Core Content

Cluster Title: Apply and extend previous understandings of arithmetic to algebraic expressions.
Standard 1: Write and evaluate numerical expressions involving whole-number exponents.
MASTERY Patterns of Reasoning
Conceptual: Understand the meaning of exponents. Understand exponential notation (e.g., $3\text{^}4 = 3^4$ and $3^4 = 3 \times 3 \times 3 \times 3$). Understand that exponential notation represents multiplication where the base always remains the same. **Procedural:** Find the value of an expression using exponential notation (e.g., $3^4 = 81$). Write exponential notation (e.g., $3\text{^}4 = 3^4$ and $3^4 = 3 \times 3 \times 3 \times 3$). **Representational:** Use multiple representations that illustrate geometric dimensions and collections of quantities such as: Represent the power of 1 as a linear model. This results in a one dimensional representation. Represent the power of 2 as area with square units. This results in a two dimensional representation. Represent the power of 3 as volume with cubic units. This results in a three dimensional representation. Represent the accumulation of quantities. Show that an exponent is notation representing repeated multiplication.

Supports for Teachers

Critical Background Knowledge
Conceptual: Repeated multiplication of similar "base" quantities (e.g., $2 \times 2 = 4$, $2 \times 4 = 8$, $2 \times 8 = 16$) **Procedural:** Skip counting Multiplication facts

Representational:
Model repeated multiplication with manipulatives, arrays, and number lines.

Academic Vocabulary and Notation

^, base number, expressions, exponents, power, superscripted numbers

Instructional Strategies Used	Resources Used
1. Have the students use manipulatives to show the number 3 after it has been doubled, which would represent 3^2=9 square units. Then have them show 3^3 = 27 cubic units with the manipulatives. 2. Play memory games with matching numerical expressions. Example: 2^3 matches with 2^3, which matches with 2 x 2 x 2, which matches with 8. 3. "I Have, Who Has?" game. Example: The first student says: "I have 2^3." The second student says: "I have 8. Who has 4^2?" The third student says: "I have 16. Who has the expression equal to 27?" Etc… Extension: Every positive integer can be expressed as the sum of four or fewer squared numbers (Note: You can use the same integer more than once.) Examples: $5 = 1^2 + 2^2$ $101 = 4^2 + 6^2 + 7^2$ $170 = 2^2 + 6^2 + 7^2 + 9^2$ Write 83 as the sum of three square numbers.	*The King's Chessboard,* by David Burch *One Grain of Rice,* by Demi http://jwilson.coe.uga.edu/EMT668/EMAT6680.F99/Martin/instructional%20unit/day4.exponential/excel/grainofrice.html *Minnie's Diner*, by Dale Ann Dodds http://www.wimp.com/makingnoodles

Assessment Tasks Used	
Skill-based Task: Evaluate: 3^2 Write five cubed times four as a numerical expression.	**Problem Task:** Certain biological cells quadruple each hour. Start with one cell at 2:00 and find out how many cells there will be by 5:00. Create a diagram to represent the cell growth. Include an equation using exponential notation.

Core Content

Cluster Title: Apply and extend previous understandings of arithmetic to algebraic expressions.

Standard 2: Write, read, and evaluate expressions in which letters stand for numbers.

a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as 5 – y.

Concepts and Skills to Master

Conceptual:

Understand that letters, called variables, represent unknown numbers.
Know that the same rules apply in operations with numbers also apply in operations with variables.

Procedural:

Translate an expression from its word form and vice versa.

Representational:

Represent variables with letters (e.g., a, x,… except e and i).

Supports for Teachers

Critical Background Knowledge

Conceptual:

Recognize that expressions use one or more mathematical symbols to represent a number or quantity.
Know that expressions do not include equal, greater than, or less than signs.

Procedural:

Perform the four operations of addition, subtraction, multiplication, and division.

Representational:

Represent a problem such as "seven plus a number is twelve" with symbolic notation, such as $7 + \Delta = 12$.
Represent the four operations with manipulatives, diagrams, and number lines.

Academic Vocabulary and Notation	
Equation, expression, notation for multiplication (e.g., 3*x,* 3(4)), variable	
Instructional Strategies Used	**Resources Used**
1. Within your classroom, have the students find situations where they can role play to compare known and unknown quantities (e.g., Student A (Dory) and Student B (Colleen); Dory says, "I have two sisters." Colleen says, "I have Dory – 1 sisters." Dory says, "You have *d* – 1 sister. You have one sister.") Use all operations. 2. Give each pair of students an expression such as *x* + 957. Challenge them to find a way to evaluate each expression for *x* = 35, 825, and 373. Then have the students write a real-life context for each expression.	http://www.mathgoodies.com/lessons/vol7/equations.html
Assessment Tasks Used	
Skill-based Task: Write each word phrase as an algebraic expression. 6 less than 3*t* the product of *w* and 8 *r* divided by 15	**Problem Task:** Hannah is 3 years younger than Katie. Joey is twice as old as Hannah. Let *k* stand for Katie's age. Write an expression to represent Hannah's age. Using *k*, write an expression for Joey's age.

Core Content

Cluster Title: Apply and extend previous understandings of arithmetic to algebraic expressions.

Standard 2: Write, read, and evaluate expressions in which letters stand for numbers.

b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*

Concepts and Skills to Master

Conceptual:

Understand that terms are values in an expression separated by addition and subtraction (e.g., $x + 3$ contains two terms; $2x - 5$ contains two terms).

Understand that multiplication and division in an expression represent a single term (e.g., $3y$ is a term, $\frac{1}{2}x$ is a term).

Know that a coefficient is a number that multiplies a variable (e.g., 3 is the coefficient of $3y$).

Procedural:

Read an expression correctly using appropriate mathematical terms (e.g., 2 (8 + 7) is read as the product of 2 times the quantity or sum of 8 + 7).

Identify parts of an expression (e.g., in the expression $2x - 5$, $2x$ is the first term where 2 is the coefficient of the variable x and 5 is the second term, a constant).

Representational:

Represent the parts of an expression with manipulatives, diagrams, or coordinate plane.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Recognize that variables represent unknown quantities.

Know the meaning of the terms sum, product, factor, quantity, and quotient.

Procedural:

Read expressions of single operations (e.g., 3*y*, *x* + 7, 4 + 3).

Representational:

Represent the four operations (addition, subtraction, multiplication and division) with manipulatives, diagrams, number lines, and story context.

Academic Vocabulary and Notation

Coefficient, Dividend, Divisor, Equation, Expression, Factor, Multiplier, Product, Quotient, Sum, Term (optional term – Constant), Parentheses

Instructional Strategies Used	Resources Used
1. Give an example of an expression, such as 4*x* + 7 + *x*, and identify the parts. The terms are 4*x*, 7 and *x*. The coefficients are 4 and 1 (since there is one x). The values 4 and *x* are both factors of the product 4*x*. Give additional examples to students to identify parts of expressions in small groups. 2. Give the students word problems such as the following and have them find the solutions: The sum of twice a number plus 13 is 75. Write the equation and find the number. 3. Use the algebra tiles to represent terms in an expression (e.g., 2x requires two x pieces) <u>Game 1:</u> Give students two sets of cards. The first set contains equations and the second set contains the academic vocabulary. Students turn over an equation card. Then students match academic vocabulary to the parts of the equation. <u>Game 2:</u> Start with a small expression, such as 2*x* - 4. Break the expression into distinct parts. Write on separate cards 2, *x*, -, and 4.	http://nlvm.usu.edu/en/nav/category_g_3_t_2.html Algebra Tiles

Code: 6.EE

Assign a child a card to hold and line the children in the front of the classroom in the order of the expression. To identify parts of the expression, have students sit down when you say the vocabulary word that corresponds with their card. Example: Teacher says “coefficient.” Student holding the 2 sits down. Teacher says “term.” Students holding 2 and *x* may link arms and sit, as well as student holding 4.	

Assessment Tasks Used	
Skill-based Task: Provide an expression (e.g., 6*x* + 4) for students to identify the parts. Write an expression that has two terms and a product. Write an expression that has a coefficient and a sum.	**Problem Task:** This is a skill-based standard. Therefore, there is no problem task.

Core Content

Cluster Title: Apply and extend previous understandings of arithmetic to algebraic expressions.

Standard 2: Write, read, and evaluate expressions in which letters stand for numbers.

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length s = 1/2.*

MASTERY Patterns of Reasoning

Conceptual:

- Know that when using an expression to solve a problem, numbers replace variables in the expression and answers will vary depending on which numbers are substituted for variables.
- Extend understanding that we use standard formulas because of their potential to efficiently express relationships to include fractional units
- Extend understanding of standard formulas through composition or decomposition to include fractional units.

Procedural:

Find and use correct units of measure in solutions to real world problems.
Substitute values for variables in expressions (e.g., If I buy 2 bags of lemon drops, the expression can be written 2*x*. If the cost of the bag is $1.59 I can substitute that value in for *x*. If the next week the price changes to $1.69 I can still use the same expression, assigning the new value to the variable).
Use order of operations including, exponents.

Representational:

Represent expressions, including standard formulas, with manipulatives, drawings, and diagrams.

Supports for Teachers

Critical Background Knowledge	
Conceptual: Recognize that variables represent unknown quantities. Standards: 6.ee.2a and 6.ee.2b • Understand that we use standard formulas because of their potential to efficiently express relationships with whole numbers. • Make sense of standard formulas through composition or decomposition using whole numbers. **Procedural:** Use order of operations. **Representational:** Create and use area models. Represent volume graphically and with manipulatives.	
Academic Vocabulary and Notation	
^, expression, equations, formula, order of operations, superscripted numbers, variable	
Instructional Strategies Used	**Resources Used**
1. Write an expression such as *x* + 6 on the board. Leave space between the symbols and numbers. Give each student an index card with a different number from 0 – 20. Have a volunteer use his or her card to cover the *x*. Have another volunteer give the value for x and evaluate the expression. Summarize by saying, for example, When *x* = 4, then *x* + 6 = 10. Repeat with other students' cards and different expressions involving any of the four operations. Vary the degree of difficulty. 2. In small groups, give students the expression 3 + 4 x 5. This represents the number of people coming to a party. There are three individuals and four families each with five members. List the steps you would use to solve this problem and justify each step. Next, have students write this expression for a variable number of	http://www.cimt.plymouth.ac.uk/projects/mepres/book7/y7s22act.pdf http://www.shodor.org/interactivate/activities/SurfaceAreaAndVolume/ http://www.math.niu.edu/~rusin/uses-math/games/krypto/

families. Allow students to reason which number becomes the variable.

Assessment Tasks Used

Skill-based Task:
Complete the table by evaluating the algebraic expression.

x	0	3	7	12
$5x + 8$				

Problem Task:
You know that you can find the area of a triangle using the formula $A = \frac{1}{2} bh$. If a triangle has an area of 48 cm^2, what can its base and height be? Draw diagrams to justify your thinking.

Core Content

Cluster Title: Apply and extend previous understandings of arithmetic to algebraic expressions.

Standard 3: Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

MASTERY Patterns of Reasoning

Conceptual:

Know that the distributive property has limitations in division ($\frac{(3+5)}{2} = \frac{3}{2} + \frac{5}{2}$ but $\frac{14}{(2+5)} \neq \frac{14}{2} + \frac{14}{5}$)

Understand that the properties used with numbers also apply to expressions with variables.

Procedural:

Apply the properties of operations with expressions involving variables to generate equivalent expressions.

Representational:

Use manipulatives or diagrams to represent the distributive property.
Show properties of operations to be equivalent with manipulatives, diagrams or story contexts.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Variables are letters that stand for numbers.
Understanding properties of operations and applying each of them in numeric representations (e.g., 3 + 2 = 2 + 3).
Multiplication is repeated addition.
Understand the associative property of addition and multiplication, the commutative property of addition and multiplication, the identity property of addition and multiplication and the distributive property of multiplication over addition or subtraction.
Recognize how to use common calculations (such as fact families, basic math facts, number bonds, composing and decomposing numbers) to generate solutions to problems.

