

Core Content

Cluster Title: Graph points on the coordinate plane to solve real-world and mathematical problems.

Standard 1: Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x -axis and x -coordinate, y -axis and y -coordinate).

MASTERY Patterns of Reasoning:

Conceptual:

- Students understand how to construct a coordinate grid using perpendicular lines.
- Students understand the role and location of the origin while plotting an ordered pair.
- Students can travel horizontally and vertically within the grid, using their understanding of number lines.
- Students will understand that the ordered pair is a set of numbers indicating direction and magnitude of movement horizontally and vertically within the coordinate plane.
- Students will understand that the horizontal axis is the x -axis and the vertical axis is the y -axis.
- Students understand that the ordered pair $(0, 0)$ refers to the origin.

Procedural:

- Students can correctly construct and label the axes and the intervals on each axis within a coordinate grid.
- Students can identify the number in an ordered pair that is the x -coordinate (which coincides with the x -axis) and the number that is the y -coordinate (which coincides with the y -axis).
- Students will match a coordinate pair with a given point within quadrant I.

Representational:

- Students can show movement along the axes coinciding with the x -coordinate and y -coordinate.
- Students will properly draw and label the axes and origin on a coordinate plane.

Supports for Teachers

Critical Background Knowledge	
<p>Conceptual:</p> <ul style="list-style-type: none"> Students have an understanding of perpendicular lines. Students understand the terms “vertical,” “horizontal,” and “intersect.” Students know how to move along a number line (including varied intervals). Students can explain how to move along paths in real-world situations involving distance. For example, which is farther from the school—the grocery store or the post office—and in what direction would you have to travel to get to there. Students understand that you begin at zero when working with a number line. Students can explain how a number line works, even if it is displayed vertically. <p>Procedural:</p> <ul style="list-style-type: none"> Students will be able to locate points on a number line despite its orientation. Students can create number lines with differing intervals. <p>Representational:</p> <ul style="list-style-type: none"> Students can represent vertical and horizontal lines. Students can represent intersecting perpendicular lines. Students can create number lines. 	
Academic Vocabulary and Notation	
perpendicular, right angle, intersect, vertical, horizontal, coordinates, x-axis, y-axis, coordinate plane/grid, origin, x-coordinate, y-coordinate, ordered pair, intervals	
Instructional Strategies Used	Resources Used
<p>Create a life-size grid in the classroom on the floor using yarn to make the perpendicular axes. Label the axes. Have students physically move along the grid, exploring movements along the x-axis and the y-axis, then discuss how that movement can be represented with ordered pairs.</p> <p>Use multimedia to explore grids and the use of ordered pairs (see resources for “Ordered Pairs”).</p>	<p>Ordered Pairs http://www.learnalberta.ca/content/me5l/html/math5.html</p> <p><i>Fly on the Ceiling</i>, by Dr. Julie Glass ISBN: 0-679-88607-9</p>

Assessment Tasks Used	
<p>Skill-Based Task: On a coordinate grid, have students identify a specific point (e.g., What are the coordinates of the point where the rabbit is located?).</p> <p>Give students ordered pairs that they must match to points on the plane within the first quadrant.</p> <p>Students label the origin, x- and y-axes, and correct intervals on graph paper with a set of perpendicular lines and a set of points already drawn. Have student match the previously drawn points to the correct ordered pair from a list of given coordinate pairs.</p>	<p>Problem Task: Give students a map of the school on a coordinate grid and ask them to identify where certain places in the school are found. Then have students tell what is located at a particular given coordinate pair.</p>

Core Content

Cluster Title: Graph points on the coordinate plane to solve real-world and mathematical problems.

Standard 2: Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

MASTERY Patterns of Reasoning:

Conceptual:

Students can interpret coordinate points in the context of real-world situations (points on a map, or data on a line graph).

Students can interpret what the axes represent in different situations (in a line graph the x -axis may represent time while the y -axis may represent temperature).

Procedural:

Students can correctly interpret real-world data and plot that data in the first quadrant of a coordinate grid.

Representational:

Students can represent real-world data on a coordinate grid. For example, graph the time (x -coordinate) and temperature (y -coordinate) during a day at the amusement park.

Supports for Teachers

Critical Background Knowledge	
<p>Conceptual: Students understand how to construct and move along a coordinate grid. Students have basic map reading skills.</p> <p>Procedural: Students can take an ordered pair, start at the origin, and move correctly along the x-axis and y-axis. Students correctly construct the coordinate grid using perpendicular lines as the axes.</p> <p>Representational: Students can create number lines using varying intervals. Students can organize and represent data in a variety of ways.</p>	
Academic Vocabulary and Notation	
<p>perpendicular, right angle, intersect, vertical, horizontal, coordinates, x-axis, y-axis, coordinate plane/grid, origin, x-coordinate, y-coordinate, ordered pair, line graph</p>	
Instructional Strategies Used	Resources Used
<p>Have students take relevant real-world data (sports stats, stock market, sales, temperatures) and plot it in a line graph. Discuss how that data relates to coordinate grids, and how that representation helps them to better understand the data.</p> <p>Have students overlay a coordinate grid on a real map. They should use their knowledge of coordinate geometry and ordered pairs to find locations and travel along the map.</p>	<p>Newspapers (e.g., <i>Scholastic News</i>) and maps as a means of gathering current real-world data:</p> <p>http://www.mathwire.com/archives/geometry.html</p> <p>http://www.pbs.org/parents/cyberchase/lessons/lessonplans/lesson1.html</p> <p>http://nces.ed.gov/nceskids/createagraph/</p> <p>Note: Spreadsheet software such as Excel, often has a way to quickly turn numerical data into a graph.</p>

Assessment Tasks Used	
<p>Skill-Based Task: Give students a map that has a coordinate grid overlaid. Ask students to find the coordinates of specific landmarks, and have them find landmarks at given ordered pairs.</p>	<p>Problem Task: The local department store has recently released its information regarding video game sales over the last nine months. In the first month they sold 75 games. In the next month they sold 72. In the third month they sold 60, and in the fourth they sold 42. In the fifth month they sold 45, in the sixth they sold 38, in the seventh they sold 56, in the eighth they sold 62, and in the ninth they sold 79. Organize this data in a chart and then plot it on a coordinate grid to help the store understand their video game sales.</p>

Core Content

Cluster Title: Classify two-dimensional figures into categories based on their properties.

Standard 3: Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. (For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.)

MASTERY Patterns of Reasoning:

Conceptual:

Students understand that two-dimensional shapes are put into categories based on their attributes (side lengths, angle measures, parallel vs. perpendicular sides), and that shapes can belong to multiple categories. Students can identify the various categories in which a specific shape may belong.

Procedural:

Students can define two-dimensional shapes based on their attributes (e.g., a rhombus is a quadrilateral with four equal sides).
Student will classify shapes according to common attributes.

Representational:

Students can draw or construct, using geoboards, specific shapes according to the definitions provided, attributes described, or categories given.

Supports for Teachers

Critical Background Knowledge	
<p>Conceptual: Students can identify properties of various shapes (e.g., what makes a triangle a right triangle). Students understand what two-dimensional shapes are. Students can define what a polygon is.</p> <p>Procedural: Students can identify acute, right, and obtuse angles. Students can identify parallel and perpendicular lines.</p> <p>Representational: Students can represent a variety of basic two-dimensional shapes.</p>	
Academic Vocabulary and Notation	
<p>polygon, angle, line, parallel, perpendicular, triangle, quadrilateral, pentagon, trapezoid, hexagon, octagon, decagon, parallelogram, rectangle, rhombus, square, isosceles, scalene, acute, right, obtuse, equilateral, two-dimensional</p>	
Instructional Strategies Used	Resources Used
<p>Give students sets of triangle-shaped cards to play “Guess My Rule.” Player 1 will look through the variety of triangles and find at least 2 that share a common attribute. (e.g., player 1 groups together all triangles that have 2 equal sides). Player 2 looks at the grouped cards to determine the common attribute. Students record the common attribute before the next person creates his/her grouping.</p> <p>After playing for a while, bring all students together to see what rules they listed. Discuss how these listed properties are properties mathematicians use to define</p>	<p>http://nlvm.usu.edu/en/nav/category_g_2_t_3.html</p> <p>http://teams.lacoe.edu/documentation/classrooms/amy/geometry/6-8/activities/quad_quest/quad_quest.html</p> <p>http://www.bbc.co.uk/schools/ks2bitesize/maths/shape_space/shapes/play_popup.shtml</p> <p>http://www.crickweb.co.uk/ks2numeracy-shape-and-weight.html#quad</p>

<p>specific triangles. Make a class poster with the definitions of the specific types of triangles that exist and discuss how, according to these rules, a triangle can fit in multiple categories.</p> <p>Repeat the activity listed above with quadrilaterals.</p> <p>Have students use geoboards (or virtual geoboards) to construct various shapes. Ask students to make a parallelogram. Discuss how a rectangle, square, and rhombus may all correctly answer that prompt.</p> <p>Ask true-or-false questions about polygons fitting into specific categories or sharing attributes (e.g., A trapezoid has two sides parallel so it must be a parallelogram—true or false?).</p>	
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Assessment Tasks Used	
<p>Skill-Based Task: A parallelogram has four sides, with both sets of opposite sides parallel. What types of quadrilaterals are parallelograms?</p> <p>Regular polygons have congruent sides and angles. Name or draw some regular polygons.</p>	<p>Problem Task: Provide a series of “sometimes, never, or always” questions, and require that students provide a written or pictorial explanation for each answer.</p> <p>Example: A parallelogram is a square—sometimes, never, or always?</p> <p>Explain how you know.</p> <p>Explain why all squares are rectangles, but not all rectangles are squares.</p> <p>My friend was sure that she had made a rectangle. The teacher said that it was not a rectangle. How can my friend check to see if it is?</p>

Core Content

Cluster Title: Classify two-dimensional figures into categories based on their properties.

