Core Content

Cluster Title: Know that there are numbers that are not rational, and approximate them by rational numbers.
Standard: Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers, show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
Concepts and Skills to Master
• Know that real numbers that are not rational are irrational. • Understand that finite decimal expansions of irrational numbers are approximations. • Show that rational numbers have decimal expansions that repeat eventually. • Convert a decimal expansion, which repeats eventually, into a rational number.

Supports for Teachers

Critical Background Knowledge	
• Understand the subsets of the real number system (natural numbers, whole numbers, integers, rational numbers). • Convert rational numbers to decimals using long division (terminating and repeating) (7.NS.2d).	
Academic Vocabulary	
decimal expansion, repeating decimal, terminating decimal, rational, irrational, square root, $\sqrt{}$, π	
Suggested Instructional Strategies	**Resources**
• Use the Pythagorean Theorem with non-perfect squares to introduce irrational numbers (8.G.7). • Use the powers of ten technique: Let $n = 0.1\overline{6}$ So, $100n = 16.\overline{6}$ Subtract $-10n = -1.\overline{6}$ $90n = 15$ $\div 90 \quad \div 90$ $n = \frac{1}{6}$	

Sample Formative Assessment Tasks	
Skill-Based Task Convert $0.35\overline{2}$ to a fraction. Group the following numbers based on what you know about the number system: 5.3, $1.\overline{7}$, $\sqrt{10}$, 2, π, 4.010010001…	**Problem Task** Suppose you have a fraction with a denominator of 7. What is the longest string of non-repeating digits that will occur in the decimal expansions of the number? (Hint: Use the long division algorithm to show that for a denominator of *n*, there are only *n* possible remainders, 0 to *n*-1.)

Core Content

Cluster Title: Know that there are numbers that are not rational, and approximate them by rational numbers.
Standard: Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.
Concepts and Skills to Master
• Compare and order irrational numbers. • Place irrational numbers on a number line. • Use approximations of irrational numbers to estimate the value of expressions.

Supports for Teachers

Critical Background Knowledge	
• Compare and place rational numbers on a number line. • Approximate irrational numbers as fractions or decimals.	
Academic Vocabulary	
rational, irrational, decimal expansion, square root, $\sqrt{}$,π, truncating, rounding	
Suggested Instructional Strategies	**Resources**
• Construct the Wheel of Theodorus to create physical lengths of the square roots of the counting numbers. Transfer those lengths onto a number line. • Find increasingly accurate estimations for square roots of numbers by guess-and-check with a calculator.	
Sample Formative Assessment Tasks	
Skill-Based Task Place the following numbers on a number line: 5.3, $1.\overline{7}$, $\sqrt{10}$, 2, $\frac{\pi}{2}$ Find between which two integers $\sqrt{42}$ lies?	**Problem Task** Explain when each approximation of π (3.14, 3, $\frac{22}{7}$) is useful in calculating the circumference of a circle. Compare the answers you would get with each approximation. (Extension: Research how different cultures have approximated pi.)

Core Content

Cluster Title: Work with radicals and integer exponents.
Standard: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example: $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$.
Concepts and Skills to Master
• Know the properties of integer exponents. • Apply the properties of integer exponents to simplify and evaluate numerical expressions.

Supports for Teachers

Critical Background Knowledge	
• Understand exponents as repeated multiplication. (6.EE.1) • Compute fluently with integers (add, subtract, and multiply).	
Academic Vocabulary	
exponent, base, power, integer	
Suggested Instructional Strategies	**Resources**
• Use repeated multiplication and division to informally derive the exponent rules. • Have students examine equivalent numerical expressions with exponents.	Birch, David. *The King's Chessboard.*

Sample Formative Assessment Tasks	
Skill-Based Task Simplify: $\frac{5^7}{5^3}$ $\left(\left((2)^2\right)^2\right)^2$ -5^0 $2^{-2} \cdot 4$	**Problem Task** Explain why $3^5 \cdot 3^2 = 3^7$ and not 9^7. Write three expressions equivalent to $3^2 \cdot 9^2$.

Core Content

Cluster Title: Work with radicals and integer exponents.
Standard: Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
Concepts and Skills to Master
• Evaluate the square roots of small perfect squares and cube roots of small perfect cubes. • Represent the solutions to equations using square root and cube root symbols. • Understand that all non-perfect square roots and cube roots are irrational.

Supports for Teachers

Critical Background Knowledge	
• Understand and use inverse operations to solve equations.	
Academic Vocabulary	
square, square root, $\sqrt{\ }$, cube, cube root, $\sqrt[3]{\ }$, radical	
Suggested Instructional Strategies	**Resources**
• Use the geometric representations of square area and cube volumes and their relation to the side length. • Use the idea of inverse operations to introduce the concept of roots.	
Sample Formative Assessment Tasks	
Skill-Based Task If a square has an area of 9/16 square inches, what is the length of a side? If a cube has a volume of 0.125 cubic meters, what are the dimensions of the cube?	**Problem Task** Is the square root of a number always smaller than the number itself? Explain.

Core Content

Cluster Title: Work with radicals and integer exponents.
Standard: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.
Concepts and Skills to Master
• Estimate numbers as a product of a single digit and a power of ten. • Compare numbers expressed as a product of a single digit and a power of ten by a scale factor.