Procedural:

Find common factors of two whole numbers.

Representational:

Model operations of addition, subtraction, multiplication and division.

Academic Vocabulary and Notation

$\frac{x}{x}$, $x \cdot \frac{1}{x}$, $x(a+b)$, associative property, commutative property, distributive property, equivalent, identity property, identity element, variable

Instructional Strategies Used	**Resources Used**
Students must be able to make connections between what they already know about the properties of operations when used with numbers and how those properties apply when used with algebraic notation (using variables). Therefore, teachers should first present the topic of equivalent expressions using numbers and then shift to variables. Have students open a package of M&Ms and use the distributive property to write expressions showing how many of each color would be in 5 bags, 10 bags etc. (e.g. There are 10 red, 15 green, and 12 yellow in one bag. That could be represented by the expression 10*r* + 15*g* + 12*y*. To represent the number in 5 bags you would obtain the expression 5(10*r* + 15*g* + 12*y*). Using the distributive property, the expression is 50*r* + 75*g* + 60*y*. So there are 50 red, 75 green, and 60 yellow M&Ms in 5 bags.)	http://www.onlinemathlearning.com/distributive-property-1.html http://www.math.niu.edu/~rusin/uses-math/games/krypto/

Assessment Tasks Used

Skill-based Task:	**Problem Task:**
Generate an equivalent expression for each of the following: 4 (*x* - 2) 15*x* - 24*y* *x* + *x* + *y* + *y* 5*x* + 2*y* 5*r* + (2*s* + 2*t*)	In one packet of nuts, there are two different types of nuts. There are 5 peanuts (*p*) and 7 cashews (*c*) in each container. I have 6 packets of nuts; write two expressions that show how many nuts I have all together. Possible answers: 6 (5 + 7) or (6 x 5) + (6 x 7)

Code: 6.EE

	Sarah says that the two expressions $3(2 + x)$ and $6 + x$ are equivalent. Is she right? If not, explain. Correct response: No, that is not correct. $6 + 3x$ is the correct response since the 3 must be distributed through both terms in the parentheses.

Core Content

Cluster Title: Apply and extend previous understandings of arithmetic to algebraic expressions.
Standard 4: Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions* y + y + y *and 3*y *are equivalent because they name the same number regardless of which number* y *stands for.*
MASTERY Patterns of Reasoning
Conceptual: Recognize equivalent expressions. **Procedural:** Substitute values into expressions to prove equivalency. **Representational:** Model equivalent expressions with manipulatives, diagrams or story contexts.

Supports for Teachers

Critical Background Knowledge
Conceptual: Variables are letters that stand for numbers. **Procedural:** Substitute values into expressions to solve an equation, **Representational:** Model algebraic expressions with manipulatives, diagrams or story contexts.
Academic Vocabulary and Notation
=, equation, equivalent, expression, variable

Instructional Strategies Used	Resources Used
Teach this standard concurrently with 6.EE.3 1. Use substitution to verify that both expressions are equivalent (e.g., $3(2 + x) = 6 + x$) is not true. Substitute any number for x; the expressions have different values, therefore they are not equivalent. (Make sure students try more than one number to make sure it works, because 0 will make this statement true.) 2. Present multiple expressions and ask students to determine if they are equivalent using substitution.	http://www.mathwire.com/numbersense/numbsense.html

Assessment Tasks Used	
Skill-based Task: Are the following expressions equivalent? $x + x + 1 + 1 = 2x + 2$ $5(x + 3) = 5x + 5$	**Problem Task:** Have students generate equivalent expressions for the number 48, using at least two operations and verifying that their notation is correct.

Core Content

Cluster Title: Reason about and solve one-variable equations and inequalities.

Standard 5: Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

MASTERY Patterns of Reasoning

Conceptual:

Understand the differences between equations and inequalities.
Know that inequalities represent a range of possible values rather than a single solution.

Procedural:

Simplify numerical expressions by substituting values for given variables.
Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Representational:

Model solutions for equations and inequalities with manipulatives, graphs, diagrams or story contexts.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Know variables can be replaced with numbers.
Understand that for an equation to be true, expressions on either side of the equal sign must be equivalent.

Procedural:

Substitute values for given variables.

Representational:

Model equations and inequalities that do not involve operations (e.g., $3 < 5$, $x > 7$, $y = 6$).

Academic Vocabulary and Notation	
>, <, equality, inequality, solution, substitution	
Instructional Strategies Used	**Resources Used**
Using a pan balance, put a bag with an unknown number of blocks and up to 10 visible blocks on one pan. Have them balance the scale by adding as many blocks as needed on the other pan. What equation is shown? Then have the students remove a block at a time from each pan to determine how many blocks are in the bag. What number makes the equation true? How do you know? Repeat the process using another bag.	Illuminations (NCTM website): "Everything Balances Out In the End" (Lesson 2: Balancing Algebraic Understanding). Algebra Tiles UEN: Evaluating Expressions Using Tiles http://nlvm.usu.edu/en/nav/frames_asid_201_g_3_t_2.html?open=instructions&from=category_g_3_t_2.html
Assessment Tasks Used	
Skill-based Task: For the inequality $4x > 16$, which of the following numbers (2, 4, 6, 8) makes it true? Explain your answer. Will the solution to $x - 5 = 34$ be greater than or less than 34? Explain.	**Problem Task:** Keith has $500 in a savings account at the beginning of the summer. He wants to have at least $200 in the account by the end of the summer. He withdraws $25 each week for food and fun. • Write an inequality that represents Keith's situation. • How many weeks can Keith withdraw money from his account?

Core Content

Cluster Title: Reason about and solve one-variable equations and inequalities.

Standard 6: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number or, depending on the purpose at hand, any number in a specified set.

MASTERY Patterns of Reasoning

Conceptual:

Recognize real-world mathematical problems can be expressed using a variable to represent an unknown.

Procedural:

Write and solve an expression that represents a real-world problem using variables.

Representational:

Use variables to represent numbers or sets of numbers when solving a real-world or mathematical problem.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Understand that a variable represents a number or a specified set of numbers.

Procedural:

Show arithmetic operations work the same way on variables as they do on numbers.
Solve real world mathematical problems using equations or expressions with numbers.

Representational:

Model authentic problems with manipulatives or diagrams.

Academic Vocabulary and Notation

$>$, $<$, $\geq$, $\leq$, constant, coefficient, solution

Instructional Strategies Used	Resources Used
1. Have students write an expression for a real-world mathematical problem in which all parts of the expression have a numerical value. Then give a similar real-world mathematical problem in which one of the parts of the expression is an unknown, resulting in one part of the expression being a variable. 2. Have students write a real-world mathematical problem and an expression that represents that problem. Then have students switch word problems only and find the expression that represents their partner's real-world mathematical problem. 3. Expose students to a variety of real-world mathematical problems. Have students work in small groups, with a partner, and eventually by themselves to write expressions to represent those situations.	Create PDF for real-world examples.

Assessment Tasks Used	
Skill-based Task: This is a problem-based standard, so there is no skill-based task.	**Problem Task:** An appliance repairman charges $50 for coming to a home for a service call and $40 an hour for the service. Write an expression to represent her earnings for h hours. Sally delivered 7 newspapers and John delivered *x* number of newspapers. Write an expression showing how many total newspapers were delivered. Write an expression to represent how many John delivered if Sally delivered seven more newspapers than John.

Core Content

Cluster Title: Reason about and solve one-variable equations and inequalities.

Standard 7: Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

MASTERY Patterns of Reasoning

This standard deals with equations where x is the independent variable, p is the constant, and q is the dependent variable.

Conceptual:

Recognize that both sides of an equation are equal, and whatever operation is performed on one side of the equation must be done on the other side to maintain the equality.

Procedural:

Solve one-step equations using all four operations with non-negative rational numbers (i.e., whole numbers, fractions, decimals).

Write and solve equations that represent real-world mathematical problems that involve non-negative rational numbers.

Representational:

Model solutions for equations of the form $x + p = q$ and $px = q$ with manipulatives, diagrams or story contexts

Supports for Teachers

Critical Background Knowledge

Conceptual:

Recognize that variables represent numbers.

Understand that variables can be operated upon in the same way as numbers.

Procedural:

Write an expression from a real-world mathematical problem.

Use substitution to determine if both sides of the equation are equal.

Representational:
Model addition, subtraction, multiplication, and division with manipulatives, diagrams or story contexts.

Academic Vocabulary and Notation

Balance, equation, equations of the form $x + p = q$ and $px = q$

Instructional Strategies Used	**Resources Used**
1. Relate the idea of equations to a balance scale. Using objects, have students balance an actual scale and relate this idea to a balanced equation. Start off with simple true/false equations balanced on a scale. Ask students if the scale "tilts" or is "balanced" (e.g. 8 = 10 - 3, 6 – 3 = 10 - 7, etc.). 2. Extend the idea of the balance scale to incorporate the idea of performing arithmetic operations on both sides of the equation to isolate the variable (i.e., $x + 5 = 8$, remove 5 from both sides of the balance, which keeps the equation balanced, so x = 3).	UEN: Algebra applies to the real world? No way! UEN: Balance or tilt NLVM: Algebra balance scales—Algebra Grades 6-8 Human Coordinate Plane: http://fcit.usf.edu/math/lessons/activities/HumanPT.htm

Assessment Tasks Used

Skill-based Task:

$4 + x = 9$	$3x = 12$
$x + 5 = 10$	$1/2x = 4/5$
$5.2 + x = 7.8$	$1/5x = 7$

Problem Task:

There were some grapes on the table. Logan ate 1/6 of them. He ate 5 grapes. Write an equation to represent the situation and solve.

Angela bought 5 shirts that each cost the same amount. She spent $34.65. How much did she spend on each shirt? (Write and solve an equation to solve the problem.)

Ronnie earned $.50, giving her a total of $3.17. Write an equation that allows you to find her beginning amount.

Core Content

Cluster Title: Reason about and solve one-variable equations and inequalities.

Standard 8: Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

MASTERY Patterns of Reasoning

Conceptual:

Recognize that infinity refers to a set of numbers that has no end, but may not include all numbers.
Recognize that a variable can stand for an infinite number of solutions when used in inequalities.
Recognize that a constraint or a condition in an inequality refers to the boundary defined in the solution set.

Procedural:

Write an inequality that represents real-world mathematical problems containing a constraint or a condition (<, >).

Representational:

Represent inequalities on a number line. Add graphic to clarify.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Understand the meanings of equality and inequality.
Recognize that a variable can stand for a number.

Procedural:

Write an inequality of the form $x > c$ or $c > x$ where x and c are rational numbers.

Representational:

Represent numbers on a number line.

Academic Vocabulary and Notation	
>, <, inequality, infinite, greater than, less than	
Instructional Strategies Used	**Resources Used**
1. Ask a question for which there are an infinite number of solutions (e.g., What are all the numbers greater than 1?). Guide students to represent that as n > 1. 2. Present real-world mathematical situations where it is apparent that multiple answers will make an inequality true (e.g., freezing occurs at 32ºF. How cold could your freezer be if you have ice cubes? 3. Represent real-world mathematical problems on a number line with students. Place an open circle on the number that is in the inequality on the number line, then draw an arrow to indicate the direction of all possible solutions.	http://fcit.usf.edu/math/lessons/activities/HumanPT.htm – adjust for inequalities http://www.education.com/activity/article/tic-tac-equations/
Extension: Tic-Tac-Toe Math	

Assessment Tasks Used	
Skill-based Task: Represent the solution to each inequality on a number line. $n > 0$ $n < 5$ $n > 3/4$ $n < -1.5$	**Problem Task:** Water boils at 100ºC. Write an inequality that represents all the temperatures at which water does not boil. Represent the solution on a number line.

Core Content

Cluster Title: Represent and analyze quantitative relationships between dependent and independent variables.

Standard 9: Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation* d = 65t *to represent the relationship between distance and time.*

MASTERY Patterns of Reasoning

Conceptual:
Recognize that a change in the independent variable creates a change in the dependent variable (i.e., as *x* changes *y* also changes).
Identify relationships between tables, graphs, and equations.
Understand that tables are useful for organizing and displaying data.
Recognize when quantitative relationships between dependent and independent variables are linear.

Procedural:
Make a table, graph, or equation to represent a problem context.
Organize and display data using tables and graphs.

Representational:
Create and analyze visual representations such as tables and graphs to justify an equation.

Supports for Teachers

Critical Background Knowledge

Conceptual:
Recognize that variables can be replaced with numbers.
For an equation to be true, expressions on either side of the equal sign must be equivalent.
Understand that graphs are useful for organizing and displaying data.
Understand that coordinates in a coordinate plane represent data from a real world context.

Procedural:
Create a graph using order pairs from a set of data (e.g., from a table).

Representational:
Represent real-world contexts by graphing points in the first quadrant of the coordinate plane.
Model values with charts, graphs, manipulatives and story contexts.

Academic Vocabulary

Graph, table, equation, variable, independent variable, dependent variable, equivalent

Instructional Strategies Used	Resources Used
Use a real-world context where students must make sense of the relationship between the dependent and independent variables, such as: The class has a jar that has 5 Skittles in it to begin with. Each student is given 7 Skittles. Begin with letting students add their 7 Skittles individually. Ask how many Skittles would be in the jar after *x* number of students. Have the students make a table where the beginning point of the table at student 0 is 5, matching what is happening in the jar. Ask the students to predict how many Skittles would be there after 10, 15, or 20 students. Have the students verbalize a rule for any number of students using the given situation and the table. Have students write an equation to represent the situation. Create a line graph showing the relationship to number of Skittles in the jar and number of students. Make sure to connect coordinate points on the graph to quantities on the table, to quantities in the equation. Have the students identify the dependent and independent variable in the table, the graph, and the equation.	
Extension: Wacky Water World	http://fcit.usf.edu/math/lessons/activities/wackyT.htm

Extension: Walk the Plank	http://fcit.usf.edu/math/lessons/activities/plankT.htm

Assessment Tasks Used	
Skill-based Task: Use this list of ordered pairs to create a table, a graph, and an equation: 0, 3 1, 5 2, 7 3, 9 4, 11	**Problem Task:** Imagine that you are training for a 13-mile race. On the first day you run 1.5 miles. Each day you run 0.5 mile longer than you ran on the previous day. How many days will it take you to work up to 13 miles? Create a table, graph, and equation and explain the relationship between the dependent and independent variables. If a jar had 4 pennies inside, and you added 7 pennies each day, how many pennies will there be after day one? Day two? Day three? Day ten? Day one hundred? Have students create a table and graph the results. Also have them identify the equation for this situation ($p = 7d + 4$).

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Core Content

Cluster Title: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Standard 1: Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lbs. of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi. and area 1/2 square mi.?

MASTERY Patterns of Reasoning:

Conceptual:

Understand how to set up a problem based on the context of the problem.
Be able to interpret what the quotient represents.
Recognize that what is known or not known is based on the type of division needed (partitive—Total / # of groups = size of groups—or quotative or measurement—Total / size of group = # of groups) model.
Create a story context using division of fractions.
Understand that multiplication and division are inverse operations regardless of the class of numbers.

Procedural:

Compute the division of fractions.
Solve a story context using division of fractions.

Representational:

Model division of fractions with manipulatives, diagrams (e.g., bar model, number line) and story contexts.
Write equations representing authentic problems involving fractions.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Know that multiplication and division are inverse operations.
Know that division is either fair sharing (partitive) or repeated subtraction (quotative).

Procedural:

Convert between improper fractions and mixed numbers.
Division by whole numbers.
Division of a whole number by a fraction.

Representational:

Model division with manipulatives, diagrams and story contexts.

Academic Vocabulary and Notation

quotient, reciprocal, inverse operation

Instructional Strategies Used	Resources Used
Use this problem: How many servings of popcorn are in 4½ cups if each person receives 3/4 cup of popcorn The teacher provides 4½ cups of popcorn. Students use a 3/4 cup measuring cup to solve the problem. Record solutions as a group. 1. Think-Pair-Draw-Share: Put students in pairs. Have one solve the problem using a picture/diagram and the other solve using the algorithm. Then they get together and compare. 2. Think-Pair-Share: Students solve the problem on their own, then get together and discuss how their solutions are the same and how they are different. 3. Four Corners: Give students a problem and the quotient. Give each corner in your room a label and have students go to the corner they think would be the correct label for the quotient.	Fraction Bars from NLVM: http://nlvm.usu.edu/en/nav/frames_asid_265_g_2_t_1.html?open=activities&from=category_g_2_t_1.html

Assessment Tasks Used	
Skill-based Task: Use representations to show that 1/4 divided by 1/2 is 1/2, that 2/3 divided by 2/5 is 5/3, that 2/3 divided by 3/4 is 8/9, and that 1½ divided by 6/4 is 1.	**Problem Task:** You have 5/8 pound of Skittles. You want to give your friends 1/4 lb. each. How many friends can you give Skittles to? Explain your answer. You have a 3/4-acre lot. You want to divide it into 3/8-acre lots. How many lots will you have? Draw a diagram to justify your solution. You have a 3/4-acre lot. You want to divide it into 2 sections. How many acres in each section will you have? Draw a diagram to justify your solution. How wide is a rectangular strip of land with length 3/4 mi. and area 1/2 square mi.?

Core Content

Cluster Title: Compute fluently with multi-digit numbers and find common factors and multiples.

Standard 2: Fluently divide multi-digit numbers using the standard algorithm.

MASTERY Patterns of Reasoning:

Conceptual:

Identify when it is appropriate to use the standard algorithm.

Procedural:

Use the standard algorithm to compute multi-digit division problems with procedural fluency.

Note: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently and appropriately (*Adding It Up*, National Research Council).

Representational:

Divide multi-digit numbers using the standard algorithm.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Understand the meaning of division.
Understand place value of multi-digit numbers.
Know that division is the inverse of multiplication.
Illustrate and explain the relationship between calculations and models for multiplying and dividing multi-digit numbers.

Procedural:

Divide with single-digit numbers.
Use compatible numbers to make an estimation to determine reasonableness of answers.
Use the standard algorithm for division.
Read division notation.

Representational:

Model division with manipulatives, diagrams and story contexts.

Academic Vocabulary and Notation	
dividend, division notation ÷, /, divisor, quotient, remainder	
Instructional Strategies Used	**Resources Used**
1. Think Aloud: Do the problem with a partner while explaining and telling what you are thinking and doing. 2. Have students identify in a problem set when they would use mental math and when they would use the standard algorithm. 3. Connect students' existing strategies for division with the standard algorithm. 4. As a starter activity, use division problems that can reasonably be solved by using mental math (e.g., 105/25), estimation (e.g., 150 ÷ 12, 227 ÷ 30), and reasoning (e.g., when I think of 105 divided by 25, I think of 4 sets of 25 with 5 left over, the 5 left over is 5/25 which is 1/5, so the answer is 4 1/5). Model for the students your thinking as you work through the problem. (Note: This strategy would not apply to complex division problems for which the algorithm is most appropriate [e.g., 4567 ÷ 192]).	http://nlvm.usu.edu/en/nav/frames_asid_197_g_2_t_1.html?open=activities&from=search.html?qt=division
Assessment Tasks Used	
Skill-based Task: 248 divided by 18.	**Problem Task:** I spent $504 on 28 tickets for a rock concert. How much did I spend on each ticket?

Core Content

Cluster Title: Compute fluently with multi-digit numbers and find common factors and multiples.

Standard 3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

MASTERY Patterns of Reasoning:

Conceptual:

Understand role of place value in the operations of addition, subtraction, multiplication, and division
Identify when it is appropriate to use the standard algorithm.

Procedural:

Add multi-digit decimals.
Subtract multi-digit decimals.
Multiply multi-digit decimals.
Divide multi-digit decimals.

Representational:

Model the operations of addition, subtraction, multiplication, and division with manipulatives, diagrams and story contexts for multi-digit decimals.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Understand decimal place values.
Know basic facts for addition, subtraction, multiplication and division.

Procedural:

Add single-digit decimals.
Subtract single-digit decimals.
Multiply single-digit decimals.
Divide single-digit decimals.

Representational:

Model the operations of addition, subtraction, multiplication, and division with manipulatives, diagrams and story contexts for single digit decimals.

Academic Vocabulary

addend, sum, difference, factor, product, divisor, dividend, quotient, remainder

Instructional Strategies Used	**Resources Used**
1-4. Connect students' knowledge of various strategies to the standard algorithm. 1-4. Have students look at student work that contains a common misconception and look at errors and discuss how to correct the error.	

Assessment Tasks Used

Skill-based Task:	**Problem Task:**
1. 242.134 + 308.02 2. 38.9 – 14.334 3. 11.82 x 2.81 4. 341.8 ÷ 1.2	The school had a bake sale and raised $75.55. If each cookie cost $0.05, how many cookies were sold? Explain how you got your answer.

Domain: The Number System Grade: 6

Core Content

Cluster Title: Compute fluently with multi-digit numbers and find common factors and multiples.

Standard 4: Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4(9 + 2).

MASTERY Patterns of Reasoning:

Conceptual:

Understand that greatest common factor and least common multiple are ways to discuss number relationships in multiplication and division.
Understand the process of prime factorization.
Understand the distributive property using sums and its use in adding numbers 1-100 with a common factor (e.g., 20 + 24 = 4(5 + 6)).

Procedural:

Compute fluently using the distributive property of multiplication over addition.
Find greatest common factor of two whole numbers less than or equal to 100.
Find the least common multiple of two whole numbers less than or equal to 12.

Representational:

Model prime factorization of whole numbers 1-100 using such tools as a number line, manipulatives, and factor trees.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Understand that a factor is a whole number that divides without a remainder into another number.
Understand that a multiple is a whole number that is a product of the number and any other factor.
Know the distributive property.

Procedural:

Compute using the distributive property
Find factors and multiples of a given number.

Code: 6.NS

Representational:
Model the distributive property of multiplication over addition using manipulatives, diagrams, and story contexts.

Academic Vocabulary and Notation

distributive property, factor, greatest common factor (GCF), least common multiple (LCM), multiple, prime factorization

Instructional Strategies Used	**Resources Used**
Find LCM and/or GCF using factor towers, Venn diagrams, and factor trees to assist in finding the prime factors of each number. Use a model to show that 4(9 + 2) is four groups of 9 and four groups of 2.	

Assessment Tasks Used

Skill-based Task:	**Problem Task:**
Find the greatest common factor of 24 and 60. Find the least common multiple of 6 and 10. Use the distributive property to show 15 + 75.	Hot dogs come in packs of 8. Buns come in packs of 12. How many packs of hot dogs and bags of buns would you have to buy to have an equal number of hot dogs and buns? You need to make gift bags for a party with the same number of balloons and candy in each bag. One package of candy has 24 pieces. One package of balloons has 20 balloons. You need to use all the candy and all the balloons. What is the greatest number of gift bags that you can make containing an equal number of items?

Core Content

Cluster Title: Apply and extend previous understandings of numbers to the system of rational numbers.

Standard 5: Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g.,temperature above/below zero, elevation above/below sea level,credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

MASTERY Patterns of Reasoning:

Conceptual:

Understand that positive and negative numbers (integers) allow us to talk about quantities that have opposite directions or values.
Understand that a negative integer is less than zero.
Understand that the meaning of zero is determined by the real world context (e.g., freezing point in the Celsius system—anything below freezing is negative, anything above freezing is positive).

Procedural:

Use integers to represent situations in real-world contexts.

Representational:

Represent integers using real-world tools such as a thermometer, balance sheet (money), etc.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Know where positive integers are on a number line.
Know the set of positive integers.
Understand that zero represents a position.
Know that number lines extend to show positive integers right and up and negative integers left and down (vertical and horizontal number lines).

Procedural:

Describe quantities having opposite values.

Representational:

Plot integer points on a number line.

Academic Vocabulary and Notation

$\rightarrow, \leftarrow, \uparrow, \downarrow, +, -$, integer, negative, positive, rational, zero

Instructional Strategies Used

Provide multiple examples of types of contexts using positive and negative integers (such as a bank account, hot air balloons, discs to show positive and negative charges, thermometer, number line) and give the students opportunities to make sense of each context.

Give students a number and have them write a real-life situation for that number and its opposite that would result in an answer of zero. Explain the meaning of zero in that situation and represent it on the number line.

Use a comparison matrix:

	Negative	Zero	Positive
Definition			
Example			
Picture			

Resources Used

http://nlvm.usu.edu/en/nav/frames_asid_334_g_2_t_1.html?from=category_g_2_t_1.html

Assessment Tasks Used

Skill-based Task:

Joe's football team had a loss of 5 yards on first down. Write an integer to represent the situation.

Problem Task:

Create a situation in which integers have opposite values and explain what zero means in this situation.

Core Content

Cluster Title: Apply and extend previous understandings of numbers to the system of rational numbers.

Standard 6: Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite.

MASTERY Patterns of Reasoning:

Conceptual:

Understand the meaning of the term *opposite*.
Recognize that the opposite of the opposite of the number is the number itself (e.g., -(-3)).
Recognize that zero is its own opposite.

Procedural:

Find the opposite of a number.

Representational:

Extend number line diagrams to include negative numbers.
Plot opposites on a number line.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Know the characteristics of a number line (extends in both directions, origin, importance of unit).
Know a number line can be used to represent real life situations.

Procedural:

Create a number line with equidistant tick marks determined by the identified unit.

Representational:

Draw a number line.
Represent real-life contexts on a number line.

Academic Vocabulary and Notation	
+, –, integer, opposite, rational number, ( ), point	
Instructional Strategies Used	**Resources Used**
Make a number line on the floor. Have a student choose an integer to stand on then call on a student to come stand on the opposite integer. Repeat process asking students to stand on the opposite of an opposite. Have students use appropriate notation to record these integers.	http://mathstar.lacoe.edu/lessonlinks/integers/integers_main.html
Assessment Tasks Used	
Skill-based Task: Locate 4 and its opposite on a number line.	**Problem Task:** This is a skill-based standard. Therefore, no problem task is offered.

Core Content

Cluster Title: Apply and extend previous understandings of numbers to the system of rational numbers.

Standard 6: Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negativenumber coordinates.

b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

MASTERY Patterns of Reasoning:

Conceptual:

Understand that the signs of numbers in ordered pairs represent a singular location on the coordinate plane.
Understand that changing the sign of one or both numbers in the ordered pair will create a reflection of the point.
Understand that a reflection on the coordinate plane is defined as a transformation of a point or shape across one or both of the axes.

Procedural:

Find reflection points across axes.
Recognize the components of the coordinate plane (Quadrant I (+,+), Quadrant II (-,+) , Quadrant III (-,-) Quadrant IV (+, -), *x* and *y* axes, origin)

Representational:

Plot points in all four quadrants for any given ordered pair.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Know locations of points in the first quadrant.

Procedural:

Identify coordinates of given points in the first quadrant.

Representational:
Plotting points in Quadrant I.

Academic Vocabulary and Notation

(*x, y*), coordinate plane, ordered pair, point, quadrant, reflection, *x*-axis, *y*-axis

Instructional Strategies Used	**Resources Used**
Have students draw and label a coordinate plane, including quadrants and axes. Make a set of cards with ordered pairs that have a matching card that is a reflection of the point. Have students get in groups and pair the cards that are reflections. Given an ordered pair, have students identify in which quadrant the ordered pair is located.	

Assessment Tasks Used

Skill-based Task:	**Problem Task:**
If you had a point graphed at (5, -3), what would be one ordered pair that is a reflection of the point? Students may use a coordinate plane to find a solution.	A town was laid out using a coordinate plane. On the city plans, the library is at (3, 2). Which of the following locations is a reflection across the *x*-axis of where the library is located? Prove your answer is correct using two different methods. School (-3, -2) Gas Station (-3, 2) Post Office (3, -2)

Core Content

Cluster Title: Apply and extend previous understandings of numbers to the system of rational numbers.

Standard 6: Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

MASTERY Patterns of Reasoning:

Conceptual:

Understand a rational number as a point on a number line.

Procedural:

Place rational numbers on horizontal and vertical number lines.
Write an ordered pair using rational numbers to represent a point on the coordinate plane.

Representational:

Plot points on a coordinate plane given an ordered pair using rational numbers.
Extend number line diagrams and coordinate axes to represent points with rational number coordinates.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Have experience with the coordinate plane.
Understand that an ordered pair is composed of two parts: the first coordinate refers to the x-axis, the second coordinate refers to the *y*-axis.

Procedural:

Identify the coordinates of plotted points.

Representational:

Plot points for given ordered pairs.

Academic Vocabulary and Notation	
+, –, coordinate plane, ordered pair, *x*-axis, *y*-axis	
Instructional Strategies Used	**Resources Used**
Play coordinate plane battleship as a whole class and then with partners.	http://www.lessonplanspage.com/MathBattleshipPlotCoordinates79.htm (Note: Modify the lesson plan to include rational numbers within a limited range; for example, halves and fourths between -2 and 2.)
Assessment Tasks Used	
Skill-based Task: Plot the following ordered pairs: (3, 2), (-4, 5), (-8, -3), (4, -6), (2½ , -5), (-9.75, 0) (Note: Approximation is appropriate.) Create a graph of several points and have students write the ordered pair for each point.	**Problem Task:**

Core Content

Cluster Title: Apply and extend previous understandings of numbers to the system of rational numbers.

Standard 7: Understand ordering and absolute value of rational numbers.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right.

MASTERY Patterns of Reasoning:

Conceptual:

Understand ordering of rational numbers, numbers are progressively smaller the further to the left you go on the number line.

Understand that a statement of inequality represents the relative position of the numbers on a number line.

Procedural:

Identify rational numbers on a number line.

Compare rational numbers using inequality symbols.

Representational:

Represent inequalities on a number line and interpret their meaning in words.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Know the words and symbols for less than and greater than.

Procedural:

Correctly read inequalities.

Representational:

Write inequalities.

Model inequalities with manipulatives, diagrams and story contexts.

Academic Vocabulary and Notation	
<, >, inequality, rational numbers	
Instructional Strategies Used	**Resources Used**
Give students a rational number and have them stand in order from least to greatest on the number line. Make flashcards of rational number and play “war.” Have the students create statements of inequality (e.g., -9 < ½) and interpret them by writing out the inequality and its meaning in a sentence or sentences, as in “Negative 9 is less than ½ means that negative nine is to the left of ½ on the number line. Negative nine is nine and one half positions to the left of ½”).	
Assessment Tasks Used	
Skill-based Task: Compare rational numbers using symbols of inequality.	**Problem Task:** On Tuesday the temperature was -7°F and on Wednesday the temperature was -5°F. Which day was colder? Write the inequality and show it on a number line. Explain how you know your answer is correct.

Core Content

Cluster Title: Apply and extend previous understandings of numbers to the system of rational numbers.
Standard 8: Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
Concepts and Skills to Master

Supports for Teachers

Critical Background Knowledge	
Academic Vocabulary	
Instructional Strategies Used	**Resources Used**
Assessment Tasks Used	
Skill-based Task:	**Problem Task:**

Core Content

Cluster Title: Apply and extend previous understandings of numbers to the system of rational numbers.

Standard 7: Understand ordering and absolute value of rational numbers.

b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write $-3^{o}C > -7^{o}C$ to express the fact that $-3^{o}C$ is warmer than $-7^{o}C$.

MASTERY Patterns of Reasoning:

Conceptual:

Understand ordering of rational numbers, numbers are progressively smaller the further to the left you go on the number line.
Understand that a statement of inequality represents the relative position of the numbers on a number line.
Understand that rational numbers found in real-world contexts can be ordered and interpreted.

Procedural:

Compare and order rational numbers in real-world contexts.
Write statements of order that reflect a real-world context.

Representational:

Model and explain statements of order for rational numbers.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Understand that rational numbers can be compared and ordered.

Procedural:

Compare and order rational numbers.

Representational:

Express comparisons of rational numbers with proper notation.

Academic Vocabulary and Notation	
<, >, rational numbers, inequalities	
Instructional Strategies Used	**Resources Used**
Have a student write a real-life situation using 2 rational numbers and have another student write an inequality to represent the situation. Then switch tasks.	
Assessment Tasks Used	
Skill-based Task: It is -20˚F in Juno, Alaska and -5˚F in Salt Lake City, Utah. Which city has the lowest temperature?	**Problem Task:** A scuba diver is 30 ft. below sea level and a submarine is 75 ft. below sea level. Jim thinks the inequality for this situation should be -30 ft. below sea level > -75 ft. below sea level. Sally thinks the inequality should be -30 ft. below sea level < -75 ft. below sea level. Who is correct? Why?

Core Content

Cluster Title: Apply and extend previous understandings of numbers to the system of rational numbers.

Standard 7: Understand ordering and absolute value of rational numbers.

c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars.

MASTERY Patterns of Reasoning:

Conceptual:

Understand absolute value of a rational number as its distance from 0 on the number line.
Understand that absolute value in a real-world context refers to the positive value of the number.
Understand that quantities may have a negative value based on context (e.g., below, debt, behind, etc.)

Procedural:

Interpret absolute value in real-world situations.

Representational:

Model absolute value with number lines and story contexts.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Understand placement of rational numbers on a number line.
Know the set of rational numbers.

Procedural:

Find the distance of a point from zero and represent it as a positive quantity.

Representational:

Model distance on a number line and represent it as a positive quantity.

Academic Vocabulary and Notation	
$\|x\|$, $\|-x\|$, absolute value	
Instructional Strategies Used	**Resources Used**
Have students move on a number line from 0 to 3 and 0 to -3 to show they move the same distance (i.e., walking on a number line on the floor or playground). Have a student choose a place on the number line and have the rest of the class write the absolute value of the number. Develop an original story problem that uses an absolute value. Justify your use of absolute value in that context.	
Assessment Tasks Used	
Skill-based Task: |-5| = If Billy owes Susie $5, express Billy's debt as an integer.	**Problem Task:** A whale swims 40 ft. below sea level. Express the whale's location as an integer and tell how many feet below the surface the whale is swimming. Explain your answers for both parts of the problem.

Core Content

Cluster Title: Apply and extend previous understandings of numbers to the system of rational numbers.

Standard 7: Understand ordering and absolute value of rational numbers.

d. Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.

MASTERY Patterns of Reasoning:

Conceptual:

Understand that as the value on a negative rational number decreases, its absolute value increases.
Understand that rational numbers can be ordered based on their magnitude.

Procedural:

Compare absolute values of rational numbers.
Order rational numbers by magnitude.

Representational:

Model comparisons of absolute values on a number line
Order rational numbers on a number line.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Have experience with number line usage.
Understand integers.

Procedural:

Compare values of integers.

Representational:

Model comparisons of integers.
Express comparisons of integers with correct notation.

Academic Vocabulary and Notation	
<, >, =, absolute value, decrease, increase	
Instructional Strategies Used	**Resources Used**
1. Make integer placards for each student (be sure to include 0 in the set). Have students line up from least to greatest as integers and then as absolute values.	http://mathforum.org/library/drmath/view/57177.html
Assessment Tasks Used	
Skill-based Task: You are $35 dollars in debt. Write your debt using mathematical symbols. How much do you need to earn to be out of debt?	**Problem Task:** A mother dolphin is 150.25 meters below sea level. Her calf is 45 meters below sea level. Which dolphin is farthest from the surface? A mother whale is at 35 meters below the surface and her calf is at the surface. How far does the calf have to swim to get to its mother? Which statement deals with absolute value? Which statement deals with ordering? Justify your answer.

Core Content

Cluster Title: Apply and extend previous understandings of numbers to the system of rational numbers.

Standard:8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

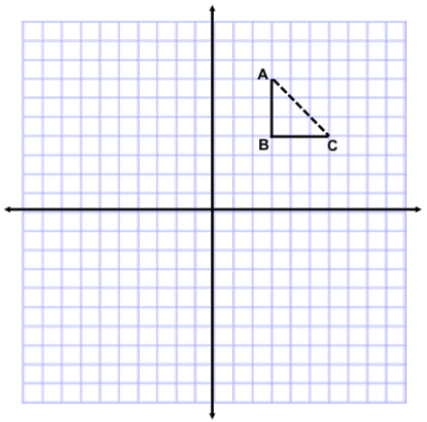
MASTERY Patterns of Reasoning:

Conceptual:

Understand that a line segment from one coordinate pair to another represents a distance.
Understand that if two coordinates have the same x or y value they are on the same line.
Understand that the distance from a point on a coordinate plane to an axis is an absolute value.
Understand that the units on a coordinate plane define the unit of distance measure.
Understand that the coordinate plane can be used to represent real world contexts (e.g. streets)

Procedural:

Find the distance between two points.
Find the length BA and BC.



Representational:

Plot points with the same first or second coordinate in all four quadrants of the Cartesian Coordinate Plane.

Supports for Teachers

Critical Background Knowledge	
Conceptual: Understand that absolute value refers to a number's distance from zero Understand that it takes two rational numbers to create a point on a coordinate plane. Procedural: Be able to use a coordinate plane to solve a real world and mathematical problems. Representational: Graph on a coordinate plane	
Academic Vocabulary	
Coordinate plane, Absolute value, Coordinate, Point	
Instructional Strategies Used	**Resources Used**
Make a coordinate grid on the floor and have students stand on points then find the distance between students. Make sure that the two points have either the same first coordinate or the same second coordinate. Have the students figure out that when you are on the same x or the same y you are on the same line.	
Assessment Tasks Used	
Skill-based Task What is the distance between (-8, 7) and (-8, -2)?	**Problem Task** Bills house is at (-4, 6), the library is at (-4, -2) and the Bakery is at (3, -2). What is the distance between Bill's house and the library? The library and the bakery? Show two different methods to find the difference.

Core Content

Cluster Title: Understand ratio concepts and use ratio reasoning to solve problems.

Standard 1: Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”

MASTERY Patterns of Reasoning:

Conceptual:

Understand the concept of a ratio as a way of expressing relationships between quantities.
Distinguish when a ratio is describing part to part or part to whole comparison.

Procedural:

Describe ratio relationships between two quantities.
Translate relationships between two quantities using the notation of ratio language (1:3, 1 to 3, 1/3).

Representational:

Communicate relationships between two quantities using ratio notation and language.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Understand the relationship between parts and wholes.

Procedural:

Translate “for every” and other meanings of multiplication into terms.

Representational:

Experience working with set and measurement models.

<table>
<tr><th colspan="3">Academic Vocabulary and Notation</th></tr>
<tr><td colspan="3">:, /, ratio, terms of ratio (i.e., the numbers used in a ratio are called its terms)</td></tr>
<tr><th colspan="2">Instructional Strategies Used</th><th>Resources Used</th></tr>
<tr><td colspan="2">Using a variety of situations, describe relationships using ratio, for example:
1. Part to part: Compare the number of girls to boys in the classroom using the different symbols for ratio (girls: boys, girls to boys, $\frac{girls}{boys}$, girls out of boys). Then compare the number of boys to girls in the same way.
2. Part to whole: Compare the number of girls to the whole class. Do the same thing for the boys in the class.</td><td>Ratio coloring activity: http://www.softschools.com/math/ratios/ratio_coloring_game/</td></tr>
<tr><th colspan="3">Assessment Tasks Used</th></tr>
<tr><td colspan="2">Skill-based Task:
There are four dogs and three cats. What is the ratio of dogs to cats and cats to dogs?</td><td>Problem Task:
The newspaper reported, “For every vote candidate A received, candidate B received three votes.” Describe possible election results using at least three different ratios. Explain your answer.</td></tr>
</table>

Core Content

Cluster Title: Understand ratio concepts and use ratio reasoning to solve problems.

Standard 2: Understand the concept of a unit rate *a*/*b* associated with a ratio *a:b* with $b \neq 0$, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."

MASTERY Patterns of Reasoning:

Conceptual:

Understand that a rate is a special ratio that compares two quantities with different units of measure.

Understand that unit rates are the ratio of two measurements in which the second term is one (e.g., *x* miles per one hour).

Understand that when using rates $\frac{a}{b}$, "*b*" cannot be 0 (because division by 0 is undefined).

Procedural:

Solve problems involving ratios.

Understand rate language (per, each, or the @ symbol).

Representational:

Correctly use ratio notation and models to represent relationships between quantities.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Understand ratio concepts from Standard 1 (6.RP.1).

Understand the meaning of equivalent ratios.

Procedural:

Simplify fractions.

Use equivalent ratios to solve problems.

Repreentational:

Communicate relationships between two quantities using ratio notation.

Academic Vocabulary and Notation	
@, each, equivalent ratio, rate, ratio, unit rate, per	
Instructional Strategies Used	**Resources Used**
1. Show examples of rates: 300 miles on 10 gallons of gas, $15 for 5 ounces, $30 for 6 hours. 2. Connect rates from number 1 with their unit rates: 30 miles per gallons, $3 per 1 ounce, $5 per 1 hour. 3. Convert rates from fraction form to written form using per, each, or @. Example $\frac{300\ miles}{10\ gallons\ of\ gas}$ = 30 miles per gallon of gas. 4. Quick write: Students brainstorm examples of unit rates in the real world (e.g., 4 candy bars per $1, 55 miles per hour, 6 points per touchdown).	UEN- Lesson "Ratio, Rate, and Proportion" Activities 1 and 2 from http://mypages.iit.edu/~smart/dvorber/lesson3.htm
Assessment Tasks Used	
Skill-based Task: Identify (given examples) the difference between a ratio and a rate.	**Problem Task:** Is the following example a ratio or rate? [60 heartbeats per minute] Explain your answer.

Core Content

Cluster Title: Understand ratio concepts and use ratio reasoning to solve problems.

Standard 3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

MASTERY Patterns of Reasoning:

Conceptual:

Understand how to make, complete, and read a table of equivalent ratios.
Understand that tools such as tables of equivalent ratios support the development of ratio and rate reasoning.
Understand that pairs of values from a table can be plotted on the coordinate plane.
Understand that establishing connections between tables and plotted points on the coordinate plane allow for extended reasoning and synthesis of the concept of ratios and rates.

Procedural:

Use a table to compare ratios.
Determine missing values using ratio reasoning.
Identify relationships in ratio tables.

Representational:

Plot pairs of values from a table to a coordinate plane.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Understand that coordinate graphs are two-dimensional and rely on two coordinate points to identify a specific location on a plane.
Understand equivalent fractions.
Understand equivalent ratio (from 6.RP.2).

Procedural:

Experience with coordinate plane graphing in quadrant 1.
Read equations

Representational:

Plotting a point on a coordinate plane when given the coordinates.

Academic Vocabulary and Notation

coordinate plane, tables of equivalent ratios (value table)

Instructional Strategies Used	**Resources Used**
1.Have students make a table given a ratio situation. They should plot those points on a coordinate plane and draw conclusions about what's happening in the ratio situation. 2.Give students a table with missing values and have them identify the missing values. 3. Have students study ratio relationships in a table.	http://www.youtube.com/watch?v=d625kdtsUIw UEN: Price-Earnings ratio http://www.uen.org/Lessonplan/preview.cgi?LPid=25290

Assessment Tasks Used

Skill-based Task:

Analyze the table below to determine the missing values. Fill in the missing values on the table below.

Swimmers	20	30	40	60	90	100
Life Guards	2	3	4	6		

Problem Task:

Graph the information from the table on the coordinate plane and explain the relationship of swimmers to life guards.

Core Content

Cluster Title: Understand ratio concepts and use ratio reasoning to solve problems.

Standard 3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

MASTERY Patterns of Reasoning:

Conceptual:

Understand that tools such as tables of equivalent ratios, tape diagrams, double number line diagrams, and equations support the development of ratio and rate reasoning.
Understand that rate problems compare two different units, such as miles to hours.
Recognize that a unit occurs when at least one of the units is one.
Understand that establishing connections between tools allow for extended reasoning and synthesis of the concept of ratios and rates (e.g., How do tape diagrams and double number lines show rate reasoning given the same context?).

Procedural:

Solve real-world problems using rate reasoning.

Representational:

Set up the unit rate correctly.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Understand ratio reasoning and relationships.
Understand equivalent fractions.

Procedural:

Use the four basic operations (+, -, x, /).

Representational:

Represent equivalent ratios with ratio notation.

Academic Vocabulary and Notation

equivalent ratios notation ($\frac{a}{b}=\frac{c}{d}$) or a is to b as c is to d; ratio; unit rate

Instructional Strategies Used	**Resources Used**
1. Identify the question being asked based on the context, and determine a method for finding the unit rate (table of equivalent ratios, tape diagrams, double number line diagrams, and equations). 2. Complete the determined tool to find the unit rate (e.g., use tool to find the ratio in which one of the units is one).	Illuminations measuring up activity: http://illuminations.nctm.org/LessonDetail.aspx?ID=L511

Assessment Tasks Used

Skill-based Task:	**Problem Task:**
If 5 CDs cost $60, what is the price of each CD?	Joe's Gas and Go has drinks for the following prices: 12 fl. oz. for $ .89 16 fl. oz. for $ .99 20 fl. oz. for $1.09 32 fl. oz. for $1.19 Which drink costs the least per ounce? You may round to the nearest cent and use a calculator if you desire.

Core Content

Cluster Title: Understand ratio concepts and use ratio reasoning to solve problems.

Standard 3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

MASTERY Patterns of Reasoning:

Conceptual:

Understand that a percent is rate per 100 and can be represented using tools such as tables of equivalent ratios, tape diagrams, double number line diagrams, and equations.
Understand that percentage-based rate problems compare two different units where one of the units is 100.
Understand that establishing connections between tools allow for extended reasoning and synthesis of the concept of ratios and rates (e.g., How do tape diagrams and double number lines show rate reasoning given the same context?).

Procedural:

Writing a percent as a rate over 100.
Finding the percent of a number using rate methods developed in 6.RP.3b.
Given the parts and a percent, determine the whole using tools identified above.

Representational:

Represent the relationship of part to whole to describe percents using model.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Understand the concept of rate as detailed in 6.RP.3b.
Understand whole and parts in the context of a ratio.

Procedural:

Use unit pricing and constant speed to solve problems.
Use unit rates to solve problems.

Representational:

Represent unit rates with models.

Academic Vocabulary and Notation

%, percent

Instructional Strategies Used	**Resources Used**
1. Model using a hundreds grid. Color in 30 units and have students write it as a fraction and percent. 2 Use double number lines and tape diagrams in which the whole is 100 to find the rate per hundred.	Coloring percent activity: http://www.softschools.com/math/percent/games/ NLVM percent virtual manipulative: http://nlvm.usu.edu/en/nav/frames_asid_160_g_2_t_1.html Tape Diagrams: http://mathgpselaboration.blogspot.com/2010/04/mp5-tape-diagrams.html

Assessment Tasks Used

Skill-based Task:	**Problem Task:**
What is 25% of 60? 72% of what number is 300?	Stop and Shop has pants for $30 with a 10% discount, while Stay and Shop has pants for $45 with a 20% discount. Which store has the pants for a better price? Use a table of equivalent values, double number line, or tape diagram to solve and explain your reasoning.

Core Content

Cluster Title: Understand ratio concepts and use ratio reasoning to solve problems.

Standard 3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

MASTERY Patterns of Reasoning:

Conceptual:

Understand that measurement units employ ratio reasoning (e.g., If 3 feet is equal to yard, then 6 feet is equal to 2 yards).
Understand that tools such as tables of equivalent ratios, tape diagrams, double number line diagrams, and equations can help scaffold understanding for converting measurement units.
Understand that establishing connections between *tools* allow for extended reasoning and synthesis of the concept of ratios and rates (e.g., How do tape diagrams and double number lines show rate reasoning given the same context?).

Procedural:

Convert customary units using ratio tools and methods.
Convert metric units by multiplying or dividing by powers of ten.

Representational:

Represent relationships between measurement units using tables of equivalent ratios, tape diagrams, double number line diagrams, and equations.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Understand customary and metric units of measurement.
Understand ratios and unit rates.

Code: 6.RP

Procedural:

Use customary and metric units of measurement.
Be able to multiply and divide by powers of 10.

Representational:

Represent multiplication and division with powers of 10 with charts, tables and manipulatives.

Academic Vocabulary and Notation
convert, 10^n (power of 10 notation)

Instructional Strategies Used	**Resources Used**
1. Use double number line, tape diagrams, tables of equivalent values, or equations to convert measurements in customary and metric units. **2.** If 4 cups equals one quart, how many cups in 12 quarts? $\frac{4\ cups}{1\ quart} = \frac{x\ cups}{12\ quarts}$	NLVM conversion manipulative: http://nlvm.usu.edu/en/nav/frames_asid_272_g_2_t_4.html?open=instructions&from=category_g_2_t_4.html

Assessment Tasks Used	
Skill-based Task: How many inches are in three feet? How many inches in two miles?	**Problem Task:** In the store a package of candy that weighs 150 grams costs $1.00. A package of 200 candies that each weigh 200 milligrams also costs $1.00. Which package is the better deal?

Core Content

Cluster Title: Solve real-world and mathematical problems involving area, surface area, and volume.

Standard 1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

MASTERY Patterns of Reasoning

Conceptual:

Classify special quadrilaterals: square, rhombus, trapezoid, parallelogram, rectangle, kite
Relate the area of triangles and the area of rectangles.
Solve problems in a real-world context.

Procedural:

Identify the relationship between bases and heights in polygons.
Determine the area of polygons.

Representational:

Recognize symbolic notation for height (dotted line).
Visually and physically decompose and compose polygons into rectangles and triangles to find area.

Supports for Teachers

Critical Background Knowledge

Conceptual:

- Recognize that perpendicular lines form right angles.
- Define and identify polygons.
- Polygons have two dimensions.
- Identify square, rhombus, trapezoid, parallelogram, rectangle, kite.
- The nature of area as an attribute.
- Since area is a different attribute it requires a different measurement unit: square units.

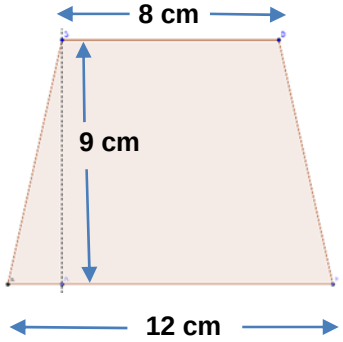
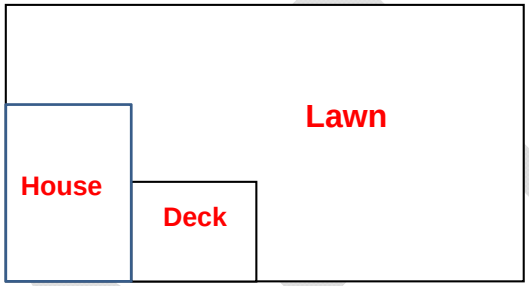
Procedural:

- Determine the area of rectangles.

Representational:

- Compose and decompose polygons.
- Identify right angles in various orientations.
- The symbol for right angles: 

Academic Vocabulary	
Compose, decompose, base, height, right triangle, polygon, special quadrilaterals, perpendicular	
Tier 1 Instructional Strategies Used	**Resources Used**
Derive the formula for triangles from rectangles.	Area Formula Graph paper
Begin teaching by decomposing a rectangle into two right triangles, and by composing a rectangle with two right triangles. • Use a Geoboard to compose and decompose polygons. • Use dot paper or grid paper to draw polygons and find the area. • Have students decompose paper polygons by cutting into triangles and rectangles.	Cutting Up Lesson Geoboards (NLVM) http://nlvm.usu.edu/en/nav/frames_asid_282_g_3_t_3.html?open=activities Dot paper
Use the “Triangle Problem” for a mathematical extension involving the area of triangles.	http://www.marktaw.com/blog/TheTriangleProblem.html

Assessment Tasks Used	
Skill-based Task: Find the area of this trapezoid by composing and decomposing the shapes.  	**Problem Task:** Mario needs to buy sod for his backyard. Here is a diagram of Mario's backyard. Determine how much sod he will need to purchase.  **Teacher Note:** Students will have to identify appropriate measures for each component and then compose and decompose to come up with an answer. If they give answers in square centimeters or inches ask them to consider if a house and yard that small would be realistic.

Core Content

Cluster Title: Solve real-world and mathematical problems involving area, surface area, and volume.

Standard 2: Find the volume of a right rectangular prism with appropriate unit fraction edge lengths by packing it with cubes of the appropriate unit fraction edge lengths (e.g., 3½ x 2 x 6) and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = Bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

Clarification: It is not intended that this be modeled physically; it should be a conceptual activity modeled with drawings and diagrams. (Note: This standard is worded differently than the official standard. There are typos and other errors in the official version. They are corrected here and a clarification is included.)

MASTERY Patterns of Reasoning

Conceptual:

- Measuring with fractional units requires students to relate volume to multiplication with fractions.
- Describe the impact of defining volume by fractional factors.

Procedural:

- Use these formulas interchangeably, $V = lwh$ and $V = Bh$.
- Make the connection that when finding volume l x w is the same as B.

Representational:

- Composing whole cubes with fractional unit cubes
- Prove that the volume formula works by creating diagrams of prisms with unit fraction edge lengths and showing how unit fraction cubes pack them.

Supports for Teachers

Critical Background Knowledge

Conceptual:

- Volume is measured with cubic units.
- The nature of volume as an attribute.
- Since volume is a different attribute it requires a different measurement unit: cubic units.
- Prisms are three-dimensional.
- Volume is filling a prism.

Procedural:

Ability to multiply fractions
Finding the area of polygons, including those with unit fraction edge lengths
Substitution for values in formulas
Finding volume of prisms with whole unit side lengths

Representational:

Use of physical models with whole-unit side lengths.
Find volume using a unit cube model.

Academic Vocabulary

volume, rectangular prism, length, width, height, base, cubic units, fraction edge length, unit fraction

Instructional Strategies Used	Resources Used

• Review of 5.MD.5:
  o Explore with cubes and arrange them into layers to create rectangular prisms. Record the dimensions of the first/base layer, add a second layer, determine new dimensions, and look for patterns to predict what will happen when a third layer is added. Add the third layer and determine if your prediction was correct. Make connections to formulas.
  o Hold up a cube and explain that the edge measures one unit and that is the standard for finding the volume of a solid figure. The volume of a solid figure is the number of same sized cubes filling the space so that there are no gaps and overlaps.
  o Make nets of rectangular prisms on graph paper. Fold and determine volume.
• Define one cubic unit in order to see fractional parts. See resource.
• Apply to formula using fractional edge lengths.	

Mathematical Task:
Jaime has the following rectangular prisms (boxes) that he would like to send to his friend Carla through the mail:

- Box 1 – 1 inches long by 4 inches wide by 6 inches height
- Box 2 – 1½ inches long by 4 inches wide by 6 inches height
- Box 3 – 3 inches long by 4 inches long by 6 inches height
- Box 4 – 1½ inches long by ½-inch wide by 6 inches height

Process between each step of the problem. How did the answer to one box lead to the answer for the next? Describe the effect of fractional edge lengths. What is the total volume of the boxes? Determine the dimensions of the smallest possible box that Jaime could use to send the 4 boxes to Carla in one shipment. How much empty space will there be? Prove your answer by drawing a representation on grid paper or constructing the boxes.

Assessment Tasks Used

Skill-based Task:
A flower box is 3 ft. long 2¾ ft. wide and ½ ft. deep. How many cubic feet of dirt can it hold?

A gallon of water uses 231 cubic in. of space. How many gallons of water are needed to fill this aquarium?

Draw a diagram to match:
l = 12½ in.
w = 8¼ in.
h = 12½ in.

Problem Task:
Build 3 rectangular prisms with the volume of 36 cubic units. At least one of the side lengths of each prism is a fractional unit. What are the dimensions of each of the rectangular prisms you built?

Core Content

Cluster Title: Solve real-world and mathematical problems involving area, surface area, and volume.

Standard 3: Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

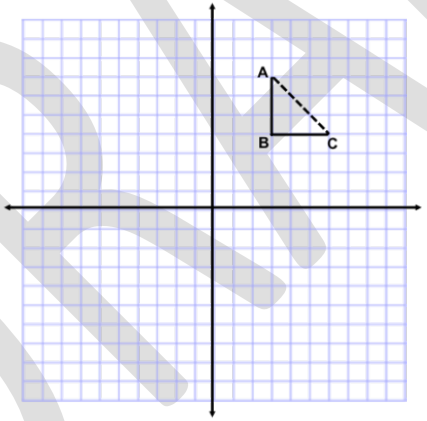
MASTERY Patterns of Reasoning

Conceptual:

Understand that a line segment from one coordinate pair to another represents a distance.
Understand that if two coordinates have the same *x* or *y* value they are on the same line.
The distance between two points on a coordinate plane is an absolute value.
The units on a coordinate plane define the unit of distance measure.
A coordinate plane can be used to represent real-world contexts (e.g., streets).

Procedural:

Find the distance between two points.
Find the length BA and BC.



Representational:

Plot points in all four quadrants of the Cartesian Coordinate Plane.
Plot a polygon in the Cartesian Coordinate Plane with given coordinates.