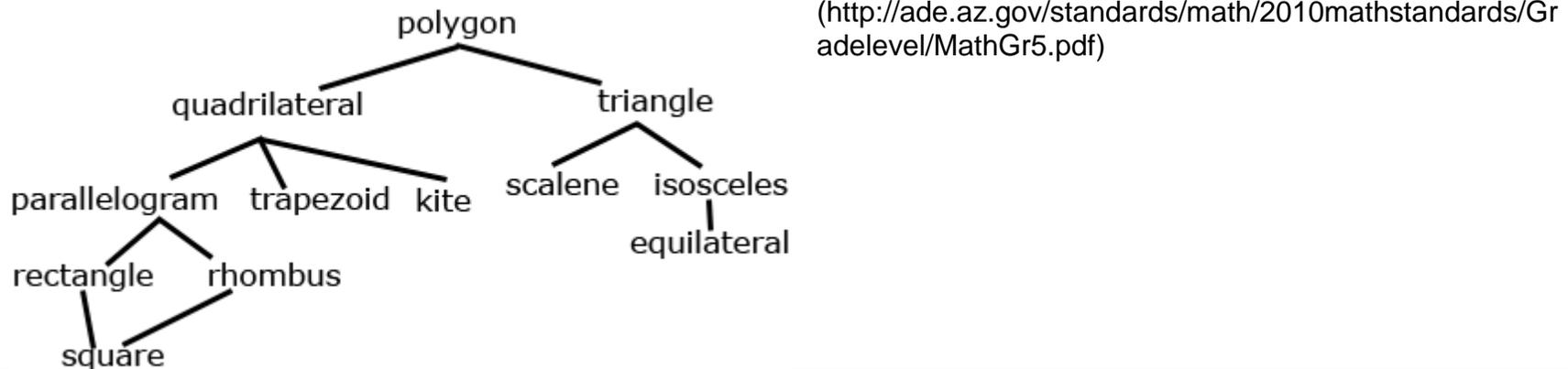
Standard 4: Classify two-dimensional figures in a hierarchy based on properties.

MASTERY Patterns of Reasoning:

Conceptual:
 Students will understand that there are terms that have broad definitions that encapsulate a wide variety of shapes, e.g., polygons.
 Students will classify shapes in a hierarchy of broad definitions to specific definitions (see graphic organizer below).

Procedural:
 Students will be able to use a graphic organizer to compare and organize shapes.

Representational:
 Students will be able to classify two-dimensional figures based on properties. See diagram below:



Supports for Teachers

Code: 5.G.4

Critical Background Knowledge	
<p>Conceptual: A shape can fit into multiple categories based on its properties. Students need to be able to identify the relevant properties of a two-dimensional shape.</p> <p>Procedural: Students can give the definitions for the various polygons.</p> <p>Representational: Students can construct various shapes, showing an understanding of the properties that define the shapes.</p>	
Academic Vocabulary and Notation	
<p>polygon, angle, line, parallel, perpendicular, triangle, quadrilateral, pentagon, hexagon, octagon, decagon, parallelogram, rectangle, rhombus, square, isosceles, scalene, acute, right, obtuse, trapezoid, equilateral, two-dimensional, hierarchy</p>	
Instructional Strategies Used	Resources Used
<p>Have students construct a mobile that displays the hierarchy. Have them start with a polygon.</p> <p>Construct a general polygon and label it.</p> <p>From the polygon hang models of quadrilaterals and triangles.</p> <p>Continue this pattern, hanging the shapes of the more specific definitions from the shape with the broader definition (much like the chart shown above).</p>	<p>http://www.ixl.com/math/standards/california/grade-5</p>
Assessment Tasks Used	
<p>Skill-Based Task: Have student identify all the polygons they can find in a piece of geometric art (http://interiorcomplex.com/accessories/20-modern-</p>	<p>Problem Task: Have students make their own piece of art, making sure to include a variety of polygons. Have students use their understanding of hierarchy to make the piece (for example, the</p>

<p>geometric-art-prints/#), and label those polygons with all the labels that fit (a rhombus should be labeled as a rhombus, parallelogram, quadrilateral, and polygon).</p>	<p>top of the picture can contain any polygons, but as it goes down it has to use more specific types of polygons).</p>
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Core Content

Cluster Title: Convert like measurement units within a given measurement system.
Standard 1: Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m) and use these conversions in solving multi-step, real world problems.
MASTERY Patterns of Reasoning:
<p>Conceptual: Students understand how to convert different-sized standard measurement units within a given measurement system (customary and metric).</p> <p>Procedural: Students can convert and apply measurement units within a given measurement system. Students can translate relationships between measurement units (e.g., 1 yd. to 3 ft. and 5 cm to 0.05 m).</p> <p>Representational: Students can use appropriate tools, models, and symbols to accurately convert from one unit of measure to another and properly label the results.</p>

Supports for Teachers

Critical Background Knowledge
<p>Conceptual: Students understand the relationship within measurement systems (capacity, length and mass).</p> <p>Procedural: Students can use conversions within a measurement system involving multi-step and real-world problems.</p> <p>Representational: Students understand appropriate use of measurement tools. Students can recognize uses of measurement in everyday situations.</p>

Academic Vocabulary and Notation	
<p>All units of measurement in the customary and metric systems, including: kilometer, meter, centimeter, millimeter, liter, milliliter, kilogram, gram, milligram, mile, yard, foot, inch, gallon, quart, pint, cup, ton, pound, and ounce as well as abbreviations for symbols (“ = in., ‘ = ft.)</p>	
Instructional Strategies Used	Resources Used
<p>A variety of situations involving measuring and converting.</p> <p>Examples:</p> <ol style="list-style-type: none"> 1. Convert yards to miles (e.g., 440 yds. = $\frac{1}{4}$ mile, 1,760 yds. = 1 mile). 2. Measure in metric units using strings, meter sticks, measuring tapes, trundles wheels, scale, pop bottles, measuring cups, etc., and convert within the system. 	
Assessment Tasks Used	
<p>Skill-Based Task: Convert the following:</p> <ul style="list-style-type: none"> • meters to kilometers • yards to miles • grams to kilograms • ounces to pounds • cups to gallons 	<p>Problem Task: A fifth grade class is running a 5k race. The class will begin practicing to increase their endurance, starting with 1,500 meters and adding 500 meters each week. How many weeks will it take to be ready for the race?</p> <p>A fifth grade class is running a three-mile race. The class will begin practicing to increase their endurance, starting with 880 yards and adding 440 yards per week. How many weeks will it take to be ready for the race?</p> <p>Zuri, the baby elephant, was born August 10, 2009 at Hogle Zoo. The calf weighed 251 lbs. at birth. If the baby elephant gains 48 ounces a day, how much will she weigh at the end of 7 days?</p>

Core Content

Cluster Title: Represent and Interpret Data
Standard 2: Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Use operations in fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.
MASTERY Patterns of Reasoning:
<p>Conceptual: Students will understand how to measure to a half, fourth or eighth of a unit and display that information in a line plot. Students will understand the use of operations with fractions to solve problems.</p> <p>Procedural: Students will measure in fractions and graph the results on a line plot. Students will be able to calculate accurately and formulate conclusions found on a line plot.</p> <p>Representational: Students will draw conclusions and communicate results from data.</p>

Supports for Teachers

Critical Background Knowledge
<p>Conceptual: Students will understand line plots, fractions, and operations with fractions.</p> <p>Procedural: Students will measure in fractional units and create a line plot. Students will analyze data to perform operations with fractional units.</p>

<p>Representational: Students will measure in fractional units with appropriate tools. Students will create line plots with collected data. Students will perform operations on fractions.</p>	
<p>Academic Vocabulary and Notation line plot, scale, intervals, benchmark fractions</p>	
<p>Instructional Strategies Used Measure various objects using fractional units with appropriate tools, and record results on a line plot. Use operations (\times, \div) of fractional parts with the data on the line plot.</p>	<p>Resources Used</p>
<p>Assessment Tasks Used</p>	
<p>Skill-Based Task: With a given set of data, students will make a line plot and answer related questions involving computation of fractions. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.</p>	<p>Problem Task: Students will design computation questions using the data displayed in a line plot.</p>

Core Content

Cluster Title: Geometric Measurement—understand concepts of volume and relate volume to multiplication and to addition.

Standard 3: Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
 a. A cube with side length of one unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.
 b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

MASTERY Patterns of Reasoning:

Conceptual:
 Students understand that a unit cube with side length of one is made up of six identical square faces and used to measure volume.
 Students understand that unit cubes fill a container without gaps or overlaps to measure volume.

Procedural:
 Students can identify pictures and objects of unit cubes.

Representational:
 Students can make a unit cube from a net of six identical square faces with the measure of one.
 Students can cover the area (the bottom of rectangular/square container) with a layer of unit cubes and then add additional layers to fill the container.

Supports for Teachers

Critical Background Knowledge

Conceptual:
 Students have knowledge of attributes of solid figures (cubes).
 Students understand that volume is the measurement of capacity.
 Students understand a cubic unit.

<p>Procedural: Students can identify faces, edges, vertices on three-dimensional figures.</p> <p>Representational: Students can use linking cubes to create a rectangular prism. Using 1 cm grid paper, create a net for the rectangular prism.</p>	
<p>Academic Vocabulary and Notation cube, unit cube (n^3), one cubic unit, volume, solid figure, overlapping (a partial face to partial face creates a gap) vs. stacking (full face to full face)</p>	
<p>Instructional Strategies Used</p> <p>Fill a clear plastic rectangular container with unit cubes and then with non-unit shapes to show how to represent volume. Example: popcorn, packing peanuts, marbles, etc.</p> <p>Have students build three different solid figures using a variety of unit cubes and find their volume.</p>	<p>Resources Used</p>
<p>Assessment Tasks Used</p>	
<p>Skill-Based Task: From a group of solids, identify a cube and recognize that each is a cubic unit used to measure volume.</p>	<p>Problem Task: How many cubic cm would it take to fill a rectangular prism with the height of 2 cm, length 3 cm, and width 5 cm?</p>

Core Content

Cluster Title: Geometric measurement—understand concepts of volume and relate volume to multiplication and to addition.
Standard 4: Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft. and improvised units.
MASTERY Patterns of Reasoning:
<p>Conceptual: Students will understand that units of measure can be different sizes (e.g., centimeter³, inches³, feet³).</p> <p>Procedural: Students can count units in three-dimensional pictures using different measures to find volume. Students can pack a container with unit cubes to find the volume.</p> <p>Representational: Students can build a solid figure using <i>unit cubes</i> to represent volume.</p>

Supports for Teachers

Critical Background Knowledge
<p>Conceptual: Students can know, understand, and write measurements in various cubic units.</p> <p>Procedural: Students can count various cubic units.</p> <p>Representational: Students will be able to stack unit cubes without gap or overlap.</p>
Academic Vocabulary and Notation
cubic in., cubic ft., cubic cm., improvised units (non-standard cubic units)

Instructional Strategies Used		Resources Used
Provide a variety of containers of different configurations and pack with unit cubes of differing measures (cm^3 , in^3 , ft^3 , improvised units).		
Assessment Tasks Used		
<p>Skill-Based Task: Identify cubic measures and transfer to volume using correct unit measurement.</p>	<p>Problem Task: Provide students with three-dimensional drawings that represent cubic units, then find volume and correctly label.</p> <p>Have students construct rectangular prisms from unit blocks varying the base and height. Determine the volume in cubic units.</p>	

Core Content

Cluster Title: Geometric measurement—understand concepts of volume and relate volume to multiplication and to addition.

Standard 5: Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

- a) Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes (e.g., to represent the associative property of multiplication).
- b) Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
- c) Recognize volumes as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

MASTERY Patterns of Reasoning:

Conceptual:

Students understand the mathematical operation for determining volume in a right rectangular prism using whole numbers.

Students understand real-world situations by recognizing that volume is the number of cubic units needed to fill a solid figure.

Students understand the volume of two or more solid figures added together equals the composite volume of the complete figure. (The sum of the parts equal to the whole.)

Procedural:

Students can pack a right rectangular prism with unit cubes and discover the numerical representation for the volume using the associative property of multiplication.

Students can explain how to relate counting cubes to the formula for finding volume.

Students can formulate separate volumes of various rectangular prisms and add shapes/volumes of the complete figure.

Representational:

Students can discover the threefold (three edge lengths) whole-number products as volume.
 Students can apply the formulas $V = l \times w \times h$ and $V = b \times h$.
 Students can separate the shape into rectangular prisms to find volume.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Students will understand the associative property of multiplication with three whole numbers.
 Students will have knowledge of attributes of solid figures.
 Students will understand a cubic unit.
 Students know and understand measurement and various cubic units.

Procedural:

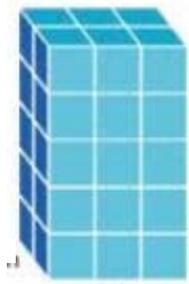
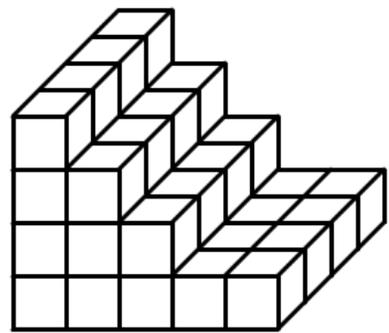
Students can demonstrate the ability to pack a right rectangular prism with unit cubes without overlap or gaps.

Representational:

Students can demonstrate an understanding of the three dimensions of a rectangular prism.

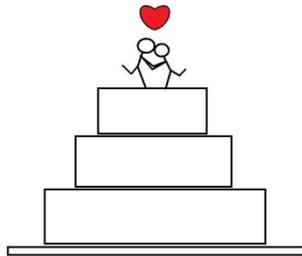
Academic Vocabulary and Notation

base/area of base, length, right rectangular prism, edge, height, formula, abbreviations of measurements (l , w , h , b , V),
 depth, additive

Instructional Strategies Used	Resources Used
<p data-bbox="178 235 1113 267">Find the volume of this shape. Record your answer in cubic units.</p> <div data-bbox="556 308 745 592" style="text-align: center;">  </div> <p data-bbox="178 738 798 771">What is the volume of this stack of blocks?</p> <p data-bbox="178 812 514 844">Answer: 44 cubic units</p> <div data-bbox="430 958 819 1291" style="text-align: center;">  </div>	

Dillan made a wedding cake with three layers.
 The bottom layer: $b = 64 \text{ inches}^2$ $h = 2 \text{ inches}$
 The middle layer: $b = 36 \text{ inches}^2$ $h = 2 \text{ inches}$
 The top layer: $b = 16 \text{ inches}^2$ $h = 2 \text{ inches}$
 What is the total volume of the cake?

Answer: 232 cubic inches



Assessment Tasks Used

Skill-Based Task:

Show a drawing of a stacked right rectangular prism. Students will find the length, width and height of the figure and calculate the volume.

Use appropriate vocabulary words to explain how to compute volume.

Accurately compute volume of solid figures composed of two non-overlapping rectangular prisms.

Problem Task:

Matthew has a treasure box with a base of 12 inches by 4 inches. The height is 8 inches. What is the volume of his treasure box?

A fifth grade class has a fish tank that is 26 inches long, 1 foot wide, and 16 inches deep. What volume of water can the tank hold?

The city swimming pool has a base area of 100 ft^2 and a depth of 12 ft. Find the volume of the pool.

Core Content

Cluster Title: Understand the place value system.

Standard 1: Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $1/10$ of what it represents in the place to its left.

MASTERY Patterns of Reasoning:**Conceptual:**

Students will recognize relationships between digits in multi-digit numbers by multiples of 10. For example, the value of 4 in 746 is four tens or 40; the value of 4 in 984 is four ones; the value of 4 in 0.46 is four tenths.

Procedural:

Students can determine the value of a digit when multiplied or divided by 10. For example $6 \times 10 = 60$ and $6,000 \div 10 = 600$ (dividing by 10 yields the same result as multiplying by $1/10$).

Representational:

Students can model the place value of a digit using a number-line, base ten blocks, and drawings. For example, the number 19 can be represented as nineteen ones or 190 tenths.

Supports for Teachers

Critical Background Knowledge**Conceptual:**

Students will know the names and position of each place value.

Students understand the value of each digit in the base 10 system.

Students will know that the value of a digit within a number increases or decreases when multiplied or divided by ten in the base ten system.

<p>Procedural: Students will be able to read and name the place value of digits in multi-digit numbers including decimals to the hundredths place. Students can multiply by 10. Students can divide by 10.</p> <p>Representational: Students can make number line representations of numbers, including decimal values. Students can model whole numbers and parts of whole numbers with drawings, base ten blocks, and other concrete models.</p>	
<p>Academic Vocabulary and Notation base ten system, decimal, names of the place values, tenth, hundredth, thousandth</p>	
<p>Instructional Strategies Used</p> <p>Have students find the difference in place value between the 2 in 542 and the 2 in 324. What is the difference in place value between the 2 in 324 and the 2 in 0.324?</p> <p>Have students explain how to represent the 2 in 542 differently than the 2 in 324, and to represent the 2 in 324 differently than the 2 in 0.324. Students can use base ten blocks or pictorial representations.</p> <p>Use a virtual manipulative to work with number lines involving decimal values.</p>	<p>Resources Used</p> <p>National Library of Virtual Manipulatives, number line with decimals. http://nlvm.usu.edu/en/nav/frames_asid_334_g_2_t_1.html?from=category_g_2_t_1.html</p>

Assessment Tasks Used	
Skill-Based Task: How many tenths are in 2.5? How much larger is 200 than 20? How much smaller is 0.3 than 3?	Problem Task: Using base-ten blocks, show at least two ways to represent the number 3.2. For example, use a flat as a unit, a stick as $\frac{1}{10}$ and a single as $\frac{1}{100}$. Show the difference between the 3 in 83.7 and the 3 in 8.37.

Core Content

Cluster Title: Understand the place value system.

Standard 2: Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

MASTERY Patterns of Reasoning:**Conceptual:**

Students will understand that when multiplying a whole number by a power of 10, the product shows an increased number of zeroes in relationship to the exponent. Students will show patterns in the number of zeros in the product of a number multiplied or divided by powers of ten.

For example, $3 \times 10^1 = 30$

$$3 \times 10^2 = 300$$

$$3 \times 10^3 = 3000$$

$$3 \times 10^4 = 30,000$$

$$3 \div 10^1 = 0.3$$

$$3 \div 10^2 = 0.03$$

$$3 \div 10^3 = 0.003$$

$$3 \div 10^4 = 0.0003$$

Students will understand that when multiplying by powers of ten, the exponent indicates how many places the decimal point is “moved” to the right in relationship to the exponent, increasing the place value of the number by 10 with each decimal place moved.

For example, $3.7 \times 10^4 = 3.7 \times (10 \times 10 \times 10 \times 10)$

$$3.7 \times 10^4 = 3.7 \times 10,000$$

$$= 37,000.0$$

Students will understand that when dividing by powers of ten, the exponent indicates how many places the decimal point is “moved” to the left in relationship to the exponent, decreasing the place value of the number by 10 with each decimal place moved.

For example, $3.7 \div 10^4 = 3.7 \div (10 \times 10 \times 10 \times 10)$

$$3.7 \div 10^4 = 3.7 \div 10,000$$

$$= 0.00037$$

Procedural:

Students can multiply and divide by powers of ten.

Representational:

Students can represent the value of a number multiplied or divided by a power of ten with a number line, base ten blocks and drawings (for example, $4 \div 10^3 = 4 \div 1000, 4 \div 1000 = 0.004$ modeled with manipulatives).

Supports for Teachers

Critical Background Knowledge

Conceptual:

Students will recognize relationships between digits in multi-digit numbers.
 Students understand the value of each digit in the base 10 system.
 Students will know that the value of a digit within a number increases when moved to the left and decreases as the number moves to the right in the base ten system.
 Students will understand that an exponent indicates the number of times a base is multiplied by itself.

Procedural:

Students can read and name the place value of digits in multi-digit numbers, including decimals to the thousandths place.
 Students will determine the value of a digit if it is moved left or right. For example, the number 19 can be written as nineteen ones or 190 tenths.

Representational:

Students can represent numbers with base ten blocks.

Academic Vocabulary and Notation

^, base ten, exponential notation, product, power of ten, exponent, base

Instructional Strategies Used

Connect multiplying and dividing numbers by powers of ten with students' prior knowledge of place value. Help them see that when a number is multiplied by a power of ten its place value is increased. The 0's added at the end

Resources Used

<p>of the number correspond to the exponent used. For example, the number 45 when multiplied by 10^2 has its place value increased by 100, becoming 4500. Help students see patterns in the increase in place value with increasing powers of 10.</p> <p>Find which power of ten could be a factor of a given large number. For example, the number 4500 could be 450×10^1, or 45×10^2, or 4.5×10^3. Have students find factors of other large numbers and explain their reasoning. Have them show patterns of increasing or decreasing powers of 10.</p>	
<p>Assessment Tasks Used</p>	
<p>Skill-Based Task: Explain why the following multiplication and division problems with powers of ten are true. $432 \times 10^3 = 432000$ $4.32 \times 10^2 = 432$ $43.2 \times 10^1 = 432$</p>	<p>Problem Task: Martha earned \$4.20 each day for ten days of babysitting. Over a year's time, she worked ten times ten days. Write an expression using exponents of 10 to show how much she earned in ten days, then in 10 times ten days. How much did she earn in one year? Justify your answer.</p>

Core Content

Cluster Title: Understand the place value system.

Standard 3: Read, write, and compare decimals to thousandths.

- a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form (e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$).
- b. Compare two decimals to thousandths based on meanings of the digits in each place, using $<$, $=$, and $>$ symbols to record the results of comparisons.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand expanded form represents a digit multiplied by its place value ($347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$).

Students will expand understanding of place value to include the thousandths place.

Procedural:

Students will read and write decimals to the thousandths place using base ten numerals, number word names and expanded form.

Students will compare two decimal numbers to the thousandths place based on the meaning of the digits in each place by using $<$, $=$, and $>$ to record the results of the comparisons.

Students will compare tenths to tenths, hundredths to hundredths, thousandths to thousandths.

Representational:

Students will model comparisons of decimals to thousandths using place value charts, grids, manipulatives and technology.

Supports for Teachers

Critical Background Knowledge	
<p>Conceptual: Students will understand the value of decimal numbers as compared to benchmark numbers such as 0, 0.5 and 1. Students will understand the value of digits to the hundredths place.</p> <p>Procedural: Students can compare decimal numbers to the hundredths place. Students can write numbers in expanded form to the hundredths place.</p> <p>Representational: Students can represent multi-digit numbers in expanded form (such as 435 as $400 + 30 + 5$).</p>	
Academic Vocabulary and Notation	
<p><, =, >, greater than, less than, equal to, equality, expanded notation, benchmark</p>	
Instructional Strategies Used	Resources Used
<p>Match the standard form to the expanded form using activities such as those found on the website for “Expanded notation games.” (See resources for “Expanded notation games.”)</p> <p>Use 0, 0.5, and 1 as benchmarks when comparing decimals.</p> <p>Have students create numbers and then arrange them in order from least to greatest, etc.</p>	<p>Expanded notation games: http://www.ehow.com/list_5880741_math-games-expanded-notation.html</p>
Assessment Tasks Used	
<p>Skill-Based Task: Write 562.376 in expanded form. Given 1.02, 1.2 and 1.002, place the numbers in order from least to greatest.</p>	<p>Problem Task: Find four numbers that are between 0.11 and 0.12 and put all six numbers in order from least to greatest.</p>

Core Content

Cluster Title: Understand the place value system.
Standard 4: Use place value understanding to round decimals to any place.
MASTERY Patterns of Reasoning:
Conceptual: Students will understand rounding procedures with decimal values.
Procedural: Students can use place value understanding to round decimals to any place.
Representational: Students can place decimals along the number line to represent the process of rounding. Students can use base-ten blocks, money, fraction bars, etc. to represent rounding decimal amounts.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand the rules and procedures for rounding whole numbers to any targeted value. Students will be able to make use of multiple opportunities for placing decimal values on number lines. Students will understand place value. Students will understand what a benchmark number is.
Procedural: Students will use benchmark numbers to compare decimals. Students will place benchmark numbers on a number line. Students will be able to round whole numbers. Students will be able to estimate whole number values.

<p>Representational: Students can model rounding of whole numbers with pictures or manipulatives.</p>	
<p>Academic Vocabulary and Notation rounding, benchmark numbers, estimate</p>	
<p>Instructional Strategies Used</p> <p>Have students play a memory game of matching cards with a decimal number to cards with that amount rounded to a certain place value.</p> <p>Play the memory game on a computer. (See resources for "Memory game.")</p> <p>Give a number such as 0.62 and have students draw it on a grid and round to the nearest tenth.</p>	<p>Resources Used</p> <p>Memory game: http://www.numbernut.com/advanced/activities/estimate_mem_20_round10th.shtml</p>
<p>Assessment Tasks Used</p>	
<p>Skill-Based Task: Round 1.069 to the nearest tenth, hundredth and one.</p>	<p>Problem Task: If the rounded number is 9.6, what could the original number have been?</p>

Core Content

Cluster Title: Perform operations with multi-digit whole numbers and with decimals to hundredths.
Standard 5: Fluently multiply multi-digit whole numbers using the standard algorithm.
MASTERY Patterns of Reasoning:
Conceptual: Students will understand that multiplication can be achieved through an algorithm using basic multiplication facts.
Procedural: Students can use the standard algorithm to calculate the product of multi-digit factors.
Representational: Students can represent multi-digit multiplication with arrays, diagrams, area models and manipulatives.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand that multiplication is repeated addition. Students will understand the significance of place value. Students will be fluent with basic multiplication facts. Students can multiply two-digit numbers using strategies based on place value and the properties of operations.
Procedural: Students can calculate single-digit multiplication. Students can calculate multi-digit addition. Students will know how to regroup when adding.
Representational: Students can model multiplication of single-digit multiplication using set, array, and area models. Students can model regrouping with multi-digit addition.

Academic Vocabulary and Notation													
product, factor, array, area model, equal groups, multiples, algorithm, partial products, multiplication symbols [$3 \times a$, $3 \cdot a$, $3 * a$, $3(a)$, $3a$]													
Instructional Strategies Used	Resources Used												
<p>Use an area model for multiplying multi-digit numbers.</p> <table style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tr> <td style="padding: 5px;"></td> <td style="padding: 5px; text-align: center;">200</td> <td style="padding: 5px; text-align: center;">20</td> <td style="padding: 5px; text-align: center;">5</td> </tr> <tr> <td style="padding: 5px; text-align: center;">10</td> <td style="padding: 5px; text-align: center; border: 1px solid black;">2000</td> <td style="padding: 5px; text-align: center; border: 1px solid black;">200</td> <td style="padding: 5px; text-align: center; border: 1px solid black;">50</td> </tr> <tr> <td style="padding: 5px; text-align: center;">2</td> <td style="padding: 5px; text-align: center; border: 1px solid black;">400</td> <td style="padding: 5px; text-align: center; border: 1px solid black;">40</td> <td style="padding: 5px; text-align: center; border: 1px solid black;">10</td> </tr> </table> <p style="text-align: center;">Area Model</p> <p>Use the area model to connect partial products to the standard algorithm.</p> <p>Use technology to show lattice multiplication as found on the Everyday Math Online website.</p> <p>Model the standard algorithm with blanks or missing digits within the partial products.</p>		200	20	5	10	2000	200	50	2	400	40	10	<p>Go to Everyday Math Online at http://emccss.everydaymathonline.com/g_login.html and click on the "Free Resources" link. Then click on "Algorithms in Everyday Mathematics" and select "Grade 5," and finally choose "Multiplication."</p>
	200	20	5										
10	2000	200	50										
2	400	40	10										
Assessment Tasks Used													
<p>Skill-Based Task: Multiply 524×639 and show your work</p>	<p>Problem Task: A bakery has 245 dozen cupcakes. How many individual cupcakes are there?</p>												

Core Content

Cluster Title: Perform operations with multi-digit whole numbers and with decimals to hundredths.
Standard 6: Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
MASTERY Patterns of Reasoning:
Conceptual: Students will understand that multi-digit division is related to single-digit division in that it involves grouping (quotative model) or fair share (partitive model). Students will understand that multiplication and division are inverse operations.
Procedural: Students can calculate whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors. Students can explain the calculation. Students can use a variety of procedures for dividing whole numbers. (For example, lucky 7, standard algorithm, partial quotient, etc.)
Representational: Students can illustrate division using equations, rectangular arrays, and area models.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand properties of operations. For example, commutative, associative, identity, etc. Students will understand both quotative and partitive division. Students will understand the significance of place value.

<p>Procedural: Students can regroup with subtraction. Students can find the whole-number quotient of whole numbers with up to four-digit dividends and one-digit divisors.</p> <p>Representational: Students will model simple division with arrays. Students will model both quotative and partitive division using arrays and area models (4.NBT.6).</p>	
<p>Academic Vocabulary and Notation</p>	
<p>dividend, divisor, quotient, area model, partitive, quotative, arrays, division notation ($\frac{a}{b}$, a/b, $a \div b$, $b \overline{)a}$)</p>	
<p>Instructional Strategies Used</p>	
<p>When instructing on the use of the traditional algorithm for long division, make sure that place value is expressly provided. For example, in the problem $324 \div 27$ set up as</p> $27 \overline{)324}$ <p>do not ask, “How many times does 27 go into 3?” when it is 3 hundred. Instead of asking how many times 27 goes into 32, ask “How many 27’s are there in 32 tens?”</p> <p>Use more than one method for dividing, such as partial quotients, lucky 7 and arrays. Work toward linking these strategies to the standard algorithm.</p>	<p>Resources Used</p>
<p>Common Core State Standards document page 89 “Common Multiplication and Division Situations”</p> <p>Understanding Division (Section 2): http://www.conceptualstudy.org/Elementary Math/Understanding Division.htm</p>	
<p>Assessment Tasks Used</p>	
<p>Skill-Based Task: Divide the following and show your work. $14 \overline{)588}$</p>	<p>Problem Task: Samantha wants to split a collection of stickers into groups of 48. Samantha has 1,008 stickers. How many groups will be created? Show two ways to find the answer.</p>

Core Content

Cluster Title: Perform operations with multi-digit whole numbers and with decimals to hundredths.

Standard 7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

MASTERY Patterns of Reasoning:**Conceptual:**

Students will understand the relationship between addition and subtraction when adding, subtracting, multiplying and dividing decimals.

Students will understand the properties of operations in relationship to adding, subtracting, multiplying, and dividing decimals to the hundredths place.

Students will understand the significance of place value when adding, subtracting, multiplying and dividing as it applies to decimals.

Procedural:

Students can add, subtract, multiply, and divide decimals to hundredths. Include decimal dividends and divisors.

Students can divide whole numbers by 0.1 and 0.01 to build understanding of the place value significance in division of decimal numbers.

Representational:

Students can use concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Students can relate the strategy to a written method and explain the reasoning used.

Supports for Teachers

Critical Background Knowledge	
<p>Conceptual: Students will understand the inverse relationship of multiplication and division. Students will understand the relationship between all operations (+ - x ÷) when working with whole numbers. Students will understand place value up to the thousandths place. Students will understand the properties of operations in relationship to adding, subtracting, multiplying, and dividing multi-digit whole numbers.</p> <p>Procedural: Students can fluently add, subtract, multiply, and divide multi-digit whole numbers using the standard algorithms.</p> <p>Representational: Students can model whole number division. Students can model multi-digit whole number multiplication. Students can accurately represent multi-digit whole numbers with concrete models and drawings.</p>	
Academic Vocabulary and Notation	
properties of operations, operation notations, multiplication symbols [3 x a, 3 · a, 3 * a, 3(a), 3a] division notation ($\frac{a}{b}$, a/b , $a \div b$, $a \overline{)b}$)	
Instructional Strategies Used	Resources Used
Build understanding of place value by dividing by 0.1 and 0.01 before moving to other tenths and hundredths. Explore how the result is related to using the powers of ten to multiply or divide. Use a variety of measurement contexts for addition and subtraction of decimal numbers.	National Library of Virtual Manipulatives: http://nlvm.usu.edu/en/nav/frames_asid_264_g_2_t_1.html?from=category_g_2_t_1.html

<p>Explain the significance of decimal places when working with measurement (time, weight, length, money, area, volume).</p> <p>Have students make up a story problem that involves $22.8 \div 6$. Ask them to share their number stories and explain how they got the answer to their problem.</p>	
Assessment Tasks Used	
<p>Skill-Based Task: Calculate the following and show your work.</p> <ol style="list-style-type: none"> 1. $3.4 + 6.2$ 2. $7.7 - 4.1$ 3. 5.6×2.4 4. $8.4 \div 2.1$ 	<p>Problem Task: A tabletop has the measurements 3.5 meters by 1.2 meters. what is the area in square meters? If your brother cut 0.3 meters off of one side, how would that affect the area of the table top. Does it matter which side is cut? Show your work, including a diagram. You can use graph paper if needed.</p> <p>I divided 6.12 by 3 and got the quotient 2.4. What did I do wrong? Give a similar problem where I might make the same error.</p> <p>In this calculation some numbers are missing. What might they be? How do you know?</p> $\begin{array}{r} 3.?? \\ - \underline{?.7?} \\ 1.?3 \end{array}$ <p>I added 3 decimal numbers together and got exactly 4. What might those 3 decimal numbers be?</p> <p>How many different ways can you make your calculator show a number with a particular decimal, such as 12.34, without pressing the decimal point button?</p>

Core Content

Cluster Title: Use equivalent fractions as a strategy to add and subtract fractions.

Standard 1: Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$. (In general, $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$.)

MASTERY Patterns of Reasoning:**Conceptual:**

Students will recognize that fractions with unlike denominators cannot be added or subtracted.

Students will recognize when one or both fractions in may need to be converted to equivalent fractions with like denominators.

Procedural:

Students can convert fractions to equivalent fractions with like denominators as needed.

Students can find the sum or difference of fractions with like denominators.

Students can convert sums and differences to equivalent fractions as appropriate. (Note: Not all fractions need to be converted to lowest terms to be considered correct. The form of the fraction will be dependent upon the problem being asked.)

Representational:

Students can create concrete and pictorial models to illustrate addition and subtraction of fractions with unlike denominators and equivalent fractions.

Students can connect models, including set models, area models, and linear models, to numerical representations.

Supports for Teachers

Critical Background Knowledge**Conceptual:**

Students will understand a proper fraction as a fraction less than 1.

Students will understand an improper fraction as a fraction greater than 1.

Students will understand mixed numbers as a whole number and a fraction or whole numbers and parts of a whole.

Students will understand and recognize equivalent fractions.

Students will understand that all addition and subtraction of fractions requires like denominators.

<p>Procedural: Students can demonstrate how to generate equivalent fractions. Students can rename improper fractions and mixed numbers. Students can add and subtract fractions and mixed numbers with like denominators.</p> <p>Representational: Students can represent fractions with models, including set models, area models, and linear models, and connect them to numerical representations. Students can represent equivalent fractions with models and connect them to numerical representations.</p>	
<p>Academic Vocabulary and Notation proper fraction, improper fraction, mixed numbers, numerator, denominator, like denominator, unlike denominator, common denominators, equivalent fractions, number lines, fraction bar, sum, difference</p>	
<p>Instructional Strategies Used</p> <ol style="list-style-type: none"> Using a variety of manipulatives (e.g., fraction strips, area models, pattern blocks, number lines), model converting pieces to equivalent fractions. Connect manipulative models with numerical representations. Help children to transition to a pictorial representation from the manipulatives to show how fractions are converted to equivalent forms. Teach the algorithm for converting fractions to like denominators. Use models to find sums or differences of fractions with unlike denominators by converting to like denominators. Using the concept of multiples, rewrite fractions in equivalent form to find common denominators. Use a multiplication table to find equivalent fractions. 	<p>Resources Used</p> <p>National Library of Virtual Manipulatives: http://nlvm.usu.edu/en/nav/frames_a_sid_106_g_3_t_1.html?from=grade_g_3.html</p> <p>http://visualfractions.com/AddUnlike/addunlike.html</p> <p>Extensions: http://www.k-5mathteachingresources.com/supportfiles/magicsquaresadditionfractions.pdf</p> <p>Equivalent Fractions: http://illuminations.nctm.org/ActivityDetail.aspx?ID=80</p>

Fraction Game:
<http://illuminations.nctm.org/ActivityDetail.aspx?ID=18>

Assessment Tasks Used

Skill-Based Task:

$$3\frac{3}{12} - \frac{1}{6} = 3\frac{3}{12} - \frac{2}{12} = 3\frac{1}{12}$$

Problem Task:

There is one quart of chocolate milk in the refrigerator. Michael drinks 1/2 of the quart. Nancy drinks 1/3 of the quart. How much chocolate milk did Michael and Nancy drink altogether? How much of the original quart is left?

John brought a pizza to a party. His friend Sally also brought a pizza to the party. At the end of the party, John had 1/4 of his pizza left. Sally had 3/8 of her pizza left. How much pizza was left at the end of the party? How much pizza was eaten?

Core Content

Cluster Title: Use equivalent fractions as a strategy to add and subtract fractions
Standard 2: Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators (e.g., by using visual fraction models or equations to represent the problem). Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$.
MASTERY Patterns of Reasoning:
Conceptual: Students can interpret word problems by using strategies (e.g., highlight key information, draw a picture). Students can determine the reasonableness of an answer using estimation and benchmark fractions.
Procedural: Students can set up an equation to represent the problem. Students can convert fractions to equivalent fractions with like denominators. Students can add or subtract fractions with like denominators. Students can use estimation and benchmark fractions to check the reasonableness of the answer.
Representational: Students can use concrete and pictorial models, including set models, area models, and linear models, and connect to numerical representations.

Supports for Teachers

Critical Background Knowledge
Conceptual: Students will understand basic problem-solving strategies. Students will understand proper fractions, improper fractions, and mixed numbers. Students will understand how to compare a fraction to a benchmark fraction. Students will understand strategies to interpret and understand word problems.
Procedural: Students can convert fractions with unlike denominators to equivalent fractions with like denominators.

Students can add and subtract fractions and mixed fractions with like denominators.
 Students can rename improper fractions and mixed numbers.

Representational:

Students can represent fractions with models including set models, area models, and linear models.

Academic Vocabulary and Notation

benchmark fractions, sum, difference, numerator, denominator, like denominators, unlike denominators, estimation, equivalent fractions

Instructional Strategies Used

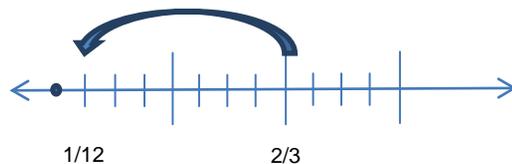
Use a word problem such as the following:

Julia was doing a science experiment in which she was comparing the heights of a bean plant at the start and end of a two-week time period. At the end of the two-week time period, the plant was $\frac{2}{3}$ of a foot tall. At the beginning of the time period, it was $\frac{1}{12}$ of a foot tall. How much did the plant grow over the two-week time period?

Ask the students to work in groups and estimate the answer. Then allow them to use either a linear model (with a number line) or a bar model to find the answer.

Examples:

Linear Model:



In this model, each tick mark represents $\frac{1}{12}$. By counting the tick marks,

Resources Used

<http://www.youtube.com/watch?v=eQa2S92ftNo&feature=related>

8 Step Model Drawing: Singapore's Best Problem-Solving Math Strategies by Bob Hogan and Char Forsten

Extensions:

<http://nrich.maths.org/2312>

<p>students determine that $\frac{2}{3} = \frac{8}{12}$. By counting backwards from $\frac{2}{3}$ students determine that the answer is $\frac{7}{12}$. So, creating the equation, $\frac{2}{3} - \frac{1}{12} =$</p> $\frac{8}{12} - \frac{1}{12} = \frac{7}{12}$	
<p>Assessment Tasks Used</p>	
<p>Skill-Based Task: Solve the problem with a visual model and equation. Also, use benchmark fractions to check the reasonableness of your answer.</p> <p>Claire took $2\frac{3}{4}$ hours to read a book. Her brother, Dan, took $\frac{2}{3}$ hour less to read his book. How much more time did Claire spend reading than Dan? Extension Question: How much time did they spend altogether reading their books?</p>	<p>Problem Task</p> <ol style="list-style-type: none"> 1. Create a word problem that could be solved by adding two specific fractions with unlike denominators. Example: $\frac{1}{2} + \frac{3}{4} =$ 2. Represent the problem using both a diagram and an equation. 3. Solve your problem. Show all your work. 4. Use benchmark fractions to explain how you know that your answer is reasonable.

Core Content

Cluster Title: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Standard 3: Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. (For example, interpret $3/4$ as the result of dividing 3 by 4, noting that $3/4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $3/4$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?)

MASTERY Patterns of Reasoning:

Conceptual:

- Students will understand that the fraction $a/b = a \div b$.
- Students will understand problem-solving strategies involving fractions.
- Students will understand that a quotient may be a whole number, mixed number, or a fraction.
- Students will understand that a remainder can be written as a fraction.

Procedural:

- Students can rewrite division expressions as fractions.
- Students can solve story problems with remainders written as a fraction.
- Students can determine between which two whole numbers a fraction lies.

Representational:

- Students can use concrete and pictorial models to show fractions represented by whole number division.