Supports for Teachers

Critical Background Knowledge	
• Understand properties of integer exponents. • Use and understand powers of ten and place value. • Estimate and round numbers.	
Academic Vocabulary	
powers of ten, estimate	
Suggested Instructional Strategies	**Resources**
• Use large and small real-life numbers (e.g. national debt). • Use base 10 blocks. • Write numbers as repeated multiplication or as the value of a power of 10 (e.g. $7 \cdot 10^3 \rightarrow 7 \cdot 1000$ *or* $7000 \rightarrow 7 \cdot 1000 \rightarrow 7 \cdot (10 \cdot 10 \cdot 10)$).	*Powersof10.com/film.* Eames Office, 2010.
Sample Formative Assessment Tasks	
Skill-Based Task The mass of the earth is 6×10^{24} kg. The mass of the moon is 7×10^{22} kg. How many times bigger is the mass of the earth than the mass of the moon? A proton has mass $2 \times 10^{-27} kg$ and an electron has mass $9 \times 10^{-31} kg$. How many times smaller is the electron than the proton?	**Problem Task** Provide two (2) original (not teacher-given) real-life situations that could be illustrated using powers of ten, one that describes a very small number and one that describes a very large number. Estimate how much larger one is than the other.

Core Content

Cluster Title: Work with radicals and integer exponents.
Standard: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.
Concepts and Skills to Master
• Add, subtract, multiply and divide with numbers expressed in scientific notation and decimal notation. • Represent very large and small quantities in scientific notation and use appropriate units. • Convert between decimal notation and scientific notation. • Interpret numbers expressed in scientific notation, including numbers generated by technology.

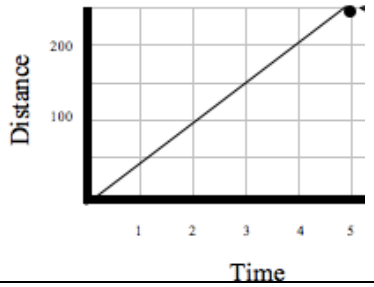
Supports for Teachers

Critical Background Knowledge	
• Use rules of exponents. • Understand powers of ten and place value.	
Academic Vocabulary	
scientific notation, decimal notation, power of ten, units of meaure	
Suggested Instructional Strategies	**Resources**
• Use examples found in science to create authentic reasons to use scientific notation.	
Sample Formative Assessment Tasks	
Skill-Based Task Evaluate and express your answers in scientific notation. • $3\times10^{5}+5.54\times10^{7}$ • $4.2\times10^{-2}-7.4\times10^{2}$ • $(3\times10^{8})(500)$ • $\frac{30}{1.5\times10^{-4}}$ Multiply 345,328,004 X 234 on your calculator and write the answer in scientific notation.	**Problem Task** Express your age at your last birthday in each of the following units: years, months, weeks, days, hours, minutes and seconds. Which values would be useful to write in scientific notation? Justify your reasoning. Compare your age to that of the 4.5 billion year old earth.

Core Content

Cluster Title: Understand the connections between proportional relationships, lines, and linear equations.
Standard: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
Concepts and Skills to Master
• Graph a proportional relationship given a table, equation or contextual situation. • Recognize unit rate as slope and interpret the meaning of the slope in context. • Recognize that proportional relationships include the point (0,0). • Compare different representations of two proportional relationships represented as contextual situations, graphs, or equations.

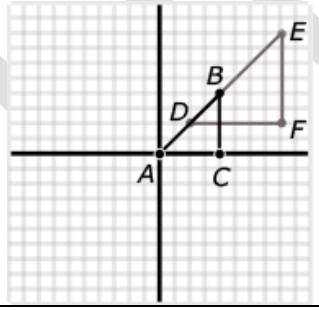
Supports for Teachers

Critical Background Knowledge	
• Use an equation to create a table and plot values on the coordinate axes. • Understand and calculate unit rates.	
Academic Vocabulary	
slope, unit rate, rate of change, *m* (slope)	
Suggested Instructional Strategies	**Resources**
• Categorize linear relationships represented in multiple ways as either proportional or not proportional. • Plot relationships generated from real-life proportional examples (e.g.,shopping) and interpret the slope in the context of the situation.	*illuminations.nctm.org/ActivityDetail.aspx?ID=124.* NCTM. (9-12 Activity: Two Terrains)
Sample Formative Assessment Tasks	
Skill-Based Task This is a graph of Susie's trip to John's cabin. John made the same trip in 4 hours. Compare their rates. Who travelled at a faster rate? How do you know?  	**Problem Task** Give examples of relationships that are proportional and relationships that are linear, but not proportional.

Core Content

Cluster Title: Understand the connections between proportional relationships, lines, and linear equations.
Standard: Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*.
Concepts and Skills to Master
• Determine the slope of a line as the ratio of the leg lengths of similar right triangles. • Explain why the slope is the same between any two distinct points on a line using similar right triangles. • Write an equation in the form $y = mx + b$ from a graph of a line on the coordinate plane.

Supports for Teachers

Critical Background Knowledge	
• Recognize similar triangles and know that they have proportional sides.	
Academic Vocabulary	
similar triangles, *m* (slope), *b* (y-intercept), linear, right triangle, origin, rise, run	
Suggested Instructional Strategies	**Resources**
• Have students draw many right triangles with the hypotenuse on the line and compare the ratio of the leg lengths. • Discuss the value of choosing easy-to-read points when determining slope. • