Some say that we are giving up out autonomy as a state by using the new Utah Common Core Standards. I disagree. We have been involved in the decision making process. We have worked as a district and a school to use the new standards to improve instruction. They teachers have received extensive professional development related to the new standards. This is not the time to take a step backwards.

Sincerely,

Grant Hansen

Grant Hansen
Principal, Gunnison Valley Elementary School
Elementary Mathematics Coordinator, South Sanpete School District

Dr. Steven K. Hirase
Superintendent of Schools

Murray City School District

August 20, 2012

Brenda Hales
Utah State Office of Education
P.O. Box 144200
Salt Lake City, Utah 84114

Dear Associate Superintendent Hales,

I am writing you this letter to share my perspective regarding the importance of the common core standards as it relates to the future success of the students in Murray School District. As the superintendent of Murray School District, I made the commitment to our school board when I was hired two years ago to make our district a "light house" district where students were provided with educational opportunities to realize their potential so they would be able to compete in a global society. As I made this commitment, I recognized the importance of the common core standards in establishing the foundation for the world-class education that our students are entitled.

As the assistant superintendent when the common core standards were adopted by the state legislature and state board of education, I had the privilege of educating our community on the importance of the standards. I met with our region PTA group, individual community councils at each of our schools, and conducted general parent meetings. After I had the chance of presenting specifics regarding the common core and answering questions that the participants had, I received overwhelming support for the state's decision to move in this direction. The greatest message I received from the participants at this meeting was their excitement regarding the level of rigor involved and how it would prepare their children to compete in a global society. Also, as I have had the chance to review the expectations of the business community as articulated in Prosperity 2020, as well as Governor Herbert's 66% of Utah's population to have obtained a postsecondary degree or certificate by 2020, our best chance to meet these goals is through the implementation of the common core standards.

I appreciate this opportunity to express my feeling regarding the common core standards to you and hope that your office as well as the Utah State Board of Education continue to support the state's adoption and implementation of these important standards. Please let me know how I can be of assistance in your efforts.

Sincerely,

Steven K. Hirase, Ed.D.
Superintendent

147 East 5065 South ■ Murray, Utah 84107
(801) 264-7400 ■ Fax (801) 264-7456

ALPINE SCHOOL DISTRICT

575 North 100 East American Fork Utah 84003-1758 (801) 610-8471
April Leder –Elementary Mathematics Specialists K-6

I am writing this letter because of my concern regarding the controversy caused by the implementation of the Mathematics Common Core State Standards in the state of Utah. I am the Elementary Math Specialist for Alpine School District and I have been in this position for the past six years. I am filled with trepidation when I hear about educational decisions that are being made without the input of teachers and other district office personnel across the state. The purpose of this letter is to express my opinion regarding the Common Core State Standards.

I do support the Common Core State Standards. Although I believe it will take time to see the effects of the standards, five to seven years according to experts to implement a change of this proportion, I do believe they will benefit the children in the state of Utah. One of the benefits is that we will be able to compare our scores of assessments to those in other states. We as a state cannot improve if we don't know where we stand compared to others.

The voices that are speaking out against the state core at this time do not represent the majority of the people in the state or across the nation. Many of the voices speaking out against the Common Core are doing so because of scare tactics that are being used. People have been made to believe that there is federal funding tied into the core. There is no proof that this is the case. They have also been made to believe that as a state, we can create our own standards with a reasonable amount of time, money and resources. This is not true! Experts throughout the nation developed these standards after much intense research and evaluation. It would be ridiculous to attempt to duplicate this work with a smaller budget, less expertise to draw from, in a shorter amount of time.

Let's give the new standards adequate time to implement. If after a reasonable amount of time it's not working, five to seven years, we can decide at that time to pull out of the adoption of the Common Core State Standards.

Sincerely,

April L. Leder
Elementary Mathematics Specialists K-6

GATEWAY PREPARATORY ACADEMY
A MONTESSORI CHARTER SCHOOL

August 17, 2012

To Whom it May Concern,

As Principal/Director of Gateway Preparatory Academy in Enoch, Utah I am writing in support of the Common Core State standards in Mathematics and Language Arts. We have been able to take advantage of week-long training for most of our staff the last two years and I have personally participated in training during the past two school years; First in literacy and then in Math and Science.

As a Montessori Charter school we have our own curriculum and performance goals. We are hands on in our approach and very student centered. We are multi-aged with a three year learning cycle rather than just one. We have some challenges in addressing the Common Core. We have invested time in developing our own scope and sequence and in correlating our curriculum map with the Common Core.

We are finding that the Common Core is rigorous and yet accessible. It challenges our students to think mathematically and to learn to express themselves and explain their reasoning. It is interdisciplinary in focus. These are all traits that we desire for our students in a Montessori program and that we feel are enhanced by the Common Core.

The Common Core has helped to give us focus and direction. Our teachers are better at their craft and our students are learning and progressing with a strong foundation and serious goals. Our concern was that the Common Core would detract from our principles and goals as a Montessori school but what we are finding is that it enhances and strengthens our work. We hope for a good assessment model that will focus on individual growth as it helps us measure ourselves against each standard

It seems that we should be focused on high standards, quality instruction and explicit learning. The Common Core promotes that. It seems that how it was developed or where, when or by who should be of less consequence than the end product; a standard that promotes rigorous learning and a quality education. That is the Common Core and that is why it has my endorsement and why it is supported by our school.

Thank you, Rob Lee

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SEVIER SCHOOLDISTRICT

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MYRON A. MICHELSON SUPERINTENDENT | *GAIL W.ALBRECHT* ASST. SUPERINTENDENT | *PATRICK D. WILSON* BUSINESS ADMINISTRAT

August 17, 2012

Dear Ms. Hales,

Sevier School District Administration and Board of Education have requested this letter of support be written for the Utah Core Mathematics Standards sometimes referred to as the Common Core for Mathematics.

The purpose of this letter is to express Sevier School District's complete and absolute support of the Utah Core Mathematics Standards sometimes referred to as the Common Core. The backing of School Board Members, District administrators, mathematics teachers and many parents became firm once training on the new core and information about the integrated model was provided. After receiving information, one board member who is an engineer stated, "That just makes sense. I wish I had been taught that way."

Mathematics teachers who attended the Common Core training provided by USOE expressed their gratitude and agreed that the integrated model was the right thing to do. Teachers who taught Math Common Core I last year, despite having to complete extensive preparation, having to teach without a text, and working to stay one step ahead of the students, still felt it was the right thing to do.

The table below shows Sevier School District's 2012 spring NWEA assessment scores for Algebra I. Notice that this spring's scores are higher than the previous year when Algebra I was taught more traditionally.

Sevier School District Spring 2011-Spring 2012 Proficiency and Growth Comparison for Algebra I			
% Met/Exceeded National Norm		% Met/Exceeded Growth Target	
2010-2011	2011-2012	2010-2011	2011-2012
78%	**80%**	**80%**	**87%**

This increase in achievement and growth scores is especially significant because usually when the core changes, there is an initial decrease in scores due to the fact that teachers aren't as familiar with the intricacies of the new core. The increase in Algebra I scores may also indicate that students benefited from the integrated approach identified by the new math core and gained a deeper understanding than they were able to get when instruction was delivered in artificial silos of "Geometry" or "Algebra." Assessment data, and anecdotal information indicate that the new math core helps enable students to see connections, make real life application, practice

problem solving skills, and use higher level thinking rather than relying on rote completion of math processes.

Simply put, going back to the previous, less rigorous core would be doing a disservice to students. It would decrease their ability to compete internationally and to be college and career ready.

Should you have questions, we could happily put you in touch with mathematics teachers, parents, or students who could speak for themselves regarding their support for the new core in mathematics.

Regards,

Myron Mickelsen
Superintendent, Sevier School District

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Nedra Call, Seth Sorensen,
Nebo School District
350 South Main
Spanish Fork, UT 84660
August 17, 2012

Members of the Utah State Legislature

Members of the Utah State Legislature:

We as Teachers, Curriculum Experts, and Administrators of Nebo School District are writing this letter to express our support for the new Utah State Standards "Common Core", and are including both testimony and data to illustrate the incredible positive impact it is having on student learning and understanding.

In Nebo School District during the 2011-2012 school year we implemented the new Utah State Standards in grades 6 and 9 for mathematics. Because of this implementation Nebo has seen an increase in student learning as measured by both district level and state level assessment as outlined in later portions of this letter.

Additional indicators of the effectiveness of this new core are the increased rigor as indicated by our teachers, and (after a combination of teacher professional development, student interventions and remediation) the lowest number of students receiving failing grades at the secondary level in mathematics in recent history.

The charts included on the following pages highlight the incredible gains made by one school, Taylor Elementary School, which is the most highly impacted school in our district. This school has the highest percentage of economically disadvantaged students, highest percentage of English Language Learners and highest percentage of minority students in Nebo school district. This data illustrates the incredible growth of all students 6th grade, resulting from the implementation of the common core.

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The charts shown for Taylor Elementary in *figure A* illustrate the overall growth and percentage of students proficient in 6th grade mathematics based on the Utah State CRT Assessment.

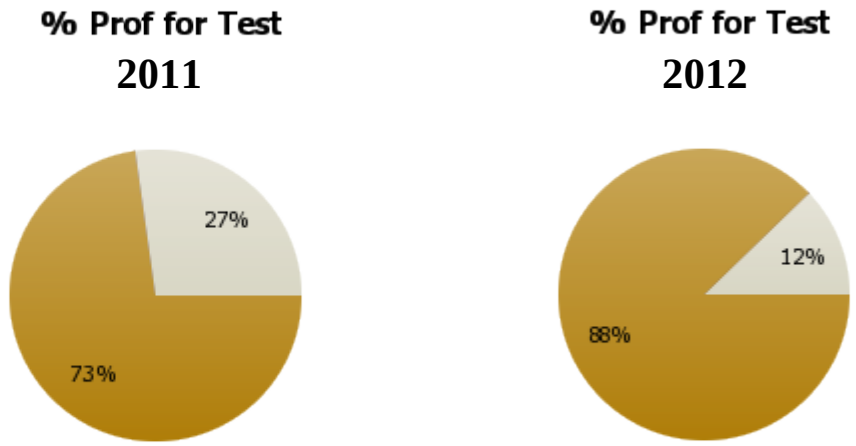

Figure A

The charts shown in *figure A* show that in 2011, 73% of students were proficient, whereas in 2012 88% of students were proficient. This is a 15% increase in the percentage of students proficient on the State CRT Assessment, which is statistically a very significant increase.

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The charts for Taylor Elementary shown in *figure B* illustrate the progress of students at every level, not just those who were not proficient, but also those who were proficient, indicating that students at all levels were benefited by the core and instruction.

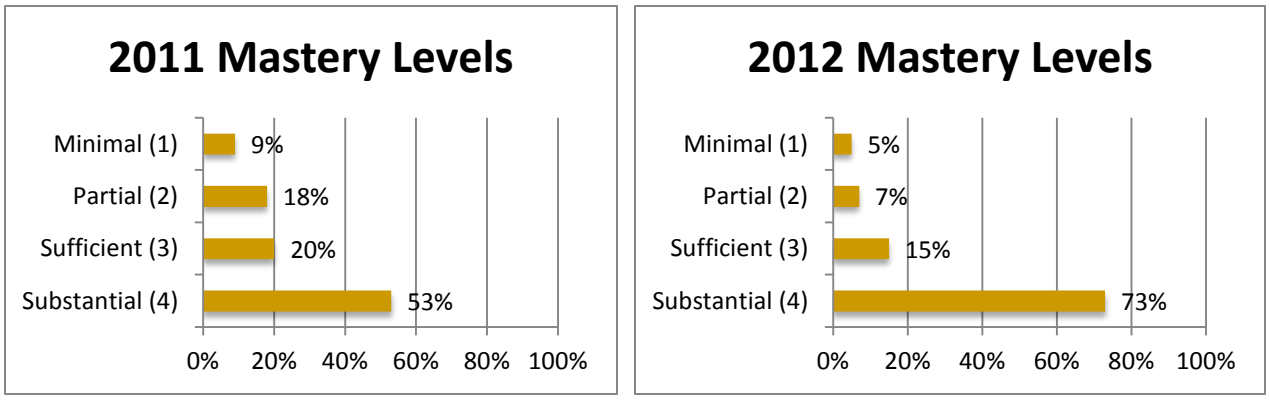

Figure B

The charts shown in *figure B* show that in 2011, 53% of students were at the highest level of proficiency, whereas in 2012 73% of students were at the highest level. This is a 20% increase in the percentage of students at the highest level of proficiency on the State CRT Assessment, which again, is statistically a very significant increase. Likewise, the two levels (Minimal & Partial) indicating that students were not proficient shrunk by a very large margin, 15%.

We have also collected data and statements from our teachers who have now implimented and used the core for a full year. We have attached a letter from Sandra Coxson, a teacher at Spanish Fork Junior High in Spanish Fork expressing her thoughts and experiences with the Common Core.

Thank you for taking the time to review our letter and please feel free to contact us with any follow up questions or comments.

Sincerely,
Nedra Call
Curriculum Director 801-354-7421

Seth Sorensen
Curriculum/Assessment Specialist 801-354-7465

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The New Common Core for Mathematics

Sandy Coxson, Teacher
Spanish Fork Junior High School
Nebo School District

The New Common Core State Standards (CCSS) and the Utah implementation of the Standards are a huge stride in the right direction for the study of math in the state of Utah. I have been a classroom math teacher for 22 years and have taught PreAlgebra, Algebra I, Geometry, Algebra II, Applied Math, inclusion classes, credit recovery classes and remedial classes. This past year, I had the privilege of teaching the new ninth grade course, Secondary Math I, and for the past year and a half, I have also been on district committees that have studied the new Core and developed materials for the secondary teachers. Having had this wide range of experience with levels of ability and content puts me in a unique position to evaluate this new core change. I would like to address four things: (1) using my experience with Secondary Math I and my knowledge of the Core from 6th grade through Secondary Math III, I will discuss the integrated model that Utah has chosen to adopt; (2) I will talk about the real-world approach to mathematics through content and the Mathematical Practice Standards; (3) I will answer the argument about the access to learning of all student; and finally, (4) I will show the freedom that teachers on the local level have to teach.

In the state of Utah, we have adopted the integrated model. For instance, Secondary Math I is the ninth grade integrated math class that contains concepts from Algebra I, Geometry, Algebra II and Statistics. In addition, the Honors Secondary math I class also contains PreCalculus concepts. Using an analogy from one of the teachers in our district, it is like how a PE teacher might teach. The teacher could teach a soccer unit, a volleyball unit, a football unit and a basketball unit separately. This would be the traditional approach. The integrated approach would have a ball handling unit that would teach and contrast the ball handling skills in each of the sports. Then, there might be a scoring unit, and a physical conditioning unit and so forth. That is the way it is in the integrated model for the new Core. In Math 6 the emphasis is on number sense, integers and unit rate. They use all of the subjects – arithmetic, PreAlgebra, Geometry and Statistics – to teach those main topics. Math 7 is centered around rational numbers, connecting arithmetic to Algebra and proportional reasoning. In Math 8, students build upon what they have learned in Math 6 and 7 and concentrate on linear relationships. Secondary Math I, II and III all deal with functions. Secondary Math I continues the discussion of linear relationships with an in-depth study of linear functions contrasting them with exponential functions. Secondary Math II adds quadratic and absolute value functions and all of the related topics and introduces concepts dealing with radical and rational functions. Secondary Math III continues the discussion of radical and rational functions and adds logarithmic functions, revisits exponential functions for an in-depth study, and studies trigonometric functions. The advantage of this model is that students study fewer topics each year and use all of the mathematical disciplines to study all aspects of each of the topics. For example, in a traditional Algebra I course, students would study rational numbers, proportional relationships, linear equations and

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functions, and quadratic functions. In the Secondary Math I class, students study linear functions and exponential functions as they appear in tables, graphs, equations, contexts such as sequences, coordinate and Euclidean geometry, and Statistics, and as used to solve real-world problems. In the integrated model, by the end of Secondary Math III, the student will have studied ***all*** of the topics in PreAlgebra, Algebra I, Geometry, Algebra II, Trigonometry and Statistics that were covered in the traditional Algebra I, Geometry and Algebra II classes. The honors classes will have included all of PreCalculus, preparing the students to take AP Calculus following Secondary Math III Honors.

In addition to the content standards, there are eight Mathematical Practice Standards that help students develop thinking and problem solving skills. These are inseparable with the Core Content Standards and cannot be ignored. The intent of the new Core, both in content and practice, is to prepare students to enter a career, either by way of college, a technical program or immediately. The jobs of today require employees to have mathematical and problem-solving skills that were not needed or required a hundred years ago. Students then could function very well on an 8th grade education. However, with the highly technical jobs of today, students need more than basic arithmetic, Algebra and Geometry skills. They must be able to use those skills to solve problems. In addition, they must be able to choose the correct tools they need to solve a particular problem. To illustrate, a class using the new Core might be given a real-world problem and told to use the skills, or math tools, they already know to solve it. ("Using appropriate tools" (5) is one of the eight Practices.) Then, they would use that same problem to explore new ways of solving the problem, thus adding new tools to their supply. From there, new questions and topics would be introduced and they might spend another class period exploring the problem in other contexts to find other applications for their knowledge. Next, they might come up with problems and situations of their own that use this same set of tools. Finally, they would practice and solidify their use of the tools, mathematically, in more traditional formats. Others of the Practices that would be addressed in these lessons would be (1) Making sense of problems and persevering in solving them, (4) Modeling with mathematics, (7) Looking for and making use of structure.

One of the main arguments against the core is that "all students do not have access to the learning" and that "it does not challenge those who are more gifted". Because each grade level addresses fewer topics than in previous Cores, there is a great deal of freedom to adjust the Core as needed to meet the needs of individual students and/or classes. For instance, this year I am teaching regular Secondary Math I classes in addition to an inclusion class, which contains higher functioning resource students as well as regular ed students who struggle, and an honors class. Although in all of the classes we will study the same curriculum with the same major skills being learned, the classes themselves will go to a different depth of learning. Even, within classes, differentiated instruction occurs to meet the needs of a wide range of abilities. Better students will be challenged with another question or way of thinking about a problem and will sometimes be asked to support struggling students, thus helping to solidify their own learning. Students who struggle will be given more support and time in their quest for learning and will not be asked to go into as much depth as other students will still learning the same topics.

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This brings up another argument against the new Core which is that it is a mandate from the Federal government and that it takes the freedom of decision away from the local school districts and classroom teachers. Reality is on the opposite end of the spectrum – it actually gives the individual teachers MORE freedom than ever before. In the first place, the origin of the new Core was with a coalition of STATES, NOT the Federal government. This is fact, regardless of what opponents are saying. Moving beyond an argument over facts, after the coalition finished developing the Core, it was up to each State to adopt it or not and then to interpret the Core for their own state school districts. Each state has done this a bit differently but the actual Core concepts are the basis for the state Cores making it possible for students to move from state to state, as is more and more common in our very mobile society, and still receive the same learning. After the states make their recommendations, it is up to each school district to form those recommendations into a format that meets the needs of the individual schools in their district. Then, teachers in each school, meeting as departments and teams, develop for their school a plan for exactly how they are going to implement the Core standards. Finally, each classroom teacher teaches each lesson, according to the plan, constantly monitoring and adjusting for the ever-changing needs of the individual students. Because there are fewer concepts, the teachers are no longer worried about "covering" all of the curriculum but can concentrate on making sure that students master each concept and skill, spending as much or as little time as is needed to accomplish this.

I have taught using three previous Cores in my 22 years of teaching and I have found this change in Core to be the most innovative, comprehensive and liberating of all of them. I simply cannot see anything in it that would be detrimental to students because it gives them an integrated approach that promotes deeper understanding and less segmented learning, it helps students develop problem-solving skills that will help them in their future lives, it gives access to the learning to all students regardless of ability, and it gives freedom to individual districts, schools and, especially, teachers to meet the needs of each student.

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Kendall Benson
Principal

Trent Nielsen
Assistant Principal

Samuel El Halta & Ashley Whiting
Counselors

Kalene Eilers
Administrative Assistant

Aug 16, 2012

To whom it may concern:

My name is Trent Nielsen. I am currently the Assistant Principal at Cedar Middle School in Iron County. I am also the District Math Coordinator. Prior to my role as Assistant Principal I taught various middle school math courses for 10 years. For the last two school years I have been working to implement the Utah Core into various grades in Iron County School District.

I am writing this letter to show my support for the new Utah Core Standards. I am very concerned about the negative comments that I have been hearing in reference to the core standards.

One of the reasons I support the new Utah Core Standards is the emphasis that it puts on critical thinking. I was an advanced math student in junior high and high school. However, when I enrolled in college I quickly learned how unprepared I was for advanced math classes. In high school I was taught how to be a "math doer." I was easily able to do the math if I was given an algorithm, but I was not taught how to problem-solve. The math standards are divided into two sections: the content standards and the Standards of Mathematical Practice. The Standards of Mathematical Practice apply to students of all grade levels. These standards help to create "math thinkers." For example, Standard One is "Make sense of problems and persevere in solving them". I believe if my high school teachers had incorporated this standard alone I would have been more prepared for my college courses.

While I was a math teacher, I was also the Mathcounts coach for my school. Mathcounts is a nationwide math competition. The majority of the problems used in Mathcounts competitions require utilization of Standard One and many of the other Standards of Mathematical Practice. In other words to be "math thinkers" and not just follow and algorithm. I recently visited with one of the students who had participated in the program. He was pleased to inform me that he scored a 36 (a perfect score) on the math section of the ACT. He credited the Mathcounts program for his success more than teaching that he had received in the math classroom. He felt it was because he was forced to problem-solve and look at the problems in different ways.

It has been exciting to work with teachers as they have implemented the new core standards this past school year. One of the changes that was implemented was the inclusion of "tasks." Students were given application problems that could be solved different ways. At first, teachers were nervous about changing their approach and forcing their students to become problem solvers. However, as the year went on, they were excited to see the progress in their students. Standard Three of the new core is "Construct viable arguments and critique the reasoning of others." These tasks allow students to share their work with the class and explain their thought

process. The class can then learn from the student or help the student adjust his work. I believe this process helps our students not only learn from each other but also prepares them for their future schooling and careers. The teachers were excited about the progress their students had made by the end of the year.

These are only a few of the reasons that I am excited about the new Utah Core Standards. I urge you to continue to support their implementation in our schools.

Sincerely,

Trent Nielsen
Assistant Principal
Cedar Middle School
District Math Coordinator
Iron County School District

Hello Senator Van Tassell.

My name is Carrie Bala and I am a secondary math teacher at Wasatch High School in Heber City. I am writing in support of the new Utah Secondary Math Core.
I have been involved in curriculum writing for the past two years and feel that the new standards are rigorous and filled with connections that were previously lacking. The standards set high expectations for all students. These new standards are also providing opportunities for me to improve my own mathematical understandings. This is good mathematics!!!
Through the Core Academies, Utah is also improving pedagogy, providing practical, research-based techniques for teachers to use in the classroom. I was able to teach Secondary I this past school year and implement the new standards and some new techniques. I was extremely excited and proud of the student achievement. As compared with traditional classes, these students had a deeper understanding of mathematics and were better able to communicate their understanding to each other and to me. I actually had moments when I was grading tests that I had to stop and share student responses with whoever was around me, because the responses were so clear and awesome!
Please continue to support the Utah Core (and teachers) in your upcoming sessions.

Thank you,
Carrie Bala

Brenda Hales,

I am in charge of getting the 1st grade program for Davis School District written. We are starting into the new core this coming 2012/2013 school year. I'm just a first grade teacher who loves analyzing everything and using data to see what's working.

My writing team has been so excited about the changes and the focus on number sense. I started implementing parts of the core last year with my class to see how things would work. I focused a lot on the specific examples of how to add and subtract listed in the core. I found great success.

We've had 1 full-day professional development to introduce the core and share the first 9 weeks of lessons in our district for first grade. The teachers are excited to cut down the broadness of our core and really go deeply into each area. We are also distributing AL abacuses to all of the first grade classes and these are used in our lessons. I'm not sure what kinds of information you are looking for but I will attach our year's scope and sequence so you can see what we are teaching. Each of these weeks represents four progressive lessons.

Rachel Jones
1st Grade Teacher
Creekside Elementary
801-402-3650

Dear Ms. Hales,

I am a teacher at South Clearfield Elementary in Davis District. My first experience with the common core was last year in math in my sixth grade class. My opinion of the common core from two angles. I am a parent of three children and a retired military member as well as a teacher.

The benefits that I foresee to use the common core as a teacher is that it allows us, teachers, to go further in-depth with each concept. Reducing the number of concepts that need to be taught makes it possible for the students to really understand the concept in a way that time did not allow before. It creates an environment that promotes more critical thinking skills by allocating time to question and discover the math concepts. It is a way to improve on the students' understanding on how they discovered the answer instead of just knowing the answer.

Furthermore it creates an education system that is standardized nationwide. When we compare the states to each other the data from the standardized testing is being used.If the tests are not created from the same curriculum for each grade it is not an accurate assessment on the value of the education system. When each state uses their own curriculum one grade may be tested on more difficult or less difficult concepts. If every state was basing their testing on the same concepts at the same grade level it would make the data from the standardized testing more accurate for comparison.

As a parent that moved from one state to the next, it was often my concern that my children would be ahead of or worse behind the concepts that were being taught each time we moved. This is a big concern for military members when they are transferred. As a teacher it is sometimes frustrating to welcome a new student into the class, and to realize that they came from a state in which they are not taught the same concepts. Math is very much building on skills. When a student misses a concept because it was not taught in the last school, it makes it difficult and frustrating on the student as well as the teacher trying to get them caught up. My school has many such cases because we are located near Hill Air Force Base.

Cathleen Gilbertson
Sixth Grade
South Clearfield Elementary
Davis District

From: mcarlson@wasatchpeak.org [mailto:mcarlson@wasatchpeak.org]
Sent: Thursday, August 16, 2012 1:35 PM
To: Hales, Brenda
Cc: Shepard, Sandy
Subject: Positive Math Common Core Experiences

On this web page under "Student Screencasts: Math" are some examples of what my students achieved using the common core inquiry/discovery based learning philosophy. These math problems were given prior to teaching the math concept relative to the assigned problem.

The first three or four months were very challenging both for myself and for the students i.e., trying to train students to be problem solvers rather than answer finders. However, as the year progressed, it became easier for the students to solve problems that they had not seen before and was very satisfying to myself as their teacher.

In my opinion, the new common core is the only way to teach math/science. It is very challenging and preparation intensive for the teacher but the payoffs are well worth it. My use of this approach was not as successful in my language arts class... But will work on implementing it there more fully this year hopefully with improved results.

If you have any questions or would like more input please feel free to contact me at mcarlson@wasatchpeak.org.

Mr. Carlson

Dear Mr. Smith,
I want to let you know why I think that the common core is great. I am 64, and I teach 3rd grade. Back in the day when I learned about math, my teachers never taught me **why** I did what I did. I just went through the motions to solve the problem. I hated story problems because I wasn't taught any strategies to help me figure out what operation to use, etc. As a result, I hated math. When I became a teacher (long ago) I realized that I needed to understand the why in order to teach, and what's more, I needed to teach my students the **why** about what they were doing. Understanding that idea has totally changed the way I feel about math--and most years, my students end up loving it too! However, I don't think that all teachers have helped their students get a good foundation in understanding the basics. The Common Core gives us a common focus on teaching why we do what we do, and further encourages manipulation and experimentation to help us realize that there may be many ways to solve a problem. I love that idea! I know that it is hard for many to have to "learn to play the game" another way, but I believe it is necessary to teach our students to think for themselves and develop stamina to work creatively to find solutions to problems. By doing that, we are giving them strategies and skills to achieve in life. We can all use all the help we can get in that regard. I may have grumbled too at the beginning, but now I embrace the idea because I believe it will help us do the job we should have been doing all along!
Marilyn Mitchell
Third Grade Teacher
South Summit School District

I am an elementary teacher writing to you to express my strong support of the new Utah Common Core Standards. Math has always been my favorite subject to teach. Through the years I have been impressed with what my students can do when challenged. The new Utah Common Core Standards are based on international bench marks which will help our students compete globally.

During the last two days I have been working with a group of teachers to pace and prepare our curriculum for this school year. Each of these teachers were enthusiastic about the new standards. Our greatest asset in our work was being able to use expert materials that was prepared by other teachers in different states. This will continue to be a good thing for teachers and students.

I believe one of the strengths of the Common Core Standards is the thoughtful structure of the Learning Progressions. It is my understanding that the developmental stages of student learning helped structure these Learning Progressions. These progressions will ensure our students master skills and are competent when they leave High School. These learning progressions have given focus to each grade level.

The Utah Common Core Standards will be a great thing for Utah students.

Kris Weiss
Instructional Coach/ Math Coach
Park City School District

I feel that the Common Core Math is very good. It lets us spend more time on fewer standards so that the children really get a deep understanding and learn the concepts very well, instead of teaching many things where you only have time to touch the "surface" and they don't remember every thing. The new common core math makes children think and explain how they get to their answers, more so than any other math standards we have had in the Utah Core for my 25+ years of teaching. I also like the common core for the fact that all the children in each state (even each city for that matter) should be learning the same things, probably close to the same time line, so that when children move around they shouldn't be behind their new classmates or way advanced either. In the past you never knew what children from other states or cities were learning. This makes it nice to know that everyone is on the same standards and objectives.
I don't understand why some people see the common core as a bad thing. They obviously are not teachers "in the trenches".

Wendy Radke Kindergarten Teacher

Davis School District
Established 1911
Dr. W. Bryan Bowles, Superintendent

Celebrating 100 Years of Learning

August 17, 2012

Education Interim Committee,

I am writing this letter in support of the common core state standards for mathematics. I have been in the education system for 28 years and I have never seen such a coherent, manageable, and clear set of math standards produced.

We are in a society such that we feel we must move as quickly as we must; we must get from point A to point B at a very fast pace. Finally, the authors have given much thought, done their research, listened to how other countries might be setting goals from research, and have produced a set of standards that are actually possible to teach with depth and rigor in one year's time. Teachers can teach concepts to mastery through real world application and know the students will remember the concept when they get ready to apply it for the increasingly more difficult curriculum in the future or the workplace.

The standards provide the connectivity between districts, states and the world.

Every administrator, teacher, and parent wants what is best for their students. This set of math common core state standards is the best step forward for our students. We want to give our future more and better than what we had – so let's give our children the benefit of what the common core math standards have to offer.

Thanks for your time again,

Dory Stevens

Dory Stevens
Davis District Elementary Math Supervisor

45 East State Street • PO Box 588 • Farmington, Utah 84025 • (801) 402-5261

August 17, 2012

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Thanks for your time again,

Dory Stevens
Davis District Elementary Math Supervisor

Business and National Support

NELSON
LABORATORIES

18 August 2012

Dr. Tami Goetz
Utah State Science Advisor
Governor's Office of Economic Development
tgoetz@utah.gov

Dear Dr. Goetz:

I wanted to take a few minutes to write you in support of the Common Core in Utah Schools. We are a science-based company that hires many Utah students. We specialize in microbiology, chemistry, physics, and perform testing for medical device, pharmaceutical, tissue, and dietary supplement companies seeking to obtain their approval and ongoing compliance with FDA and/or international notified bodies.

The ongoing success of our business in Utah depends on our ability to hire capable, intelligent scientists from Utah's educational systems. Unfortunately, we have already had to begin to compensate for a lack of skills and knowledge that our employees bring to the job. We have instituted a remedial writing program to help reteach the basics of grammar, writing, and editing. We also must regularly review fundamental mathematical principles such as rounding, scientific notation, and the proper use of significant figures.

This remedial teaching is a cost of business that detracts from our ability to produce for our customers. In a competitive national and international landscape, this poses a threat to our ability to compete. If other states and nations can more effectively empower students with fundamental math and language skills, I am sure this will eventually have economic consequences for our company and for Utah as well.

The Common Core seems like a logical and reliable way to ensure that our students are receiving the knowledge and skill that they need to be successful in today's workplace. I would ask that you continue to fight the good fight to maintain and even strengthen this commitment to a consistent educational standard.

We appreciate and enjoy the favorable business climate in Utah, but we must ensure that we have ready-access to a trained and capable workforce. I remain hopeful that our system can deliver this essential resource so that we might not only grow our business, but attract more science-based businesses to the state. Thank you for your ongoing efforts,

Sincerely,

Jeffery R. Nelson, M.B.A. SM(NRCM)
President/CEO
Nelson Laboratories, Inc.

P.O. Box 17557 | Salt Lake City, UT 84117–0057 · 6280 S Redwood Road | Salt Lake City, UT 84123–6600
www.nelsonlabs.com · Telephone 801 290 7500 · Fax 801 290 7998 · sales@nelsonlabs.com

Aug 17, 2012

Tami Goetz
Govorners Office of Economic Development
60 East South Temple
Salt Lake City, UT 84111

RE: Statement of Support for the Science and Math Education in Utah and for the Common Core

Dear Tami,

Thank you very much for your efforts on behalf of Utah technology businesses. High-tech business can only thrive in an environment that produces a steady stream of science and engineering graduates ready to tackle the new challenges found at the cutting edge of any technical field. We are privileged to operate in Research Park in Salt Lake City and we have grown from the 11 workers that moved down from Idaho Falls in 1999 to over 370 scientists, engineers, and administrative workers today. I am confident we would not have been able to achieve as high a level of growth if we had continued in Idaho because of Utah's superior investment in science and math education.

This summer, we employed over 50 interns from local high schools and universities and we are pleased to help contribute to the education of the next generation of technology workers. We are also grateful to the state for its efforts to develop superior educational programs and requirements. The prerequisites for work in biotechnology are not arbitrary and I we believe that the Common Core helps produce workers that understand mathematics which lies at the base of all science. Almost all of our employees use math daily. Whether in computer programming, statistical analysis of experimental results or production processes, or in business and accounting disciplines, everyone here needs a significant level of math literacy. We have half a dozen workers with graduate degrees in mathematics.

Asking for increased state investment in education may seem self-serving for a company like ours – of course, we need a skilled technical workforce. But our company also serves a powerful humanitarian mission. We are devoted to producing technologies that save lives. Our diagnostic systems allow hospital laboratories to rapidly identify deadly disease organisms so that doctors can more effectively treat their patients. If that isn't worth a substantial investment, I don't know what is.

Sincerely,

Kirk Ririe
CEO Idaho Technology Inc.

Idaho Technology Inc.
Innovation Amplified
390 Wakara Way, Salt Lake City, UT 84108
phone 1-801-736-6354 | fax 1-801-588-0507
www.idahotech.com

THE WALL STREET JOURNAL

June 23, 2011

The Case for Common Educational Standards

By Jeb Bush and Joel Klein

The success of today's students will determine our nation's destiny. America's economic strength and standing in the world economy are directly linked to our ability to equip students with the knowledge and skills to succeed in the 21st-century economy. Students are no longer competing with their peers in other cities—they are competing with students across the globe. Business leaders have become champions of education reform, recognizing the role that rigorous academic standards have on their success.

Yet in all but a very few states, current academic standards of educational performance are too low. A recent analysis by ACT, the respected national organization responsible for college admissions tests, concluded that three-fourths of the young men and women entering colleges "were not adequately prepared academically for first-year college courses." In other words, their high-school diplomas weren't worth the paper they were printed on.

This is not a winning formula. We must insist on standards that will prepare our high-school graduates for the demanding challenges they will face. And, while education is a national priority, the answer here does not appear to be a new federal program mandating national standards. States have historically had the primary responsibility for public education, and they should continue to take the lead.

The good news is that the states are stepping up. Recognizing our great need for more rigorous academics, state leaders and educators have come together to create model content standards. The Common Core State Standards were voluntarily developed by the National Governors Association and the Council of Chief State School Officers, in collaboration with teachers, school administrators and math and English experts. The standards provide a framework of clear and consistent skills for math and English. Already some 43 states and the District of Columbia have signed up to adopt the standards for English and math in lieu of their previous requirements.

We must insist on standards that will prepare our high-school graduates for the demanding challenges they will face.

The Common Core State Standards define what students need to know; they do not define how teachers should teach, or how students should learn. That is up to each state. And they are built on what we have learned from high-performing international competitors as well as the best practices in leading states.

For example, the Common Core State Standards focus on the math that matters most to achieve college and career readiness. The literacy standards require students to make arguments with evidence rather than just restate their own opinions or experiences.

Thus, research shows that rigorous mastery of fractions is crucial for later math performance. The Common Core State Standards provide clear grade-by-grade goals for what students should know about fractions, built on the best practices of high-performing countries. In literacy, what most predicts college readiness is the ability to read and understand complex texts. The Common Standards set clear benchmarks for each grade for students reading sufficiently complex texts in English, history/social studies, science and technical subjects.

New standards demand new measurements for achieving them and, once again, states are leading the way. Two consortia of states are currently working together to develop assessments of whether students are mastering the material.

It is the states' responsibility to foster an education system that leads to rising student achievement. State leaders, educators, teachers and parents are empowered to ensure every student has access to the best curriculum and learning environment. Governors and lawmakers across the country are acting to adopt bold education reform policies. This is the beauty of our federal system. It provides 50 testing sites for reform and innovation. The Common Core State Standards are an example of states recognizing a problem, then working together, sharing what works and what doesn't.

Mr. Bush, the Republican governor of Florida from 1999 to 2007, is now chairman of the Foundation for Excellence in Education and the Foundation for Florida's Future. Mr. Klein, a former chancellor of New York City's public schools, is the CEO of News Corporation's educational division.

About the Common Core State Standards

IN THEIR WORDS

"The Common Core State Standards are an example of states recognizing a problem, then working together, sharing what works and what doesn't."[1]

- Jeb Bush, former Governor of Florida
and **Joel Klein**, former Chancellor of the New York City Public Schools
June 23, 2011

"As a former CEO of a Fortune 500 company, I know that common education standards are essential for producing the educated work force America needs to remain globally competitive. Good standards alone are not enough, but without them decisions about such things as curricula, instructional materials and tests are haphazard. It is no wonder that educational quality varies so widely among states."[2]

- Craig Barrett, former CEO, Intel Corp.
April 6, 2010

"The Common Core State Standards are a building block in our state's education system meant to ensure that teachers and districts can innovate within a framework of high expectations and accountability. They are based on the fundamental belief that every child in every classroom deserves an education that will properly equip them with the skills they need for college and a career. Our aggressive implementation of these standards in partnership with districts will ensure that our children have an education that will serve them well in the next stages of their lives." [3]

- Chris Christie, Governor of New Jersey
September 13, 2011

"Since the release of A Nation at Risk in 1983, it has been increasingly clear that despite incremental reforms and progress in some states and school districts, academic expectations for American students have not been high enough. K-12 education in the United States leaves far too many students unprepared for postsecondary education and the 21st century workplace. There is no reason why students in the United States should not achieve at the same levels as their international peers in high-performing countries."[4]

- John J. Castellani, President and CEO
Business Roundtable (BRT)
March 10, 2010

"Anything which encourages or forces students and teachers to aim higher and expect more of themselves is great in education these days. It is important to know, with clarity, how our kids are doing compared with students in this country and abroad."[5]

- Mitch Daniels, Governor of Indiana
September 2011

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[1] *Wall Street Journal:http://online.wsj.com/article/SB10001424052702304070104576399532217616502.html*
[2] *Wall Street Journal*: http://online.wsj.com/article/SB10001424052702304017404575165682594015398.html?_requestid=110713
[3] Office of the Governor of New Jersey:http://www.state.nj.us/governor/news/news/552011/approved/20110913a.html
[4] *Business Wire*: http://www.businesswire.com/portal/site/home/permalink/?ndmViewId=news_view&newsId=20100310006782&newsLang=en
[5] *GOVERNING Magazine*: http://www.governing.com/papers/Impacting-the-Future.html

"Like the Interstate Compact on Educational Opportunity for Military Children that provides common guidelines for states to follow in handling issues that impact children of military families as they transition between schools, the rigor of the proposed academic Common Core Standards will be a benefit to military dependent students everywhere."[6]

- **Gen. Benjamin C. Freakley**, Commanding General,
United States Army Accessions Command

"Under my leadership and the leadership of the State Board of Education and Superintendent of Public Instruction, Arizona adopted more rigorous standards in late June 2010. These standards represent a significant improvement over Arizona's current standards and were developed by governors and other education leaders from the best standards across the nation through the "common core" program."[7]

- **Jan Brewer**, Governor of Arizona

"It only makes sense that we have some platform of expectations in terms of rigorous common standards in core subjects across this nation if we're going to be able to reasonably compare achievement, progress, and learning."[8]

- **Kay Persichitte**, Dean, University of Wyoming College of Education
April 5, 2010

"The best and most exciting points that the standards establish is what students need to learn, but they will not dictate how teachers should teach. Instead, schools and teachers will collaborate and decide how best to assist students in reaching their highest potential through critical thinking and problem solving skills to reach the standards. The consistency of the standards translates into what all parents want for their students, the ability to graduate from school prepared to succeed, and build a strong future for themselves and the country."[9]

- **Karin Brown**, President, Florida Parent Teacher Association
March 22, 2010

"Today, all students—not just top students—need to master math, science, English and social studies and learn skills in problem solving, thinking creatively and communicating clearly. Our goal can't just be for Iowa students to be best in the nation again. We must make sure Iowa students can compete with young people in countries with the highest-performing schools. Our youngsters deserve to be as well educated as those in Canada, Japan and Australia, among other places,"[10]

- **Terry Branstad**, Governor of Iowa
July 25, 2011

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[6] http://www.corestandards.org/assets/ccsi_statements/StatementArmy.pdf
[7] http://janbrewer.com/on-the-issues/improving-our-schools
[8] *Casper Star-Tribune*: http://trib.com/news/state-and-regional/article_525c3d8e-40b9-11df-9236-001cc4c03286.html
[9] *Sun-Sentinel*: http://weblogs.sun-sentinel.com/educationblog/2010/03/guest_blogger_pta_mom.html
[10] *Des Moines Register:* http://blogs.desmoinesregister.com/dmr/index.php/2011/07/25/branstad-calls-for-increasing-iowas-academic-standards/

Statements of Support for the Utah Core Standards

August 23, 2012

Utah Organizations that support the Core

Governor's Commission of Education Excellence
Utah Board of Regents
Utah Board of Regents Math Majors Committee
Utah Parent Teacher Association
Salt Lake Chamber of Commerce
Prosperity 20/20
Utah Technology Council
United Way
Utah Society of Superintendent's Association
Utah School Boards Association

National support for the core standards

NGA (a membership organization representing all 50 governors)
CCSSO (a membership organization representing all 50 state superintendents)
National PTA
Former Gov. Jeb Bush
Gov. Mitch Daniels (IN)
Condoleezza Rice
Governor Chris Christy (NJ)
Association of American Colleges and Universities
Dr. Hung-His Wu, University of California, Berkley

"Phoenix Rising: Bringing the Common Core State Standards to Life" by Hung-Hsi Wu http://www.aft.org/pdfs/americaneducator/fall2011/Wu.pdf

http://www.corestandards.org/assets/2010_6_10_WU.pdf

Dr. Bill Schmidt, Michigan State University

http://news.msu.edu/story/7929/http://news.msu.edu/story/7929/

A joint public statement of the National Council of Teachers of Mathematics (NCTM), the National Council of Supervisors of Mathematics (NCSM), the Association of State Supervisors of Mathematics

(ASSM), and the Association of Mathematics Teacher Educators (AMTE)

http://www.corestandards.org/assets/k12_statements/NCTM-NCSM-ASSM-AMTE-joint-statement.pdf

American Statistical Association

http://www.corestandards.org/assets/CommonCoreStateStandardsforStatisticsProbability.pdf

Fordham Foundation: Grade A-

"Common Core provides admirable focus and explicitly requires standard methods and procedures, enhancements that would benefit Utah's standards."

http://www.edexcellencemedia.net/publications/2010/201007_state_education_standards_common_standards/CommonCore_Math.p

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

FRANCIS (SKIP) FENNELL
Past President

August 17, 2012

Larry Shumway
State Superintendent of Schools
Utah State Office of Education
250 E. 500 South
PO Box 144200
Salt Lake City, UT 84114-4200

Deborah Roberts, President
Utah State Board of Education
Utah State Office of Education
250 E. 500 South
PO Box 144200
Salt Lake City, UT 84114-4200

Dear Dr. Shumway and Ms. Roberts:

I have been following the news regarding Utah's decision to withdraw from the SMARTER Balanced consortia assessment program and continue to read online indications about the possibility that your state will also withdraw from its decision to adopt the Common Core State Standards.

I am writing to you as a mathematics educator who has had the opportunity to serve as member of the writing team for the National Council of Teachers of Mathematics' Principles and Standards for School Mathematics (NCTM, 2000) and the Curriculum Focal Points (NCTM, 2006). I presented the Curriculum Focal Points in testimony to your legislature in 2006. More recently I have served as a member and content task group chair of the National Mathematics Advisory Panel (2006-2008) as a writer and reviewer of the Common Core State Standards – Mathematics, and as chair of the United States National Commission for Mathematics Instruction (2007-2009). Since the release of the CCSS I have written and spoken widely about their potential and the necessity of a common set of rigorous mathematics expectations for all students. The fact that 45 states (including yours) and the District of Columbia are now implementing the Common Core is, indeed, testimony to their promise.

I am writing to support the efforts of the Utah State Office of Education in improving mathematics education through **YOUR** collective efforts in adopting and moving toward implementation of the Common Core State Standards in mathematics. Your state's standards are focused, rigorous and coherent. Additionally, your state team has invested countless hours in a transition and implementation plan that will not only assist but engage teachers in this critical process.

This letter is, in no way, an attempt to usurp your statewide responsibilities. However, it is an attempt to support your efforts to this point and to encourage you to continue to move forward, with confidence, in your adoption of the Common Core State Standards. This history-making move is grounded on important mathematics within a focused and coherent curriculum that has been internationally benchmarked.

Feel free to contact me for additional information as needed.

Sincerely,

Francis (Skip) Fennell
L. Stanley Bowslbey Professor of Education and Graduate and Professional Studies
McDaniel College
Westminster, MD 21157
&
Past President
National Council of Teachers of Mathematics
http://ffennell.com
Project Director - Elementary Mathematics Specialists and Teacher Leaders Project

McDaniel College
Education Department
Westminster, MD 21157
TEL: (410) 857-2509
FAX: (410) 857-2515

A joint public statement of the National Council of Teachers of Mathematics (NCTM), the National Council of Supervisors of Mathematics (NCSM), the Association of State Supervisors of Mathematics (ASSM), and the Association of Mathematics Teacher Educators (AMTE)

Mathematics Education Organizations Unite to Support Implementation of Common Core State Standards

The release of the Common Core State Standards (CCSS) is a welcome milestone in the standards movement that began more than 20 years ago when the National Council of Teachers of Mathematics published *Curriculum and Evaluation Standards for School Mathematics*. By initiating the development of the CCSS, state leaders acknowledged that common K–grade 8 and high school standards culminating in college and career readiness would offer better support for national improvement in mathematics achievement than our current system of individual state standards. The CCSS provides the foundation for the development of more focused and coherent instructional materials and assessments that measure students' understanding of mathematical concepts and acquisition of fundamental reasoning habits, in addition to their fluency with skills. Most important, the CCSS will enable teachers and education leaders to focus on improving teaching and learning, which is critical to ensuring that all students have access to a high-quality mathematics program and the support that they need to be successful.

Greater Coherence Built on a Strong Foundation

The National Council of Teachers of Mathematics (NCTM), the National Council of Supervisors of Mathematics (NCSM), the Association of State Supervisors of Mathematics (ASSM), and the Association of Mathematics Teacher Educators (AMTE) support the goal of the CCSS to describe a coherent, focused curriculum that has realistically high expectations and supports an equitable mathematics education for all students. Many aspects of the central elements of the CCSS echo the longstanding positions and principles of our organizations:[1]

- All students need to develop mathematical practices such as solving problems, making connections, understanding multiple representations of mathematical ideas, communicating their thought processes, and justifying their reasoning.

[1] As articulated in NCTM's Standards publications (1989, 1991, 1995, 2000), NCTM's *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence (2006)*, NCSM's *Principles and Indicators for Mathematics Educators (PRIME) Leadership Framework* (2008), NCTM's *Focus in High School Mathematics: Reasoning and Sense Making* (2009), *and* AMTE'*s Standards for Elementary Mathematics Specialists State Certification* (2010).

- All students need both conceptual and procedural knowledge related to a mathematical topic, and they need to understand how the two types of knowledge are connected.
- Curriculum documents should organize learning expectations in ways that reflect research on how children learn mathematics.
- All students need opportunities for reasoning and sense making across the mathematics curriculum—and they need to believe that mathematics is sensible, worthwhile, and doable.

Supporting and Facilitating Implementation

The collective strengths of our organizations give us the potential to generate the momentum necessary to implement the CCSS effectively. Together, our organizations represent mathematics teachers, mathematics education leaders at the school, district, state, and national levels, researchers, and mathematics teacher educators in schools and colleges of education and departments of mathematics, who collectively have the expertise to lead implementation efforts.

The critical first steps will be to help educators interpret and understand the CCSS and to support the development and implementation of comprehensive, coherent instruction and assessment systems. To this end, we intend to do the following:

- Work with our local, state, and national affiliates to feature the CCSS in our professional development opportunities, including annual and regional conferences, academies, and seminars, and infuse them into our teacher education classes.
- Support the development and implementation of the corresponding assessment system, particularly with respect to preparing teachers, leaders, and teacher educators to use assessment results effectively to inform instruction and to incorporate formative assessment practices in the classroom.

Finally, we strongly encourage and support both research about the standards themselves (e.g., research on specific learning trajectories and grade placement of specific content) and their implementation, as well as periodic review and revision based on such research.

June 2, 2010

ASSOCIATION OF STATE SUPERVISORS OF MATHEMATICS
ASSM

A joint public statement of the National Council of Teachers of Mathematics (NCTM), the National Council of Supervisors of Mathematics (NCSM), the Association of State Supervisors of Mathematics (ASSM), and the Association of Mathematics Teacher Educators (AMTE)

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The release of the Common Core State Standards (CCSS) is a welcome milestone in the standards movement that began more than 20 years ago when the National Council of Teachers of Mathematics published *Curriculum and Evaluation Standards for School Mathematics*. By initiating the development of the CCSS, state leaders acknowledged that common K–grade 8 and high school standards culminating in college and career readiness would offer better support for national improvement in mathematics achievement than our current system of individual state standards. The CCSS provides the foundation for the development of more focused and coherent instructional materials and assessments that measure students' understanding of mathematical concepts and acquisition of fundamental reasoning habits, in addition to their fluency with skills. Most important, the CCSS will enable teachers and education leaders to focus on improving teaching and learning, which is critical to ensuring that all students have access to a high-quality mathematics program and the support that they need to be successful.

Greater Coherence Built on a Strong Foundation

The National Council of Teachers of Mathematics (NCTM), the National Council of Supervisors of Mathematics (NCSM), the Association of State Supervisors of Mathematics (ASSM), and the Association of Mathematics Teacher Educators (AMTE) support the goal of the CCSS to describe a coherent, focused curriculum that has realistically high expectations and supports an equitable mathematics education for all students. Many aspects of the central elements of the CCSS echo the longstanding positions and principles of our organizations:[1]

- All students need to develop mathematical practices such as solving problems, making connections, understanding multiple representations of mathematical ideas, communicating their thought processes, and justifying their reasoning.
- All students need both conceptual and procedural knowledge related to a mathematical topic, and they need to understand how the two types of knowledge are connected.
- Curriculum documents should organize learning expectations in ways that reflect research on how children learn mathematic

[1] As articulated in NCTM's Standards publications (1989, 1991, 1995, 2000), NCTM's *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence (2006),* NCSM's *Principles and Indicators for Mathematics Educators (PRIME) Leadership Framework* (2008), NCTM's *Focus in High School Mathematics: Reasoning and Sense Making* (2009), *and* AMTE's *Standards for Elementary Mathematics Specialists State Certification* (2010).

- All students need opportunities for reasoning and sense making across the mathematics curriculum—and they need to believe that mathematics is sensible, worthwhile, and doable.

Supporting and Facilitating Implementation

The collective strengths of our organizations give us the potential to generate the momentum necessary to implement the CCSS effectively. Together, our organizations represent mathematics teachers, mathematics education leaders at the school, district, state, and national levels, researchers, and mathematics teacher educators in schools and colleges of education and departments of mathematics, who collectively have the expertise to lead implementation efforts.

The critical first steps will be to help educators interpret and understand the CCSS and to support the development and implementation of comprehensive, coherent instruction and assessment systems. To this end, we intend to do the following:

- Work with our local, state, and national affiliates to feature the CCSS in our professional development opportunities, including annual and regional conferences, academies, and seminars, and infuse them into our teacher education classes.
- Support the development and implementation of the corresponding assessment system, particularly with respect to preparing teachers, leaders, and teacher educators to use assessment results effectively to inform instruction and to incorporate formative assessment practices in the classroom.

Finally, we strongly encourage and support both research about the standards themselves (e.g., research on specific learning trajectories and grade placement of specific content) and their implementation, as well as periodic review and revision based on such research.

June 2, 2010

Appendix B

Peter Trapa

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August 17, 2012

Board of Education
Utah State Office of Education
Salt Lake City, UT 84114

Members of the Board:

I am writing in my capacity as Chairman of the Department of Mathematics at the University of Utah, a decorated department ranked by US News and World Report in the top thirty among all Mathematics Departments nationwide (and in the top fifteen of those at public institutions). Before joining the faculty at Utah, I received my Ph.D. at MIT and held appointments at the Institute for Advanced Study in Princeton and at Harvard University. At each stage of my career I have been keenly interested in issues of mathematics education.

One of the most pressing educational challenges facing Utah is increasing the production of high quality college graduates, particularly in STEM disciplines where the need for a competent workforce is greatest. I am deeply involved and committed to meeting this challenge head-on. In the past year, I led two large-scale efforts in this direction: the creation of a Technology Intensive Concurrent Enrollment (TICE) course for high school seniors entitled "Mathematical Decision Making for Life"; and the implementation of a new Engineering Math sequence which all future Engineering students at the University of Utah will take. Each required aligning high school curricula and college curricula with the ultimate aim of increasing college-readiness and improving retention rates among students on campus.

I have therefore examined the new Utah Core Standards in Mathematics carefully, and I am convinced that the **successful implementation of the Core Standards in Mathematics will increase the production of mathematically well-prepared students across the board, and will lead to increased numbers of high quality college graduates in STEM disciplines in Utah.**

The Utah Core Standards in Mathematics represent a major step forward for mathematics education in Utah, and ultimately present an opportunity to address some of the State's most pressing economic concerns. It is an opportunity which Utah cannot afford to squander.

Sincerely,

Peter E. Trapa, Ph.D.
Professor of Mathematics
Chairman, Department of Mathematics

Appendix C

Algebra

Algebra Topics in the 2010 Utah Core Mathematics Standards

Utah's 2010 Core Standards spread the topics traditionally appearing in an Algebra I course over several years, with the bulk of the topics appearing in 8th grade. This coherent trajectory through algebra supports greater depth of knowledge in algebra topics and allows for integration with geometry, statistics, and modeling through the students' secondary experience. This chart represents ONLY those topics from the 2007 Algebra I course. It is not a representation of all algebra topics in the 2010 Core which is more comprehensive than those which appear here.

2010 Utah Core Standards Placement	Topics formerly appearing in Utah's Algebra I course of study, as explicated in the 2007 Utah Core Standards for Mathematics.
5th Grade	Evaluate and simplify numerical expressions using the order of operations.
	Write algebraic expressions or equations to generalize visual or numerical patterns.
6th Grade	Define a rational number as a point on the number line that can be expressed as the ratio of two integers.
	Place rational numbers on a number line between two integers.
	Evaluate and simplify numerical expressions containing rational numbers.
	Compute solutions to problems, represent answers in exact form, and determine the reasonableness of answers.
	Apply the properties of operations to generate equivalent expressions and identify when two expressions are equivalent.
7th Grade	Solve single-variable linear equations.
	Solve single-variable linear inequalities.
	Solve proportions.
8th Grade	Define numbers that are not rational as irrational.
	Classify numbers as rational or irrational, knowing that rational numbers can be expressed as terminating or repeating decimals and irrational numbers can be expressed as non-terminating, non-repeating decimals.
	Place rational and irrational numbers on a number line between two integers.
	Classify pi and square roots of non-perfect square numbers as irrational.
	Simplify expressions using positive exponents.
	Solve quadratic equations that can be simplified to the form $x^2 = a$ where $a \geq 0$ by taking square roots.
	Identify the slope of a line.
	Interpret the slope of a linear function as a rate of change in real-world situations.
	Determine the effect of changes in slope or y-intercept in y-mx+b.
	Determine and explain the meaning of slopes and intercepts using real-world examples.
	Represent linear equations in slope-intercept form $y=mx+b$, and standard for $Ax+Bx=C$.
	Distinguish between linear and non-linear functions.
	Solve linear equations.
	Solve systems of two linear equations graphically and algebraically.

8th Grade (Continued)	Determine the number of possible solutions for a system of two linear equations.
	Calculate the measures of the sides of a right triangle using the Pythagorean Theorem.
	Collect, record, organize and display a set of data with at least two variables.
	Determine whether the relationship between two variables is approximately linear or non-linear by examination of a scatter plot.
	Characterize the relationship between two linear related variables as having positive, negative, or approximately zero correlation.
	Interpret the slope and y-intercept of a line through data.
Secondary I	Solve equations for a specified variable.
	Write the equation of a line.
	Graph linear relations and inequalities.
	Graph a system of linear inequalities and identify the solution.
	Identify the slope of a line given points, a graph, or an equation.
	Estimate the equation of a line of best fit to make and test conjectures.
	Predict *y*-values for given *x*-values when appropriate using a line fitted to bivariate numerical data.
Secondary II	Solve quadratic equations using factoring.
	Simplify polynomials.
	Add, subtract, and multiply polynomials.
	Factor trinomials, difference of two squares, and perfect square trinomials.
	Write a quadratic equation.
Not explicitly in UCS	Simplify, add, subtract, multiply, and divide expressions with square roots.
	Identify horizontal and vertical lines given the equations or slopes.
	Find the greatest common monomial factor of a polynomial.

Appendix D

Singapore Math Example

Secondary Mathematics and Additional Mathematics

- Why are the books for grades 7-10 labeled 1-4?

  In Singapore, students attend Primary school for 6 years, then Secondary school usually for 4 years, and then can attend junior colleges for 2-3 years. Grades 7-11 in the U.S. are roughly equivalent to Secondary 1-4 in Singapore.

- Where does Additional Mathematics fit in?

  In Singapore, some students do a second math class in Secondary 3 and 4 called Additional Mathematics. The books we sell with Additional Mathematics in the title are used for these classes over a 2-year period. Since these books cover pre-calculus and calculus topics, they could be used after the Secondary 1-4 books.

- How do these books correlate with the math sequence in the U.S.? Which is pre-algebra, algebra 1, geometry and so on?

  These books have an integrated approach. They progress sequentially through various topics in algebra, geometry, and some trigonometry, with minimal review of earlier levels. Please see the scope and sequence. As a rough comparison:
  Secondary 1: pre-algebra, some algebra 1 and geometry
  Secondary 2: algebra 1 and geometry
  Secondary 3: some algebra 2, geometry, some trigonometry
  Secondary 4: some advanced topics and review
  Additional Mathematics: first half is mostly pre-calculus including trigonometry, second half is mostly calculus.

- What is the new Discovering Mathematics Common Core Standard Series? How does it compare?

  This series for 7th and 8th grade is a revision of the original Discovering Mathematics series in order to include the Common Core Standards. A few topics were added to some chapters, and some topics were rearranged between the levels. There is some US customary measurement, but metric measurement still predominates. Terms have been changed to reflect the common use in the US, such as gradient to slope and indices to exponents. American English spelling is used. Otherwise, it is the same as the original series. No topics were removed. It therefore follows the Singapore Mathematics Framework as well as covers the Common Core Standards.

- Will the new Discovering Mathematics Common Core Standard Series be less advanced or rigorous than the original in order to accommodate the Common Core Standards?

  No, it will not. The two levels will together still include all the topics in the original. The 8th grade will in fact include a few topics from Discovering Mathematics 3 in order to cover all topics considered to be Algebra 1, such as the quadratic formula and fractional exponents (radicals). It will still include topics such as factorization of algebraic expressions and solving quadratic equations that are not part of the Common Core Standards for 8th grade but are part of the Singapore syllabus at that level.

- Will there still be a teacher's edition for the new Discovering Mathematics Common Core Standard Series that includes worked out solutions?

  Yes, there will. It will be similar to the current one, with some notes to teachers and worked out solutions for all the problems.

- How does the format of the secondary level books compare to Primary Mathematics? How do the workbooks correlate with the textbooks? What is the structure of a lesson in the textbook and how long does it take?

  The textbooks contain both the lesson and the practice, instead of having the practice in a separate workbook. The workbook is now a supplementary, non-consumable book with additional problems that can be done after each chapter. The lessons include some activities that are similar to the learning tasks in Primary Mathematics, but there are also more worked examples with solutions provided and more written explanations. Some lessons are meant to take several days, with exercises only at the end of the lesson.

- Do you have placement tests?

  We have assessment tests for some levels. Please click here for Placement Tests. These tests cover the material in the level indicated on the test.

- Do you have a scope and sequence for these books?

  Yes. Please click here for Scope and Sequence.

- Do you have a list of contents? Sample pages?

  Yes. In the list of products, click on the picture of the product or on the words 'more info'. Then click on the tab that says Contents_Sample and scroll down. There are links to sample pages for each book.

- Are there teacher's guides available? Tests? Answer keys? Solutions?

  All secondary level textbooks and workbooks have answers to most, but not all, of the problems at the back. Depending on the series, there are various teaching resources available with additional answers or solutions. See the chart below for comparison between the series. However, none of the teacher guides have detailed, already prepared daily lesson plans.

- Which series should I use?

  It depends on your preference. New Elementary Mathematics is an older series and less modern in its format, but it is more challenging than Discovering Mathematics. Discovering Mathematics has fully worked solutions for all levels including the workbook problems.

- Can I switch between series?

  You can switch between New Elementary Mathematics to Discovering Mathematics between years. You can also switch after the Discovering Mathematics Common Core Standards Series 8th grade to the original Discovering Mathematics3. There will be some repetition of two topics (fractional exponents and quadratic formula).

If my student is not able to easily answer all the problems or get most of the problems on reviews or tests correct, does that mean the program does not teach the material well?

The problems have a range of difficulty level to allow all students to work to their maximum potential. If all the problems were easy, then the student is not working to his or her full potential. Some of the problems in the exercises are simply practice, but some are truly problems that allow a student to gain more depth of knowledge by reasoning through them and applying concepts in new ways. Also, in Singapore most students are expected to score between 50% and 75% on tests. Only better students will be able to score above 75%. If your student can answer all the problems easily, he or she would score well on such a test, but not being able to answer every problem easily does not necessarily indicate lack of understanding of the concepts. The grading scale in Singapore for the secondary level is:

A1: 75% and above
A2: 70% to 74%
B3: 65% to 69%
B4: 60% to 64%
C5: 55% to 59%
C6: 50% to 54% (passing grade)
D7: 45% to 49%
E8: 40% to 44%
F9: Below 40%

- What if I have more questions or need more help with the content or choosing which books to get?

  Please visit our forums and post your questions or concerns. You can also email the curriculum advisor.

- It says in the chart below that some activities in Discovering Mathematics use Geometer's Sketchpad. What is that and where can I get it?

  Geometer's Sketchpad is a dynamic geometry software program produced and sold by Key Curriculum Press. (We are not associated with Key Curriculum Press and we are not responsible for any purchases made with this company.) You can use other dynamic software programs for the same activities, such as Geogebra but the steps will be different.

	New Elementary Mathematics (NEM)	Discovering Mathematics original (DM)	Discovering Mathematics CCS (DMC)
Publication date	First published 1991. New edition in 1996. This is based on the syllabus used prior to 2001 and is one of the texts used by students taking the TIMSS international test. This series is no longer used in Singapore	First published in 2008.	First published 2012. Based on the original Discovering Mathematics series.
Textbook organization	Textbook pages are black and white. Lessons in the textbook consist of explanations, worked examples, and occasional class activities which allow students to learn through discovery. At the end of each chapter there is a summary listing the concepts learned in the chapter, but not a review exercise. Each chapter is followed by an optional "Challenger" and a "Problem Solving" exercise. After 3-4 chapters there is a set of 5 short cumulative review exercises, and an optional Miscellaneous Exercise and Investigation for exploring some concepts in more depth. There are two practice cumulative assessments at the end of each textbook.	Textbook pages are more visually attractive than NEM and in color. There are two textbooks for each level, A and B. Lessons in the textbook consist of explanations and worked examples. Each worked example is followed by a similar question (Try It!) that students can do to see if they understood the example. Many lessons include one or more class activities, which allow students to learn through discovery. Some of these use Geometer's Sketchpad. Some lessons are meant to take several days. Each chapter is followed by a summary listing the major concepts, a review exercise, an open-ended Extend Your Learning Curve problem, and one or two questions for journal writing. At the end of each textbook is a list of problem solving heuristics with some examples corresponding to the topics for that level. There are no cumulative review exercises or assessments, but the chapter reviews tend to integrate concepts from earlier chapters.	See the entry under DM.
Exercise organization	Exercises vary in length and tend to be long so it is possible to select problems rather than do all of them. The type of problems depends on the lesson; some exercises are all computation or skill problems and some are all word problems. More challenging problems are marked with an asterisk.	The problems in each lesson's exercise are divided by difficulty level into Basic Practice, Further Practice, Maths@Work (application) and Brainworks (challenge). The cheaper review exercises are not divided by difficulty level.	See the entry under DM.
Answers to the textbook problems.	There are answers to the regular exercises, the periodic Miscellaneous and Review exercises and the Assessments in the back of the textbook.	There are answers to the Try It! problems in the lesson, the lesson exercises except for Brainworks section, and the reviews at the back of the textbook.	See the entry under DM.
Teacher resources and fully worked solutions	The Teacher's Manual has a weekly schedule and fully worked solutions for the Challengers, Problem Solving, and Investigations. The solutions manual has fully worked solutions to the rest of the exercises.	The Teacher's Guide for each level has a weekly schedule, brief notes for the teacher, and fully worked solutions to all problems in the textbook, including class activities.	The Teacher Notes and Solutions for each level has a weekly schedule, brief notes for the teacher, and fully worked solutions to all problems in the textbook, including class activities.
Workbook	The workbook is supplementary and contains additional problems for each chapter, practice test papers every 2	There is one supplementary workbook for each level. It has a set of problems for each chapter divided into Basic Practice, Further	See the entry under DM. There is a separate Workbook Solutions with fully worked solutions.

	chapter and 2 mid-term and final term assessments. Only the test papers have room in the book to work the problems. Answers are in the back. There are no fully worked solutions.	Practice, Challenging Practice, and Enrichment (even more challenging). Answers are in the back. There is a separate Teacher's Edition of the workbook with fully worked solutions.	
Tests	There are two sample tests at the end of each textbook and practice test papers for every 2 chapters, midterms, and semester test in the workbook. These are quite challenging, as the workbook is somewhat for enrichment.	The workbook has mid-year and end-of year sample tests. There is a separate Question Bank book for each level with test questions arranged by difficulty level. Questions will have to be selected, copied, and transferred to a test paper, since answers are included on each page in the Question Bank.	To be determined.

Appendix E

Research

Utah STATE OFFICE of Education

UTAH CORE STATE STANDARDS
for
MATHEMATICS

SAMPLE OF WORKS CONSULTED

Sample of works consulted list reprinted in its entirety with permission from CCSSO.

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The Common Core State Standards in Mathematics

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Background

In 2009, 48 out of the 50 states in the U.S. came together under the leadership of the National Governors Association (NGA) and the Council of Chief State School Officers (CCSSO) to write school standards for Mathematics and English Language Arts that would ensure students leaving high school are ready for college and career. The Common Core State Standards (CCSS) were released on 2 June 2010, and they have been adopted by 45 states.[1]

Twenty years earlier, in 1989, the National Council of Teachers of Mathematics made the first move in the modern era towards a common understanding of school mathematics [7]. Before that time curriculum varied among the nation's numerous school districts. The NCTM standards were not themselves an act of government, but in response to them the governments of the 50 states started developing their own standards, bringing a measure of consistency to the mathematics curriculum within states. However, consistency between states proved elusive and in the years after 1989 state standards diverged greatly. For example, Table 1 shows the 2006 distribution of grade levels at which 42 state standards introduced addition and subtraction of fractions:

Grade	1	2	3	4	5	6	7	8
Number of states	2		7	22	9	1	1	

Table 1: Distribution of grade levels where state standards introduce addition and subtraction of fractions[12].

The situation was famously described as follows in 1997:

> There is no one at the helm of mathematics and science education in the U.S. ... No single coherent vision of how to educate today's children dominates U.S. educational practice ....
>
> These splintered visions produce unfocused curricula and textbooks ... [that] emphasize familiarity with many topics rather than concentrated attention to a few ... [and] likely lower the academic

[1] A 46th state, Minnesota has adopted the English Language Arts Standards.

> performance of students who spend years in such a learning environment. *Our curricula, textbooks, and teaching all are "a mile wide and an inch deep."* [emphasis added] [14]

The mile wide inch deep curriculum is partly a natural result of the U.S. system of local control, in which there is no central Ministry of Education, authority over education is delegated to the states and, on many questions of policy, to the 16,000 school districts. Another cause was the math wars, an ideological conflict about almost every question related to mathematics education: curriculum, assessment, methods of teaching, the nature of mathematics itself. The fragmented system of local control allowed this debate to rage unchecked, drawing school boards, parent groups, curriculum reformers, policy makers, and university faculty into a draining conflict about curriculum materials at the expense of work on other important problems in mathematics education.

Various efforts in the last decade have had a primary or secondary goal of improving this situation by bringing the different sides together and aligning state standards with each other and with international standards: the American Diploma Project, Finding Common Ground in K–12 Mathematics Education, Adding it Up, the NCTM publications *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* and *Focus in High School Mathematics*, reports by ACT and College Board, and the report of the National Mathematics Advisory Panel ([11], [6], [5], [8], [9], [1], [2], [10]).

These efforts came to a head with the surprisingly rapid consensus around developing common standards in 2009. The initiative was spearheaded by NGA and CCSSO, two new actors in the world of standards writing, who showed agility in bypassing old stalemates. Key to the success of the endeavor in political terms was that unlike previous efforts, the Common Core effort was led by state policymakers, not the federal government. The standards drew on many sources, including the reports and publications mentioned earlier, and also including standards of U.S. states and high achieving countries, particularly in East Asia. A recent analysis has shown that the standards are closely aligned with the standards of the A+ countries, a group of countries that formed a statistically significant group of top achievers on TIMSS 1995, and that state achievement in mathematics on the National Assessment of Education Progress is correlated with the closeness of previous state standards to the Common Core[13].

With the great majority of states adopting CCSS, the U.S has entered an unprecedented time of opportunity, with long-standing arrangements in mathematics education now open to renegotiation. For over 20 years discourse in mathematics education has been dominated by the math wars, in which apparently divergent views of mathematics competed for dominance; it was difficult, for example, to advocate both that students acquire fluency with algorithms for addition, subtraction, multiplication, and division in elementary school, and that they engage in serious work with statistics in high school, without being viewed askance by both camps. But CCSS incorporates both stances, viewing the two as integral parts of a coherent progression in skill and understanding.

What are the opportunities? First, the opportunity for curriculum developers to produce more focused and coherent materials, without having to attend to diverse demands for topic placement made by different state standards; second, the opportunity for teacher preparation and professional development to become less generic and more focused on the mathematics taught at at a given grade level; and finally, the opportunity for teachers from across the country to share tools for implementation based on common standards.

What should standards look like?

In countries with a fully functioning education system, they can look like Figure 1. I was one of the lead writers of the Common Core; we sometimes dreamed of the ability to make simple bulleted lists lists like this. How does Singapore get away with this? we asked ourselves. The answer is that Singapore has a Ministry of Education that produces curriculum and exams; their standards document is a description of that system, not a prescription for it.

Figures 2 and 3 show two images of standards in the U.S. The NCTM standards have 14 "standards" (bulleted items) for Number and Operations, Grades 6–8, followed by 7 pages of narrative. To a certain extent the NCTM had an education system available as well, or rather two systems: the system of commercial and NSF-funded textbook projects for producing curriculum, and the system of 50 state departments of education for producing exams. The system was divergent, chaotic, and voluntary.

O Level Mathematics Syllabus — Secondary One

Secondary One	
Topic/Sub-topics	**Content**
Algebraic representation and formulae	Include: • using letters to represent numbers • interpreting notations: * ab as $a \times b$ * $\frac{a}{b}$ as $a \div b$ * a^2 as $a \times a$, a^3 as $a \times a \times a$, a^2b as $a \times a \times b$, ... * $3y$ as $y + y + y$ or $3 \times y$ * $\frac{3 \pm y}{5}$ as $(3 \pm y) \div 5$ or $\frac{1}{5} \times (3 \pm y)$ • evaluation of algebraic expressions and formulae • translation of simple real-world situations into algebraic expressions • recognising and representing number patterns (including finding an algebraic expression for the nth term)
Algebraic manipulation	Include:

Figure 1: A page from the Singapore secondary standards

Number and Operations

Standard for Grades 6–8

Instructional programs from prekindergarten through grade 12 should enable all students to—

Expectations

In grades 6–8 all students should–

Understand numbers, ways of representing numbers, relationships among numbers, and number systems	• work flexibly with fractions, decimals, and percents to solve problems; • compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line; • develop meaning for percents greater than 100 and less than 1; • understand and use ratios and proportions to represent quantitative relationships; • develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation; • use factors, multiples, prime factorization, and relatively prime numbers to solve problems; • develop meaning for integers and represent and compare quantities with them.
Understand meanings of operations and how they relate to one another	• understand the meaning and effects of arithmetic operations with fractions, decimals, and integers; • use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals; • understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems.
Compute fluently and make reasonable estimates	• select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods; • develop and analyze algorithms for computing with fractions, decimals, and integers and develop fluency in their use; • develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of the results; • develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.

Figure 2: A page from the 2000 NCTM Principles and Standards for School Mathematics

MATHEMATICS STANDARD ARTICULATED BY GRADE LEVEL

GRADE 6

Strand 1: Number Sense and Operations

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Number Sense Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.
PO 1. Express fractions as ratios, comparing two whole numbers (e.g., ¾ is equivalent to 3:4 and 3 to 4). PO 2. Compare two proper fractions, improper fractions, or mixed numbers. PO 3. Order three or more proper fractions, improper fractions, or mixed numbers. PO 4. Determine the equivalency between and among fractions, decimals, and percents in contextual situations. PO 5. Identify the greatest common factor for two whole numbers. PO 6. Determine the least common multiple for two whole numbers. PO 7. Express a whole number as a product of its prime factors, using exponents when appropriate.

Figure 3: A page from a typical set of state standards, 2008

The document illustrated in Figure 3 (which is representative of state standards at the time) has 82 standards for Number and Operations in Grade 6 alone, and no pages of narrative. This is much more detailed and performance-based than the NCTM standards. Unlike the NCTM standards, state standards have direct policy and legal consequences, and are used a basis for writing assessments. They are flat lists of performance objectives of even grain size, designed to be delivered into the hands of assessment writers without the need for too much discussion or interpretation.

It was against this background that the Common Core State Standards were written. On the one hand they were commissioned by the states and therefore had to be the type of document states were used to: detailed bulleted lists describing what we want students to know and be able to do. On the other hand, were were being asked to do something new, to break out of the system that produced the mile wide inch deep curriculum.

Design of the Standards

The fundamental design principles for the Standards are focus, coherence, and rigor.

Focus means attending to fewer topics in greater depth at any given grade level, giving teachers and students time to complete that grade's learning.

Coherence means attending to the structure of mathematics and the natural pathways through that structure, where "natural" means taking into account both the imperatives of logic and the imperatives of cognitive development in designing the sequence of ideas. Since these two imperatives are sometimes in conflict, attaining coherence is a complex exercise in judgement, requiring a certain amount of professional craft and wisdom of practice not easily obtained from any one source.

Rigor means balancing conceptual understanding, procedural fluency, and meaningful applications of mathematics. Here the word rigor is used not in the way that mathematicians use it, to indicate a correct and complete chain of logical reasoning, but in the sense of a rigorous preparation for a sport or profession: one that exercises all the necessary proficiencies in a balanced way.

Organization of the standards

The Standards are divided into Standards for Mathematical Content and Standards for Mathematical Practice. The content standards are further subdivided into K–8 standards and high school standards. The K–8 standards are specified by grade level and organized into *domains*, topics which follow a coherent progression over a certain grade span (see Figure 4).

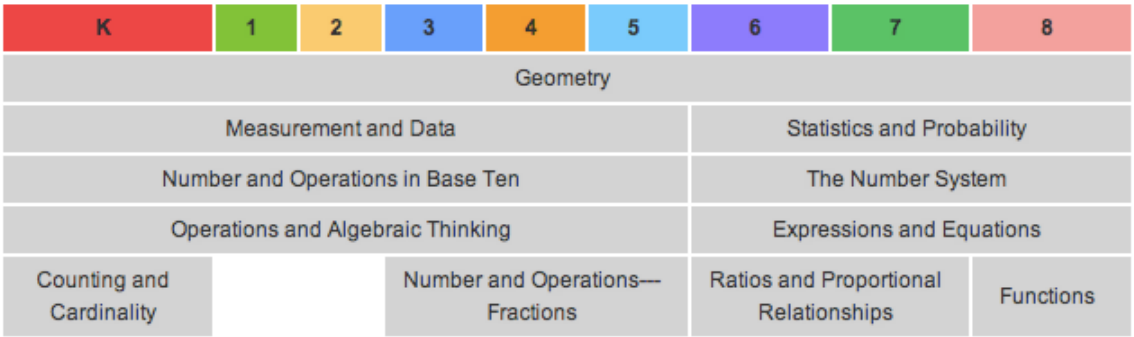

Figure 4: Domains in the Common Core, Grades K–8

The organization by domains is different in an important way from the organization by strands typical of previous state standards. Under the latter scheme, four or five strands (e.g., Number and Operations, Algebra and Functions, Data and Measurement, Geometry) would extend from Kindergarten to

Grade 12. The homogeneity of this scheme with respect to time is at odds with the progressive nature of mathematics, and resulted in a tendency to fill in every cell of the grade-by-strand matrix, one of the causes of the mile wide inch deep curriculum.