Supports for Teachers

Critical Background Knowledge	
<p>Conceptual: Students will know that division is fair sharing. Students will understand long division. Students will understand fractions can be less than 1 or more than 1. Students will understand the relative sizes of fractions and mixed numbers and their places on a number line. Students will know that multiplication and division are inverse operations.</p> <p>Procedural: Students can divide whole numbers where quotients have a remainder.</p> <p>Representational: Students can draw pictorial representations showing fair sharing.</p>	
Academic Vocabulary and Notation	
numerator, denominator, proper fraction, improper fraction, mixed number, quotient, divisor, dividend, remainder, fair sharing	
Instructional Strategies Used	Resources Used
<p>Divide 12 equal-sized pizzas among 4 students ($12/4 = 12 \div 4$). Ask: How many pizzas does each student get?</p> <p>Have students suggest how to divide 2 pizzas equally among 3 students ($2/3 = 2 \div 3$). Explain how to divide the pizzas in smaller pieces using fraction circles. Show that each pizza is divided equally into the number of parts which represent the number of students. So each pizza is divided into 3 smaller pieces. Show that dividing 6 smaller pieces of pizza among 3 students means each student gets 2 pieces each. Because a pizza comprises 3 equal pieces, each student gets $2/3$ of a pizza.</p>	

Assessment Tasks Used	
<p>Skill-Based Task: Write a word problem to show that $\frac{3}{4}$ is a division problem. Draw a model to illustrate the story problem.</p> <p>Write a word problem with a fraction less than 1 used as a division problem. Draw a model to illustrate the story problem.</p> <p>Write a word problem with a fraction greater than 1 used as a division problem. Draw a model to illustrate the story problem.</p>	<p>Problem Task: Draw a picture and write an equation to solve the following problem:</p> <p>Six teachers need to equally share 27 boxes of pencils. How many boxes of pencils will each teacher receive?</p>

Core Content

Cluster Title: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Standard 4: Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand a number sentence can be restated as a word sentence. Examples: $5 \times 3/4$ is the same as 5 groups of $3/4$, $1/2 \times 1/2$ is the same as $1/2$ of a group of $1/2$.

Students will understand that a whole number multiplied by a fraction can be represented as repeated addition.

Example: $6 \times 3/4 = 3/4 + 3/4 + 3/4 + 3/4 + 3/4 + 3/4$.

Students will be able to create a story context for an equation of the form $(a/b) \times q$.

Procedural:

Students can multiply a fraction by a whole number.

Students can multiply a fraction by a fraction including improper fractions and mixed numbers.

Representational:

Students can use area models to represent multiplication of a fraction by a whole number and a fraction by a fraction.

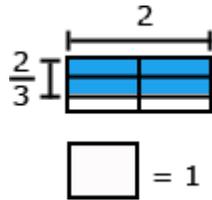
Supports for Teachers

Critical Background Knowledge	
<p>Conceptual: Students will understand multiplication and division of whole numbers. Students will understand 5×7 is 5 groups of 7.</p> <p>Procedural: Students can multiply and divide whole numbers fluently. Students can solve a two-step problem. Students can represent a whole number as a fraction. For example, $12 = 12/1$.</p> <p>Representational: Students can represent problems with visual fraction models.</p>	
Academic Vocabulary and Notation	
partition, factors, products, numerator, denominator, fraction, whole number	
Instructional Strategies Used	Resources Used
<p>Teacher’s Note: When multiplying a problem such as $3/5 \times 6$, the operations can be thought of in more than one way. Examples: $3 \times (6 \div 5)$ or $(3 \times 6)/5$ OR $(3 \times 6) \div 5$ or $18 \div 5$ ($18/5$)</p> <p>Represent problems such as: $1/3 \times 5$ as finding $1/3$ of 5 or $1/2 \times 1/2$ as finding half of a half.</p> <p>Students create a story context for $3/5 \times 6$. Examples: Isabel had 6 feet of wrapping paper. She used $3/5$ of the paper to wrap some presents. How much does she have left?</p>	3-5 Fractions-Rectangle Multiplication: http://nlvm.usu.edu/en/nav/frames_a_sid_194_g_2_t_1.html?from=topic_t_1.html

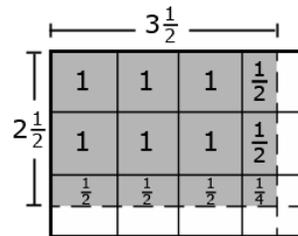
Every day Tim ran $\frac{3}{5}$ of mile. How far did he run after 6 days? (Interpreting this as $6 \times \frac{3}{5}$.)

Use area models. Examples:

$$2 \times \frac{2}{3} = \frac{4}{3}$$



$$2\frac{1}{2} \text{ groups of } 3\frac{1}{2}$$



Using the standard algorithm, the problem $\frac{2}{3} \times \frac{4}{5}$ can be represented as $\frac{2 \times 4}{3 \times 5}$.

Visual fraction models can be used to represent the following example:

Three-fourths of a class is girls. Two-thirds of the girls are having pizza for lunch. What fraction of the girls is eating pizza?

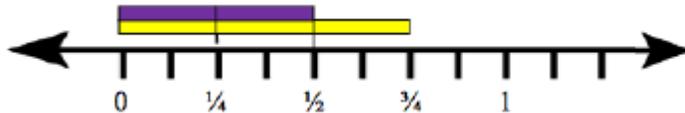
This question is asking what $\frac{2}{3}$ of $\frac{3}{4}$ is, or what is $\frac{2}{3} \times \frac{3}{4}$.

Examples of visual fraction models:

Area Model:



Number Line Model:



Many of the images and explanations in the Instructional Strategies are from the site below:
<http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/5th.pdf>

Assessment Tasks Used

Skill-Based Task:

Interpret the product with a visual model.

$\frac{1}{3} \times 5 =$

$2\frac{1}{4} \times 3 =$

$\frac{1}{3} \times \frac{7}{8} =$

Problem Task:

Find the product and create a story context for this problem:
 $\frac{4}{5} \times \frac{3}{4}$.

Use a visual model to solve this problem:
 George drank $\frac{3}{4}$ of $\frac{1}{2}$ gallon of milk. How much of the gallon did he drink? How much of the gallon is left?

Core Content

Cluster Title: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Standard 4: Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

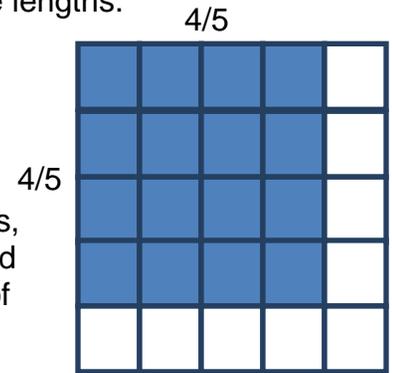
b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths.
 Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

MASTERY Patterns of Reasoning:

Conceptual:
 Students will understand multiplication of a whole number by a fraction and a fraction by a fraction.
 Students will understand that area of a rectangle is measured in unit squares.

Procedural:
 Students can find the area of a rectangle with fractional side lengths by using fraction tiles.
 Students can find the area of a rectangle with fractional side lengths by multiplying side lengths.

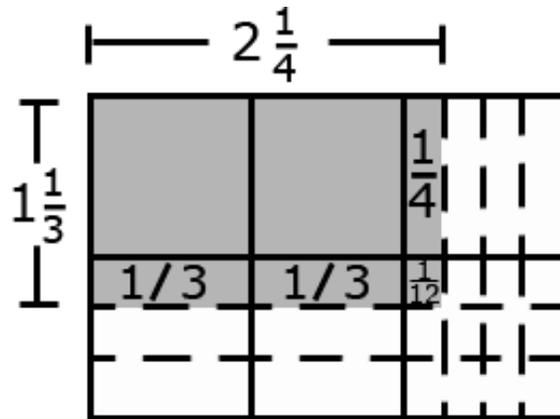
Representational:
 Students can use concrete manipulatives (fraction tiles) to demonstrate area.
 Students can use graph paper/dot paper to make pictorial representations of area.
 Students can use graph paper/dot paper to show fraction products as rectangular areas, e.g., show $16/25$ square units by creating a rectangle of 25 square units (the whole) and coloring in 16 square units (the part) and then determining the fractional side lengths of the resulting rectangle.



Supports for Teachers

Critical Background Knowledge	
<p>Conceptual: Students will understand that area is the number of square units in a figure. Students will understand the meaning of multiplication of a whole number by a fraction.</p> <p>Procedural: Students can multiply a whole number by a fraction. Students can use the formula for area: Area = length x width.</p> <p>Representational: Students can find area using the dimensions of a rectangle. Students can use tiles and arrays to calculate area.</p>	
Academic Vocabulary and Notation	
<p>area, length, width, $A = lw$, square units, x means OF, array, dimension</p>	
Instructional Strategies Used	Resources Used
<div style="text-align: center;"> </div> <p>Numerical Representation: $\frac{1}{2} \times 4 = 2$</p>	

Example of Area Model:



Numerical Representation: $1\frac{1}{3} \times 2\frac{1}{4} = 3$

Assessment Tasks Used

Skill-Based Task:

Find the area of a rug that is $3\frac{1}{2}$ feet by $2\frac{1}{2}$ feet.

Problem Task:

Mr. Brown is building a sandbox that is $6\frac{1}{2}$ feet by $4\frac{1}{2}$ feet. Draw a model of the sandbox, labeling all dimensions. Find the total area of the sandbox. Explain your answer.

Core Content

Cluster Title: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Standard 5: Interpret multiplication as scaling (resizing) by:

- a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand that when you increase or decrease one of the factors in a multiplication problem, the product will also increase or decrease.

Students will understand that multiplying greater factors results in greater products and multiplying lesser factors results in lesser products.

Procedural:

Students can compare products mentally. For example, 4×9 is greater than 2×9 because 4 is greater than 2.

Example: $6 \times 9 = 54$

$3 \times 9 = 1/2 \times 54$ because 3 is $1/2$ of 6

$6 \times 24 = 144$

$2 \times 24 = 1/3 \times 144$ because 2 is $1/3$ of 6

Representational:

Students can draw a model to compare the size of products based on various factors.

Supports for Teachers

Critical Background Knowledge	
<p>Conceptual: Students will understand relationships between the size of the factors and products.</p> <p>Procedural: Students can multiply whole numbers.</p> <p>Representational: Students can multiply using area models or arrays.</p>	
Academic Vocabulary and Notation	
scaling, array, factors, products, x means “of”	
Instructional Strategies Used	Resources Used
<p>Put students in small groups to solve problems such as those found below. Have each group present their answers to the class and justify their answers using models and equations.</p> <p>Example 1: Mrs. Jones teaches in a room that is 60 feet wide and 40 feet long. Mr. Thomas teaches in a room that is half as wide, but has the same length. How do the dimensions and area of Mr. Thomas’ classroom compare to Mrs. Jones’ room? Draw a picture to prove your answer.</p> <p>Example 2: How does the product of 225×60 compare to the product of 225×30? How do you know? Possible strategy/answer: Since 30 is half of 60, the product of 225×60 will be double or twice as large as the product of 225×30.</p>	<p>Numbers and Operations 3-5— Rectangular Multiplication: http://nlvm.usu.edu/en/nav/grade_g_1.html</p>

Assessment Tasks Used	
Skill-Based Task: Without multiplying, which product is larger and why? 12 x 12 or 12 x 48	Problem Task: Joey has a bedroom that is 12 feet by 8 feet long. His sister, Mary, has a bedroom that is 10 feet by 12 feet long. Which bedroom has the greatest area? Justify your reasoning with a model and an equation.

Core Content

Cluster Title: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Standard 5: Interpret multiplication as scaling (resizing), by:

- b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand that multiplying a fraction greater than 1 and a given number results in a product greater than the given number. Examples: $2 \times \frac{5}{4} = \frac{10}{4}$ OR $2 - \frac{1}{2}$ $\frac{10}{4} > 2$ OR $2 - \frac{1}{2} > 2$

Students will understand that multiplying a fraction less than 1 and a given number results in a product smaller than the given number. Examples: $3 \times \frac{1}{4} = \frac{3}{4}$ $\frac{3}{4} < 3$

Students will realize that multiplying a fraction less than 1 times a fraction less than 1 results in a number less than either fraction. Examples: $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ $\frac{1}{4} < \frac{1}{2}$

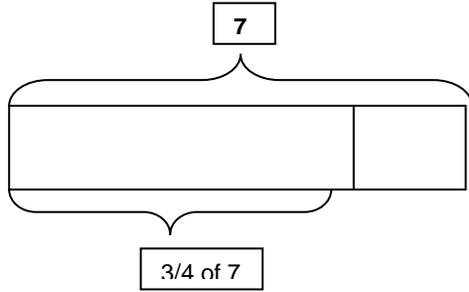
Procedural:

Students will practice multiplying fractions less than 1 by a whole number or fractions greater than 1 by a whole number to show how the products are affected. For example, $\frac{1}{6} \times 9$ is less than $\frac{1}{3} \times 9$ because $\frac{1}{6}$ is less than $\frac{1}{3}$.

Representational:

Students can draw models of multiplication of fractions by given numbers to compare the size of products.

Supports for Teachers

Critical Background Knowledge	
<p>Conceptual: Students will understand relationships between the size of the factors and products. Students will understand that a denominator indicates the number of pieces into which the whole is broken; therefore, as the size of the denominator increases, the size of the pieces decreases.</p> <p>Procedural: Students can multiply whole numbers.</p> <p>Representational: Students can multiply using area models or arrays.</p>	
Academic Vocabulary and Notation	
increase, decrease, product, factors, arrays, fraction greater than 1, improper fraction, fraction less than 1, proper fraction, mixed number	
Instructional Strategies Used	Resources Used
<p>Draw area models to illustrate the size of products in relationship to the size of the factors. Use estimation to check the reasonableness of your answers.</p> <p>Examples:</p> <p>$\frac{3}{4} \times 7$ is less than 7 because 7 is multiplied by a factor less than 1, so the product must be less than 7.</p> <div style="display: flex; align-items: center; justify-content: center;">  </div> <p>$2\frac{2}{3} \times 8$ must be more than 8 because 2 groups of 8 is 16 and $2\frac{2}{3}$ is almost 3 groups of 8. So the answer must be close to, but less than 24.</p>	<p>Numbers and Operations 3-5— Fractions Rectangular Multiplication: http://nlvm.usu.edu/en/nav/frames_asid_194_g_2_t_1.html?from=category_g_2_t_1.html</p>

Assessment Tasks Used	
<p>Skill-Based Task: Without multiplying, which product is larger and why?</p> <p>$12 \times \frac{1}{5}$ or $6 \times \frac{1}{5}$</p>	<p>Problem Task: Mrs. Bennett is planting two flower beds. The first flower bed is 5 feet long and $1\frac{1}{5}$ feet wide. The second flower bed is 5 feet long and $\frac{5}{6}$ feet wide. How do the areas of these two flower beds compare? Is the value of the area larger or smaller than 5 square feet for each flower bed? Draw pictures to prove your answer.</p>

Core Content

Cluster Title: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Standard 6: Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand strategies to interpret word problems involving multiplication of fractions.

Procedural:

Students can make sense of a real-world problem.

Students can write an equation to represent a word problem and solve the equation.

Representational:

Students can use concrete and pictorial area models to represent a real-world problem (e.g., unit bars, number lines, area models, linear models, pattern blocks, fraction circles).

Supports for Teachers

Critical Background Knowledge

Conceptual:

Students will understand multiplication of whole numbers.

Students will understand that 5 times 7 is 5 groups of 7, so 5 times $\frac{1}{2}$ is 5 groups of $\frac{1}{2}$.

Procedural:

Students can multiply whole numbers fluently.

Students can solve a two-step problem.

Students can represent a whole number as a fraction. (For example, $12 = \frac{12}{1}$.)

<p>Representational: Students can use visual fraction models.</p>	
<p>Academic Vocabulary and Notation equation, factor, product, fraction, mixed number</p>	
<p>Instructional Strategies Used</p>	
<p>Resources Used</p>	
<p>Examples:</p> <p>Evan bought 6 roses for his mother. $\frac{2}{3}$ of them were red. How many red roses were there?</p> <p>Using a visual, a student divides the 6 roses into 3 groups by color and counts how many are in 2 of the 3 groups.</p>  <p>A student can use an equation to solve. $\frac{2}{3} \times 6 = \frac{12}{3} = 4$ red roses.</p>	
<p>Assessment Tasks Used</p>	
<p>Skill-Based Task: Solve with visual fraction models and an equation to represent the problem:</p> <p>Tasha finished a job in $\frac{3}{4}$ hour. Megan finished the same job in $\frac{4}{5}$ of the time Tasha took. How long did Megan take to finish the job?</p>	<p>Problem Task: Given the problem $\frac{3}{5} \times 1\frac{1}{2}$, write a real-world problem to represent this expression and solve.</p>

Core Content

Cluster Title: Apply and extend previous understanding of multiplication and division to multiply and divide fractions.

Standard 7: Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

- Interpret division of a unit fraction by a non-zero whole number and compute such quotients. For example, create a story context for $(1/3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(1/3) \div 4 = 1/12$ because $(1/12) \times 4 = 1/3$.
- Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.
- Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share $1/2$ lb. of chocolate equally? How many $1/3$ -cup servings are in 2 cups of raisins?

MASTERY Patterns of Reasoning:

Conceptual:

Students will understand division of a unit fraction by a non-zero whole number.

Students will understand division of a whole number by a unit fraction.

Students will understand division of a unit fraction by a non-zero whole number and also division of whole numbers by unit fractions in real-world problems.

Procedural:

Students can develop an equation to represent the division of a unit fraction by a non-zero whole number or division of a whole number by a unit fraction.

Students can develop strategies to make sense of real-world problems.

Representational:

Students can use visual fraction models to illustrate division of a unit fraction by a non-zero whole number or division of a whole number by a unit fraction. (For example, 4 students are sharing $1/2$ of a cookie. How much of a cookie does each student receive? Use fraction circles to represent the problem and solve $(1/2) \div 4 = 1/8$).

Students can also use number lines, grids, and graph paper to illustrate real-world problems.

Supports for Teachers

Critical Background Knowledge

Conceptual:

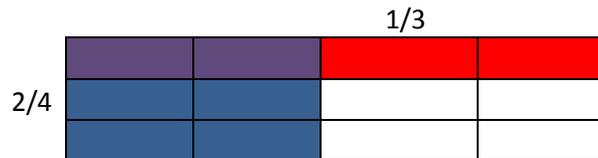
- Students will understand how to multiply whole numbers.
- Students will understand how to multiply a whole number by a fraction.
- Students will understand the meaning and use of inverse operations.
- Students will demonstrate understanding of fractional representations.
- Students will understand the number of unit fractions in a whole.
- Students will understand division involves equal groups/sharing.

Procedural:

- Multiply whole numbers by unit fractions.
- Identify the number of unit fractions within the whole.

Representational:

- Placing fractions on a number line.
- Use a visual model for multiplying fractions.



Academic Vocabulary and Notation

unit fractions, whole numbers, quotient, dividend, divisor, equation, inverse operations

Instructional Strategies Used

Provide real-world problems and then provide manipulatives so that all students can access the content.

$1/3$ of a pan of brownies is on the table. 8 friends want to share the brownies. How much of the total pan does each person get?

Resources Used

<http://www.ixl.com/math/grade-5/divide-fractions-by-whole-numbers>

<p>Bill is going to run an 8-mile race. There are check points every $\frac{1}{3}$ of a mile. How many check points are there in the race?</p> <p>Give students an equation. Have them come up with a real-world problem that represents the equation. Example: $5 \div \frac{1}{6} = ?$</p> <p>Have students create their own real-world problems. Students can then trade problems with a partner and solve each other's problems.</p>	
<p>Assessment Tasks Used</p>	
<p>Skill-Based Task: $\frac{1}{9} \div 3 = n$ $12 \div \frac{1}{4} = n$</p>	<p>Problem Task: $\frac{1}{2}$ of a pie is shared between 3 friends. How much of the original whole pie does each person get?</p> <p>Jacob has a 40-gallon gas tank. If it fills up $\frac{1}{8}$ of a gallon every minute, how long will it take to fill an entire tank?</p>

Core Content

Cluster Title: Write and interpret numerical expressions.
Standard 1: Use parentheses, brackets, or braces in numerical expressions, and solve expressions with these symbols.
MASTERY Patterns of Reasoning:
<p>Conceptual:</p> <ul style="list-style-type: none"> Students will understand the steps in the order of operations. Students will understand the purposes of parentheses, brackets, or braces in numerical expressions. Students will understand the difference between numerical expressions and numerical equations. <p>Procedural:</p> <ul style="list-style-type: none"> Students will build on third grade knowledge of the order of operations, adding the parentheses, brackets, braces to it in fifth grade. Students will solve multi-step problems using parentheses, brackets, or braces. Students will use a variety of examples to model the importance of grouping symbols. For example: $[32 \div 4] + [27 \div 3] = n$. Note: If a student didn't use grouping symbols and didn't understand order of operations, he/she might try to solve the problem going from left to right. Example: $32 \div 4 + 27 \div 3 = n$ $8 + 27 \div 3 = n$ $35 \div 3 = 11 \text{ R. } 2$ (Incorrect Answer) $[8] + [9] = 17$ <p>Representational:</p> <ul style="list-style-type: none"> Students will use physical models, pictures, drawing, diagrams, etc. to represent grouping items using parentheses, brackets, or braces. Note: There is no particular significance for when to use parentheses, brackets or braces. The different grouping symbols are an efficient way to keep track of the different parts of a problem. Round parentheses are the most commonly used, but square brackets and curly braces may also be used.