Supports for Teachers

Code: 6.G

Domain: Geometry Grade: 6

Critical Background Knowledge
Conceptual: Understand ordered pairs (*x*,*y*). Recognize that a coordinate plane has four quadrants. **Procedural:** **Representational:** Identify the *x*- and *y*-axes. Know how to plot a point using an ordered pair. Plot coordinates in the first quadrant of the Cartesian coordinate plane.
Academic Vocabulary
Coordinate planes, origin, *x*-axis, *y*-axis, Quadrant I, Quadrant II, Quadrant III, Quadrant VI, vertex, vertices, coordinates, polygons, *x*-coordinate, *y*-coordinate, length, Cartesian coordinate plane

Instructional Strategies Used	Resources Used
Have the students explore how to determine the side length of a polygon. Discuss conjectures and test the findings. Solidify the two different methods: 1) Students can find the side length of a polygon by counting. 2) Students can find the side length of a polygon by subtracting *x* coordinates or *y* coordinates.	Illumination Lesson: Finding Your Way Around
Archeologists use the coordinate plane to note the locations of artifacts found. They select a point at the dig site to be the origin. From that point they mark the grid with 1-meter squares. When an artifact is found in a square the ordered pair is recorded. Ask the students to use coordinate grids to locate the following objects: a piece of pottery at (5, 4), a maize grinding stone at (5, -5), a doll at (-2, -5), a cup at (-2, 4). Have the students give you the following distances: • From the cup to the piece of pottery • From the doll to the cup	Learn NC: Archeological Example

Code: 6.G

- From the piece of pottery to the grinding stone.

Ask the students to identify the polygon and to give its perimeter.

Have the students create their own archeological digs and plot the points for at least four items that might be found in such a dig. Have them identify the polygon and give the length of each side of the polygon and the perimeter. Stress that the points connected must have the same first coordinate or the same second coordinate.

Mathematical Task:
Nicole wants to landscape her yard with at least 4 trees located 15 meters apart, with two in the front yard and two in the back yard. Her yard is a rectangle with sides 40 meters by 50 meters. Use a coordinate grid to determine the placement of the trees. Give the coordinates of each tree as an ordered pair.

Assessment Tasks Used

Skill-based Task:
Plot ordered pairs to form a polygon. Determine one of the side lengths.

Example:
Plot the ordered pairs: A (2,6) B (2,2) C (-4,4). Find the side length AB.

Problem Task:
Given the coordinates A (2,5), B (-4,5), C (-4,1), and D (2,1) Jose says that the distance between A and D can be found by subtracting 2 from 5. Prove or disprove. Explain your answer with words, pictures, and equations.

Core Content

Cluster Title: Solve real-world and mathematical problems involving area, surface area, and volume.

Standard 4: Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

MASTERY Patterns of Reasoning

Conceptual:

The surfaces of three-dimensional shapes are composed of two dimensional faces.
Understanding surface area using nets can be used in real-world contexts (e.g., wrapping presents, packaging).

Procedural:

The area of two-dimensional shapes can be used to find the surface area of the three-dimensional shape.
Transitioning from three dimensions to two dimensions requires spatial reasoning.

Representational:

Use a net to represent a 3-D figure.
Use a net to find the surface area of a 3-D figure made up of rectangles and triangles (polyhedron).
Compose and decompose a polyhedron using rectangles and triangles.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Area is covering the surface of a two-dimensional shape.
Area is measured with square units.

Procedural:

Find the area of a rectangle and triangle.

Representational:

Polygons can be decomposed.

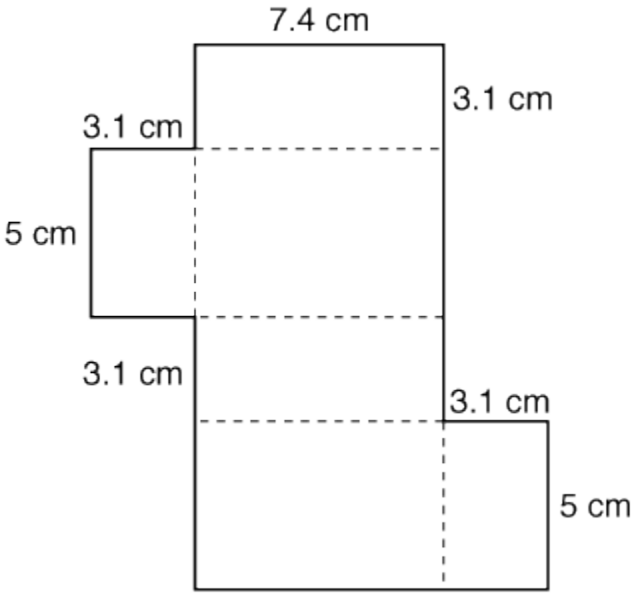
Domain: Geometry Grade: 6

Academic Vocabulary	
Net, three-dimensional figures, surface area, vertices, face, edge, length, width, base, height, polyhedron, prism, pyramid	
Instructional Strategies Used	**Resources Used**
Make polyhedrons from given nets. Recognize the rectangles and triangles that compose the polyhedron. Find the area of each polygon and add together to find the total surface area of the polyhedron. Have nets on graph paper to aid in finding the area of polyhedrons. **Mathematical Task:** Use the real-world problem at Figure This: Real World Application (paper version attached). Give the students three small ice blocks and an ice block the equivalent to the size of all three smaller blocks. Have the students find the surface area of the block and each cube. Have them create a hypothesis about which will melt faster–the intact ice block or the cubes. Have the students perform the experiment by observing and measuring the time it takes for the block to melt and for all three cubes to melt. How does the melting time compare to the surface area exposed? Generalize the relationship. Extension: Can you create a formula to show the relationship?	Interactive Nets: 3D Nets and Surface Area Nets and Surface Area (scroll to page 106) Graph Paper Nets: Shodor Interactive Surface Area
Assessment Tasks Used	
Skill-based Task: Find the surface area.	**Problem Task:** Belinda had two boxes to wrap for a birthday party. Box A has a length of 12 in, width of 8 in, and height of 6 in. Box B has a length of 11 in, width of 9 in, and height of 7 in. Which box will require the least amount of wrapping paper?

Code: 6.G

Domain: Geometry

Grade: 6



Code: 6.G

Core Content

Cluster Title: Develop understanding of statistical variability.
Standard: 1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*
MASTERY Patterns of Reasoning:
Conceptual: Understand that data generated from statistical questions will vary. Recognize that responses to statistical questions have variations that can be used to draw conclusions about the data set **Procedural:** Identify the difference between a statistical and non-statistical question. Write simple statistical questions **Representational:** Create models that represent the anticipated data from statistical questions such as charts and tables**.**

Supports for Teachers

Critical Background Knowledge
Conceptual: Know the difference between a statement and a question **Procedural:** Be able to formulate a question **Representational:** Know how to set up and use charts and tables for representing data.
Academic Vocabulary and Notation
data, expectation, statistics, variability

Instructional Strategies Used	Resources Used
Provide examples and non-examples of statistical questions such as: Example: Over the course of the month, what time did Billy eat breakfast each day? Non-example: What time did Billy eat breakfast today? Direct students to generate questions, and then as a class decide whether they are statistical questions or not.	http://www.math.wichita.edu/history/topics/stat.html

Assessment Tasks Used	
Skill-based Task	**Problem Task**
Given a list of questions, students will categorize them as statistical or non-statistical. For example, what color is my pencil? (non-statistical) What are the colors of the pencils in this class and how many of each are there? (statistical)	Students will create their own statistical and non-statistical questions and address how the data might vary in response to that question. Encourage students to create story contexts for the questions given.

Core Content

Cluster Title: Develop understanding of statistical variability.

Standard: 2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

MASTERY Patterns of Reasoning:

Conceptual:

Understand that data collected in response to a statistical question can be analyzed by its distribution.

Understand that data distribution can be viewed by its center (mean, median, and mode), spread (range), and overall shape

Procedural:

Describe a set of data using center and spread

Representational:

Represent a set of data using center and spread

Supports for Teachers

Critical Background Knowledge

Conceptual:

Understanding of statistical questions.

Understanding of data collection.

Procedural:

Organize a set of data in a table or chart

Representational:

Represent quantities with tables and charts.

Academic Vocabulary and Notation

Center, shape, spread

<table>
<tr><th>Instructional Strategies Used</th><th>Resources Used</th></tr>
<tr><td>The teacher will pose a statistical question (e.g. the length of pinky fingers, hours of TV students watch a week, the number of video games they have in their home, the number of people in their family). They will survey the class and, using the data generated, assist students in creating a line plot. The teacher will guide a discussion about the center, spread, and overall shape of the data on the line plot.</td><td>Picking data activity:
http://www.bbc.co.uk/education/mathsfile/shockwave/games/datapick.html</td></tr>
<tr><th colspan="2">Assessment Tasks Used</th></tr>
<tr><td>Skill-based Task
Students will identify the center, spread, and overall shape of data graphed by teacher.</td><td>Problem Task
Provided a box score from a college or professional basketball game, have the students pick out the points scored by each player. The students will find the center of the data (in this case, let’s use the median), and the spread of the data. Have the students graph the data and describe the overall shape. Then have the students answer the following questions:
All players who don’t score at or above the median points scored have to ride a stationary bicycle for 20 minutes. List the players who have to ride the bicycle.
The coach is trying to get the team to play more as a team. He is using the spread of the data as a way to determine if they are playing as a team. How might the coach use the spread to accomplish his goal?</td></tr>
</table>

Core Content

Cluster Title: Develop understanding of statistical variability.

Standard: 3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

MASTERY Patterns of Reasoning:

Conceptual:

Understand that the mean of a set of numerical data is a measure of center of that data summarized by a single number and represents the arithmetic average of the data.

Understand that the median of a set of numerical data is a measure of center of that data summarized by a single number and represents the point at which fifty percent of the data is greater than or equal to that number and fifty percent is less than or equal to that number.

Understand that the mode of a set of numerical data is a measure of center of that data summarized by a single number and represents the most frequent value of a set of data.

Understand that the range of a set of numerical data is a measure of how the data varies summarized by a single number and represents the difference between the highest and the lowest numbers in that set.

Procedural:

Given different numerical data sets, students will determine appropriate center (mean, median and/or mode) and variation (range).

Representational:

Create models such as graphs and data charts that show the range in a set of data.

Use models such as data charts to indicate a measure of center in a set of data.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Understand that graphs and tables can organize data and allow that data to be interpreted.

Procedural:

Basic computation skills using all four operations.

Ordering numbers.

Representational:
Use models such as graphs and tables to organize data from a numerical set.

Academic Vocabulary and Notation

Center, Mean, Median, Mode, Range, Variability, $\bar{x}$

Instructional Strategies Used	**Resources Used**
Create a human graph using the amount of letters in the students' names. Gather the data and summarize in a data chart. Use the amstat.org website at the right for activities related to this graph. See pages 29 and 30 in the guide.	http://amstat.org/education/gaise/GAISEPreK12_LevelA.pdf http://serc.carleton.edu/sp/cause/conjecture/examples/reasoningcenterandspreadactivity.html

Assessment Tasks Used

Skill-based Task	**Problem Task**
Students will identify mean, median and mode given different data sets. Students identify range given different data sets.	The students create statistical questions that have meaning to them (e.g. how much allowance they get, how far they walk or ride to school) in groups. Students survey students in other grade levels and/or classes to gather data, and they then graph the data. Have them then analyze and summarize the data using the vocabulary in this lesson.

Core Content

Cluster Title: Summarize and describe distributions.

Standard: 4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

MASTERY Patterns of Reasoning:

Conceptual:

Understand that data can be organized in graphs in order to analyze the data.
Understand the decisions that must be made in order to create a useable data display (e.g., how much data is there, what comparisons need to be made)
Know when data is best represented on number lines, dot plots, histograms or box plots.

Procedural:

Students will create dot plots (line plot), histogram, and box plots (box-and-whisker) including labeling and scaling axes appropriately.

Representational:

Represent a set of numerical data accurately on a number line, dot plots, histograms and box plots.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Experience with the use of horizontal and vertical axes.

Procedural:

Choose appropriate and consistent scale for a given data set.
Choose appropriate interval for a given data set.

Representational:

Set up an accurate and useable number line with correct labels.