By contrast, domains operate at a finer level (there are 12 domains in the K–8 standards), and have a beginning and an end, each preparing for and eventually giving way to higher domains that both build on and encapsulate previous work. Domains allow for convergence and consolidation of ideas, as when the K–5 number work in the domains Operations and Algebraic Thinking, Number and Operations in Base Ten, and Fractions, is consolidated into a unified understanding of The Number System in Grades 6–8. The abbreviated life time of a domain also allows for the delineation of foundational domains that support more than one future domain: the work on Fractions in Grades 3–5 is a basis for The Number System, but also for the work on Ratios and Proportional Relationships in Grades 6–7, leading to Functions in Grade 8.

The high school standards are arranged into the broad *conceptual categories* shown in Table 5, which are further divided into domains as in the K–8 standards. However, the high school standards are not arranged into grade levels, and so the domains do not always exhibit a temporal progression. Some of the domains are conventional topics (e.g. Congruence is a domain in Geometry); others describe ways of thinking that help students bind their mathematical knowledge into coherent packages rather than trying to remember innumerable different formulas and techniques. For example, the Algebra category has a domain Seeing Structure in Expressions, which undergirds a student's work during the entire high school experience, from linear functions to logarithms.

Number and Quantity
Algebra
Functions
Modeling
Geometry
Statistics and Probability

Figure 5: High school conceptual categories in the Common Core

Just as the content standards attempt to describe the complex structure of mathematical knowledge, the Standards for Mathematical Practice (see Figure 6) describe the contours of mathematical practice; the various ways in which proficient practitioners of mathematics carry out their work. These are not intended to free floating proficiencies, unconnected with content, nor are they uniformly applied over all the work that students do. Just as a rock climber's various skills are called on differently during different parts of a climb, so specific aspects of practice become salient in specific pieces of mathematical work. For example, students learning how to complete the square in a quadratic expression benefit from consciously looking for structure and seeing the regularity in reasoning with a sequence of well-chosen examples (SMP 7 and 8); students

constructing geometric proofs will learn to critique arguments and use precise language (SMP 3 and 6); students designing a study to see if there is a connection between athletic and academic proficiency will construct a statistical model and choose appropriate methods and technologies (SMP 4 and 5).

SMP.1	Make sense of problems and persevere in solving them
SMP.2	Reason abstractly and quantitatively
SMP.3	Construct viable arguments and critique the reasoning of others
SMP.4	Model with mathematics
SMP.5	Use appropriate tools strategically
SMP.6	Attend to precision
SMP.7	Look for and make use of structure
SMP.8	Look for and express regularity in repeated reasoning

Figure 6: Standards for Mathematical Practice

Taking focus seriously

Four out of the six domains in K–5 deal with number and operations (see Figure 4): Counting and Cardinality (Kindergarten), Number and Operations in Base Ten (K–5), Operations and Algebraic Thinking (K–5), and Fractions (3–5).

The focus on number and operations in elementary school is even stronger than this count would suggest, because many standards in the other domains are designed to support the focus on number and operations. For example, the following data standard in Grade 2 supports the principle work with addition and subtraction of whole numbers:

> **2.MD.10.** Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

As another example, many geometry standards in elementary school deal with composing and decomposing figures, and dealing with composite figures, supporting the unit fraction approach to fractions starting in Grade 3.

In order to make room for the focus on number and operations, some topics are given much less time in elementary school than was the case with previous state standards. This was a necessary step to make good on the promise of repairing the mile wide inch deep curriculum. For example, standards on data, patterns, and symmetry are reduced to a trickle in elementary school. This

was one of the more controversial shifts in the Common Core, and it is worth looking at in a little more detail. Debate about curriculum in the United States has suffered from an all-or-nothing quality, and nowhere is this seen more clearly than in the debate about data and statistics in elementary school: it has seemed that the only choices were embracing a rich stream on data work in elementary school, as advocated by the GAISE report[4], or drying it up to nothing. In contrast, the Common Core is based on progressions that start with a trickle before they grow into the full flow of a domain. Thus the data standards in elementary school are neither to be ignored nor to be given undue prominence. In due time, in high school, statistics and probability becomes a major topic.

The function concept is another topic that is delayed compared to previous state standards. There is a trickle of pattern standards in elementary school, carefully worded to support the emergence of an incipient notion of function:

3.OA.9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.
4.OA.5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.
5.OA.3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.

Figure 7: The trickle of pattern standards in elementary school

In middle school, further preparation for functions is provided in the domains Ratios and Proportional Relationships and Expressions and Equations. The function concept finally makes its appearance in its own domain in Grade 8, and becomes a major conceptual category in high school.

As the examples of statistics and functions illustrate, taking focus seriously means delaying favored topics until their time, which will be a difficult shift for the educational system in the U.S.

The payoff for this approach occurs in high school, where the subject matter focus broadens as the foundations developed in K–8 allow for a variety of work in number and quantity, algebra, functions, modeling, geometry, statistics, and probability. Focus in high school means not so much a small number of topics as a concentration of skills and practice into a small number of underlying principles.

Preserving coherence

The act of writing standards for a subject is inherently in conflict with the goal of showing the structure of the subject. In [3] this is likened to shattering

an intricately decorated Grecian urn into pieces and expecting the shape and decorative details to be visible in the pieces:

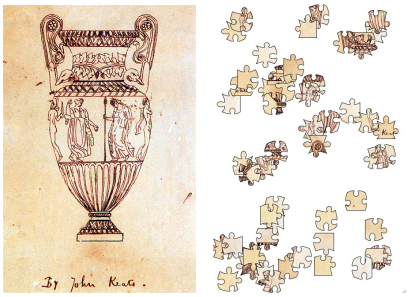

Figure 8: Standards for a Grecian Urn

In order to avoid this problem and preserve a coherent view of the subject, both in the broad contours and in the small details, the Common Core breaks with a long-standing tradition that each individual standard should have the same "grain-size". Mathematics itself does not come in pieces of equal grain-size, and neither should a description of it. Consider, for example the Grade 2 cluster of standards shown in Figure 9.

Understand place value

2.NBT.1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a 100 can be thought of as a bundle of ten tens—called a "hundred."

b The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

2.NBT.2. Count within 1000; skip-count by 5s, 10s, and 100s.

2.NBT.3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

2.NBT.4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using $>$, $=$, and $<$ symbols to record the results of comparisons.

Figure 9: Grade 2 cluster of standards on place value

The first standard is large and fundamental, figures in much of the work of

elementary school, and will show up again and again in a curricular implementation of the standards, reinforced and deepened by work in later grades. The second standard is a discrete performance objective that, once secured, recedes from importance.

This example illustrates another feature of the standards designed to provide coherence: the clusters and cluster headings. In the Common Core, the individual statements of what students are expected to understand and be able to do (the "standards") are embedded within cluster headings, which are in turn embedded in domains. "The Standards" refers to all elements of the document's design, including the wording of domain headings, cluster headings, and individual statements. In this case, the cluster heading "Understand place value" says clearly the fundamental purpose of this cluster, in a way that is not completely captured by any individual standard within the cluster.

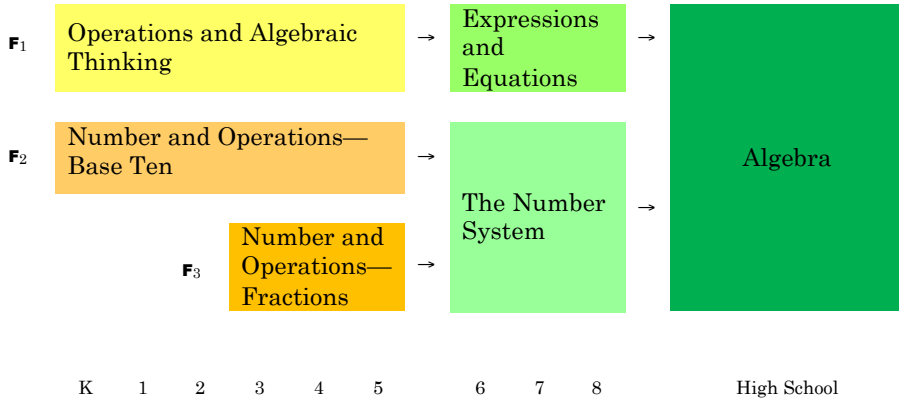

Figure 10: Flows leading to algebra

Another aspect of coherence is the flow of domains across grade levels, described on page 7. As a further example of this, Figure 10 shows the flow of domains to high school algebra. Building a viable ramp to algebra was a design requirement implied by the mandate to write standards that prepared students for college and career.

Balancing understanding, fluency, and applications

State standards before the Common Core were often formulated in terms of concrete observable performances, following a hierarchy of verbs, in which some verbs describe higher levels of performance than others (e.g., memorize, interpret, formulate, analyze). One verb in particular was often avoided, however, the verb "understand." Many times during the writing of the standards we were told we could not use this verb because standards had to be measurable, and understanding was ill-defined and either impossible or very difficult to measure.

Nonetheless, if the goal of standards is to express our desires for our children's achievements, it is hard to argue that understanding is not among them. The Common Core calls explicitly for understanding in a number of standards and cluster headings (see Figure 11).

- Understand and apply properties of operations and the relationship between addition and subtraction (Grade 2)
- Understand concepts of area and relate area to multiplication and to addition (Grade 3)
- Understand ratio concepts and use ratio reasoning to solve problems (Grade 6)
- Understand congruence and similarity using physical models, transparencies, or geometry software (Grade 8)
- Understand solving equations as a process of reasoning and explain the reasoning (High School)
- Understand and evaluate random processes underlying statistical experiments (High School)

Figure 11: Selected cluster headings using the word "understand"

Other standards explicitly call for fluency with addition and multiplication facts and with standard algorithms for addition, subtraction, multiplication, and division. These are capstone standards, occurring after adequate groundwork in earlier grades on strategies and algorithms based on place value and the properties of operations.

Yet other standards call for students to apply the mathematics they have learned. Modeling with mathematics is one of the Standards for Mathematical Practice (see Figure 6), and many of the high school standards are flagged as particularly important venues for modeling with a special symbol. The elementary and middle school standards build towards this with a progression of standards from simple word problems involving addition of whole numbers in Grade 2 to the following culminating standard in Grade 7:

7.EE.3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Concluding thoughts

Let me conclude by mentioning two projects designed to round out the description of the standards:

- The Progressions Project, ime.math.arizona.edu/progressions/
- The Illustrative Mathematics Project, illustrativemathematics.org

The standards were founded on progressions, narrative descriptions of domains provided by experts on the working team. The original progressions documents did not keep up with the rapid revision process, and the Progressions Project aims to produce final versions of them. The progressions provide another view of the standards, useful for curriculum designers, teacher educators, and could support research aimed at making recommendations for revisions to the standards.

The Illustrative Mathematics project is collecting sample tasks to illustrate the standards. It uses a community based approach, in which tasks are submitted, reviewed, edited and finally published by a growing community of experts who gain discernment and craft by participating in the process. It aims to become a permanent virtual destination that is both a repository of materials and a place where an expert community works.

I have tried to give some idea of what the standards look like, but ultimately a close reading of the standards is necessary to gain a complete picture. The standards are not designed to be easy reading, but they are designed to be read. The promise of the Common Core is having a shared text that, whatever its virtues and flaws, provides the basis of disciplined innovation in curriculum and shared tools for teaching.

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