Supports for Teachers

Critical Background Knowledge	
<p>Conceptual: Students will understand what numerical equations are (e.g., $3 + 4 = 7$). Students will understand the steps of the order of operations without exponents or parentheses.</p> <p>Procedural: Students can solve numerical equations using the order of operations without exponents or parentheses. Students can use numerical equations to solve problems.</p> <p>Representational: Students can use equations with symbols or models to represent the four operations. Students can use physical models, pictures, drawings, etc. to represent numerical equations and their solutions.</p>	
Academic Vocabulary and Notation	
<p>expression, parentheses, bracket, brace, operation, order of operations, $()$, $[\]$, $\{ \}$</p>	
Instructional Strategies Used	Resources Used
<p>Give the students a multi-step problem to solve on their own, and then discuss their answers. Why are their answers different?</p> <p>Use this as a way of establishing a need for rules regarding the order in which operations occur. For example, the expression $5 + 8 \times 2$ could equal 21 or 26 if no regard is paid to order of operation.</p> <p>As a class, use that discussion to make a nonverbal representation (poster, skit, etc.) defining their rules for order of operations, including use of parentheses, brackets, and braces.</p> <p>Have groups create multi-step problems and exchange them with another group. Compare answers</p>	<p>Exploring Krypto: http://www.illuminations.nctm.org</p> <p>http://www.mathgoodies.lessons/vol7/order_operations.html</p> <p>http://www.azed.gov/wp-content/uploads/PDF/MathGr5.pdf</p>

<p>with another group.</p> <p>Bring in two calculators—one scientific calculator that will calculate with the order of operations and an inexpensive calculator that will not calculate with the order of operations. Have the same problem completed on each calculator and compare the results. Discuss the importance of the order of operations and technological applications.</p>	
Assessment Tasks Used	
<p>Skill-Based Task: Solve: $2 \times [5 + (3 \times 2)]$ or $2 [5 + (3 \times 2)]$</p> <p>Solve: $7 + 8 \times 3 = 45$</p> <p>Where do the parentheses have to be placed for this equation to be true?</p> <p>Extension: Write to explain the order of operations. Explain to someone else the order of operations.</p> <p>Compare: $15 - 7 - 2 = \underline{\quad}$ to</p> <p>$15 - (7 - 2) = \underline{\quad}$</p> <p>Compare: $3 \times 100 \div 25 + 7 = \underline{\quad}$ to</p> <p>$[3 \times (100 \div 25)] + 7 = \underline{\quad}$</p>	<p>Problem Task: Monique went to the store to buy groceries for her party. She bought 5 bananas for 50 cents each. She also bought 4 cartons of ice cream for \$3.00 each. At check-out, she was given 10 cents off the bananas. Write an expression that represents the problem. You may use models if you choose to do so. Then solve the problem to determine how much Monique spent in all. Explain your reasoning.</p> <p>$[(5 \times .50) - .10] + (4 \times 3.00) = \\14.40</p>

Core Content

Cluster Title: Write and interpret numerical expressions.
Standard 2: Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.
MASTERY Patterns of Reasoning:
<p>Conceptual: Students will understand that the word “then” implies one operation happens after another and parentheses are used to indicate the order of operations. Example: “Add 8 and 7, then multiply by 2” can be written as $(8 + 7) \times 2$. Students will understand how to write a real-world problem as an expression. Students will recognize that $3 \times (18,932 + 921)$ is three times as large as $18,932 + 921$, without having to solve. Students will recognize that $3(18,932 + 921)$ means the same thing as $3 \times (18,932 + 921)$.</p> <p>Procedural: Students can write expressions using the correct numerical and symbolic notation in the proper order.</p> <p>Representational: Students can use numerical and symbolic notation to represent an expression from a problem.</p>

Supports for Teachers

Critical Background Knowledge
<p>Conceptual: Students will understand the rules of the order of operations, especially the use of parentheses.</p> <p>Procedural: Students can record single-step expressions not involving parentheses or brackets.</p> <p>Representational: Students can represent simple operations using the proper notation (+, -, \times, \div).</p>
Academic Vocabulary and Notation
expression, parentheses, bracket, brace, order of operations, terminology for operations (e.g. sum, add, multiply, difference), “then”

Instructional Strategies Used	Resources Used
<p>Introduce this concept by providing students with a variety of real-world situations in which multi-step calculations occur.</p> <p>Ask them to use their understanding of operations and notations to express the situation mathematically. For example, “Jan purchased 8 pencils for school. She later decided she needed 7 more. She has made the same purchase at the beginning of school for the last 6 years. Show how many pencils she has purchased with a numeric expression.”</p> <p style="text-align: center;">Expression: $(8+7) \times 6$</p> <p>Give the students an expression and see if they can come up with a real-world situation that would match.</p> <p>Use a math picture book to present different problems. Stop at various points to have students write an expression that matches the situation in the book.</p>	<p>http://illuminations.nctm.org/LessonDetail.aspx?id=L803</p> <p><i>Alexander, Who Used to Be Rich Last Sunday</i> by Judith Viorst</p> <p><i>The Grapes of Math</i> by Greg Tang</p>

Assessment Tasks Used	
<p>Skill-Based Task: Students write an expression for calculations given in words such as the following:</p> <p>“Divide 144 by 12, and then subtract 7.”</p> <p>They write $(144 \div 12) - 7$.</p>	<p>Problem Task: Mara bought 6 bags of Skittles at \$0.85 each and 9 packs of gum at \$1.20 each for a sleepover. Express what she purchased mathematically.</p> <p>Answer: $(6 \times 0.85) + (9 \times 1.20)$</p> <p>Extension: Mara bought 6 bags of Skittles at \$0.85 each and 9 packs of gum at \$1.20 each for a sleepover. She still didn't think she would have enough snacks. She went to the store the next day and bought the same number of items again. Express what she purchased mathematically.</p> <p>Answer: $2 \times [(6 \times 0.85) + (9 \times 1.20)]$</p> <p>or $2 [(6 \times 0.85) + (9 \times 1.20)]$</p> <p>Explain your reasoning. How do you know your expression fits the problem?</p>

Core Content

Cluster Title: Analyze patterns and relationships.

Standard 3: Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.

MASTERY Patterns of Reasoning:**Conceptual:**

Students will understand numerical patterns.

Students will draw conclusions based on identified patterns.

Students will understand how patterns can be represented on a coordinate plane.

Procedural:

Students can generate numerical patterns.

Students can identify relationships between corresponding terms from two numerical patterns.

Students can form ordered pairs from corresponding terms.

Students can graph ordered pairs on a coordinate plane.

Students can discuss the relationship of the corresponding terms.

Representational:

Students can represent numerical patterns and identify corresponding terms.

Students can use physical models of a coordinate plane and place objects at locations specified by ordered pairs of corresponding terms.

Students can use models of a coordinate plane to extend the above pattern.

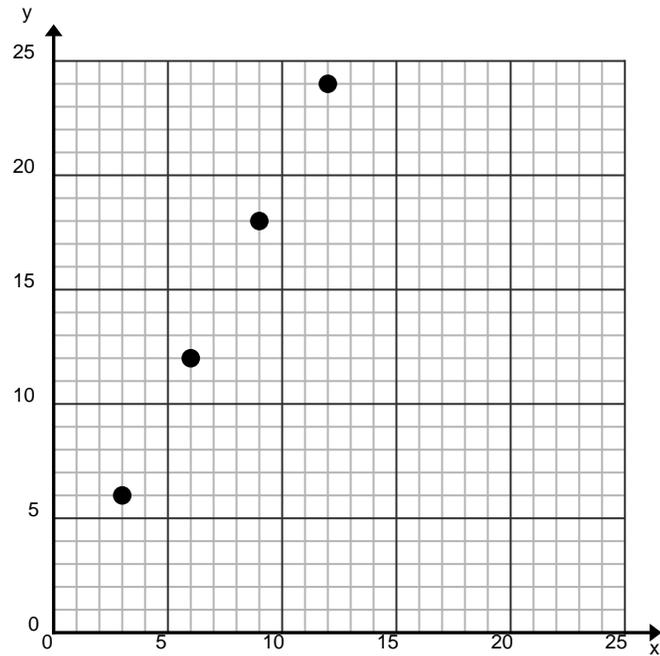
Note: At the fifth grade level, only Quadrant I of the coordinate plane is used.

Supports for Teachers

Critical Background Knowledge	
<p>Conceptual: Students can identify single numerical patterns (e.g., “Add 3,” “Multiply by 4”). Students will understand how a coordinate plane is formed (e.g., origin, x-axis, y-axis). Students can graph points in Quadrant I.</p> <p>Procedural: Students can plot points on a coordinate plane. Students can perform basic operations.</p> <p>Representational: Students can record numerical patterns.</p>	
Academic Vocabulary and Notation	
<p>corresponding terms, coordinate plane, ordered pair, coordinates, pattern, relationship, graph, origin, x-axis, y-axis</p>	
Instructional Strategies Used	Resources Used
<p>Have students generate two numerical patterns. Example: “Add 2” and “Add 4.”</p> <p>Add 2: 2, 4, 6, 8, 10, 12, 14... Add 4: 4, 8, 12, 16, 20, 24, 28...</p> <p>Instruct the students to:</p> <ol style="list-style-type: none"> Complete the patterns. Find corresponding terms. Form ordered pairs from corresponding terms. Graph ordered pairs on a coordinate grid. <p>Use word problems such as the following to teach this concept.</p>	<p>Patterns and Grids Resources: http://www.mathwire.com http://www.oswego.org/ocsd-web/games/BillyBug/bugcoord.html http://www.counton.org/games/virtualmathfest/dinosaur.html</p> <p><i>One Grain of Rice</i> by Demi (picture book)</p>

John and Maren are flying model airplanes. Both planes take off at the same time. John's plane climbs 3 feet every second. Maren's plane climbs 6 feet every second.

John	3	6	9	12
Maren	6	12	18	24



Assessment Tasks Used																											
<p>Skill-Based Task: Complete the table below. Identify the pattern for each row. Form ordered pairs from corresponding terms. Graph ordered pairs on a coordinate grid.</p> <table border="1"> <tr> <td>8</td> <td>16</td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>2</td> <td>4</td> <td></td> <td></td> <td></td> <td></td> </tr> </table>			8	16					2	4					<p>Problem Tasks: Russell bought 3 movie tickets for a total of \$21. Catherine bought 5 movie tickets for a total of \$35. Create a table to show the pattern of the prices of movie tickets. How much is 1 ticket, 2 tickets, and 4 tickets? Graph the corresponding terms as ordered pairs on a coordinate plane. What pattern do you see? Explain why.</p> <p>Answer:</p> <table border="1"> <tr> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>7</td> <td>14</td> <td>21</td> <td>28</td> <td>35</td> </tr> </table>			1	2	3	4	5	7	14	21	28	35
8	16																										
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