Academic Vocabulary and Notation

1^{st} Quartile (Q_1), 2^{nd} Quartile (Q_2), 3^{rd} Quartile (Q_3), 4^{th} Quartile (Q_4), Box plot (box-and-whisker), Distribution, Dot plot (line plot), Histogram, Interquartile range, Upper quartile, Lower quartile, Median, Upper endpoint (upper extreme), Lower endpoint (lower extreme)

<table>
<tr><th>Instructional Strategies Used</th><th>Resources Used</th></tr>
<tr><td>Gather scores from the last quiz, and then create plots with the given data.</td><td>http://www.deltastate.edu/docs/math/Mitchell3.pdf

Box Plot:
http://www.shodor.org/interactivate/activities/BoxPlot/

6.SP.4 reaction time activity (see below)</td></tr>
<tr><th colspan="2">Assessment Tasks Used</th></tr>
<tr><td>Skill-based Task

Given a data set, create the following:

<table><tr><td>Data Set:</td><td>Dot Plot:</td></tr><tr><td>Histogram:</td><td>Box Plot:</td></tr></table></td><td>Problem Task

Have students count the number of steps they take to get to school. If they ride in a car or a bus they would count the steps they take to get to the vehicle and then into the school. Graph the data using all graphs in this standard.</td></tr>
</table>

Domain: Statistics and Probability Grade: 6

Reaction Time Activity

Equipment Needed: Meter stick or rulers
Graph paper

Essential Questions:

1. What is reaction time and why does it vary from person to person? Why would people want to know what the expected variance of a population would be?
2. How can statistics be used to predict reaction times for an entire population?

Statement of the Problem:

A reaction time is a measure of how long it takes you to do something (*such as step on the brakes)* upon receiving a certain signal (*such as see a child start across the street in front of you.)* Reaction times vary from one person to the next. Reaction times, from one measurement to the next, also vary for the same person.

In this experiment you will measure reaction times for a "specific event" of catching a dropped ruler or meter stick. Then based on the data for your partnership and your class, you will calculate measure of measure of center and variation, explain what these measures mean, and then graph the data in a box plot.

Procedure:

a. The reaction time to be measured in this experiment is how long it takes you to stop a stick that starts falling between your finger and thumb. The class will be broken into partnerships. One partner will hold the ruler or meter stick just above the finger and thumb of the other partner in order to test his/her reaction time. The ruler or meter stick should have the 0 at the bottom with the numbers going up. The person holding the ruler or meter stick will let it go allowing it to drop between the finger and thumb of their partner. The other person, looking only at the bottom of the stick, catches the stick as quickly as possible by pressing thumb and forefinger together.
b. The length of the meter stick from the 0 centimeter end to the "catch" position is related to the elapsed time from moment of "drop" to moment of "catch". Thus, you can use this length as a "measure" of the reaction time. Be sure to read the number visible just above the finger or thumb each time.

Code: 6.SP

c. Have each member of the partnership catch three “drops”. Record the meter stick readings on a sheet of data paper.
d. Each person will calculate their own mean (average) measure and record this value on the board. These readings become the data set for the class.

- Adapted from the Laboratory Activities in *Unit 19 Working With Statistics* CORD Applied Mathematics 1989; pp. 21-33.

Calculations:

a. Record your findings in the chart below

Student	Trial #1	Trial #2	Trial #3
#1			
#2			

b. Determine the mode, median and mean of the reaction times for the group.

Mode _____

Median _____

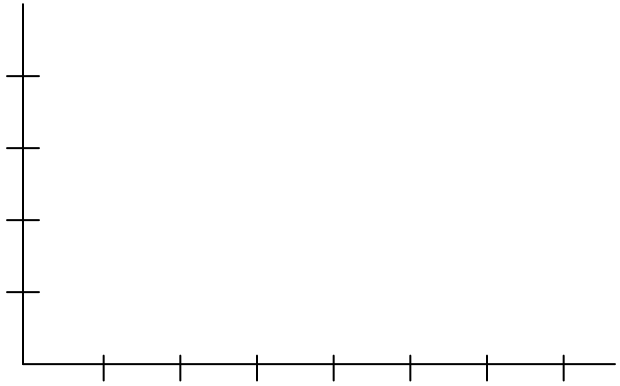
Mean _____

c. Divide the reaction times into approximately 5 intervals. Make a histogram of the reaction times for your group.

What intervals did you choose?

Why did you choose these intervals? (Use a complete sentence.)

Complete the histogram below.



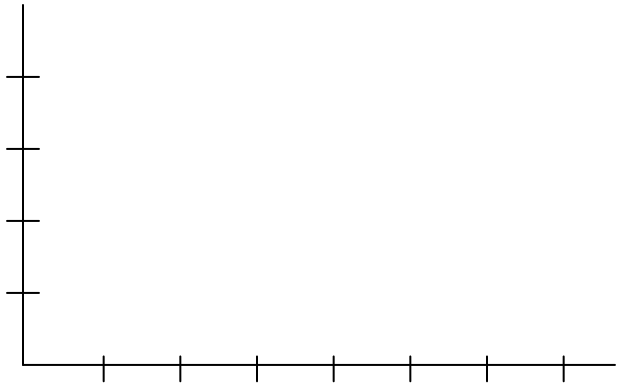
d. Determine the mode, median and mean of the reaction times for the entire class.

Mode _____

Median _____

Mean _____

e. Divide the reaction times into approximately 5 intervals. Make a histogram of the reaction times for your class.



f. How do the mean and standard deviation for your group compare with those for the class?
Mean:

Standard Deviation:

g. How does the histogram for your group compare to the class histogram?

h. Does the data shown in the histograms “fit” a normal curve?
What is the reason for your conclusion?

i. Create a box plot of the class data.

* Adapted from the Laboratory Activities in *Unit 19 Working With Statistics* CORD Applied Mathematics 1989; pp. 21-33.

Core Content

Cluster Title: Summarize and describe distributions.

Standard: 5. Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations.

MASTERY Patterns of Reasoning:

Conceptual:

Understand what an observation is (e.g. sample size, n size) and how it relates to numerical data sets.
Understand and explain why the number of observations is important to summarizing numerical data sets.

Procedural:

Find and report the total number of observations given a plot.

Representational:

Show where the number of observations is or can be represented in a data display (line plot, histogram, box plot) and explain why that representation is efficient or not efficient.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Understand how to collect data.
Understand how to find data sets in media such as newspapers and webpages.

Procedural:

Be able to count the number of observations.

Representational:

Represent data in line plots, histograms, and box plots.

Academic Vocabulary and Notation

Data set, n - size, Observation, Sample size

Instructional Strategies Used	Resources Used
Show examples of plots and ask students to find the total observations. Be sure to clarify the difference between intervals and observations.	http://www.ade.az.gov/standards/math/2010MathStandards/Gradelevel/MathGr6.pdf page 39

Assessment Tasks Used	
Skill-based Task Students will find total observations given different plots.	**Problem Task** Have students use data found in newspaper or other media to interpret total number of observations in that data set. Have them explain why the number of observations is important for that set of data.

Core Content

Cluster Title: Summarize and describe distributions.

Standard: 5. Summarize numerical data sets in relation to their context, such as by:

b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

MASTERY Patterns of Reasoning:

Conceptual:

Understand how the data was gathered.
Justify the appropriateness of the process used for data collection.
Understand the importance of the units used in the data sets.

Procedural:

Identify and describe the attribute being measured.
Describe how the data was gathered.

Representational:

Interpret labels given on the plot including horizontal and vertical axes, the number line, title, and legend.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Experience with horizontal and vertical axes.
Experience with units of measurement.

Procedural:

Set up a plot including the axes, the number line, the title, and the legend.

Representational:

Represent intervals within units of measure

Academic Vocabulary and Notation

Abbreviations for common measurements, Attribute, characteristic, investigation

Instructional Strategies Used	Resources Used
Bring in samples of plots from media and have students identify attributes.	http://www.ade.az.gov/standards/math/2010MathStandards/Gradelevel/MathGr6.pdf page 39

Assessment Tasks Used	
Skill-based Task Students will identify attributes and unit of measurement of a given data set.	**Problem Task** Students will gather data, and defend measurement technique and choice of unit.

Core Content

Cluster Title: Summarize and describe distributions.

Standard: 5. Summarize numerical data sets in relation to their context, such as by:

c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

MASTERY Patterns of Reasoning:

Conceptual:

Interpret a set of numerical data beyond measures of center and variability by noticing and describing patterns and deviations.
Understand the representation of a set of numerical data in the context of the data.
Understand mean absolute deviation.

Procedural:

Determine variability such as interquartile range (use mean absolute deviation as an extension).
Describe the overall pattern of data.
Describe any striking deviations from the overall pattern (outliers).

Representational:

Create and use data plots to interpret a set of data.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Understand measures of center and how they are used to interpret a set of data.
Understand measures of variability and how they are used to interpret a set of data.
Understand absolute value.

Procedural:

Calculate mean, median, and range of a data set.

Representational:

Represent data with plots

Academic Vocabulary and Notation	
1^{st} Quartile (Q_1), 2^{nd} Quartile (Q_2), 3^{rd} Quartile (Q_3), 4^{th} Quartile (Q_4), Box plot, Distribution, Dot plot, Histogram, Interquartile range, Upper quartile, Lower quartile, Median, Upper endpoint, Lower endpoint, Deviation	
Instructional Strategies Used	**Resources Used**
Have each student choose two to three books from the library and have them record the number of pages in each. Graph the data as a class on a line plot, then interpret the data and create a box plot. Use the box plot to show variation of data, and then interpret deviation from mean using specific data from the pages recorded.	http://www.deltastate.edu/docs/math/Mitchell3.pdf http://www.ade.az.gov/standards/math/2010MathStandards/Gradelevel/MathGr6.pdf pages 41-43
Assessment Tasks Used	
Skill-based Task Students will gather and summarize data using a box plot.	**Problem Task** They will also interpret differences in given values compared to the mean (deviation from the mean), noting patterns or deviations in the data display.

Core Content

Cluster Title: Summarize and describe distributions.

Standard: 5. Summarize numerical data sets in relation to their context, such as by:

d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

MASTERY Patterns of Reasoning:

Conceptual:

Understand how the context of the data gathered may determine the measures of center and variability chosen to interpret the data.
Understand how the shape of the data distribution may determine the choice of measure of center and variability.
Understand that measures of center emphasize different attributes for the data set. (e.g., mean income v. median income)

Procedural:

Choose the most appropriate measure of center (mean or median) and variability (range).

Representational:

Represent a set of numerical data in order to show the shape of the data distribution (e.g. in a dot plot).

Supports for Teachers

Critical Background Knowledge

Conceptual:

Know definitions of mean, median, and range.

Procedural:

Accurately plot a set of numerical data in a dot plot.

Representational:

Represent a set of numerical data in a dot plot (line plot).

Academic Vocabulary

Context, data distribution

Instructional Strategies Used	**Resources Used**
Expose students to data from a company. Students must choose the best measure of center for different situations (such as choosing between mean and median when showing profit or loss).	http://www.ohiorc.org/pm/math/richproblemmath.aspx?pmrid=62 This is a rich mathematical problem from the Ohio Resource Center which addresses all four subparts of standard 6.SP.5
Assessment Tasks Used	
Skill-based Task Given a data plot graph with background information, students will identify the context of the data and the measure of center used and will explain why that measure of center was chosen.	**Problem Task:** Interpret the following statement in terms of how the context of the data gathered could have influenced the shape of the data (how more music students were on the high end of the honors and awards and grades). "According to the National Education Longitudinal Study of 1988, music students received more academic honors and awards than non-music students. A higher percentage of music participants received As, As/Bs, and Bs than non-music participants." *(Source: NELS:88 First Follow-up, 1990, National Center for Education Statistics, Washington D.C.)*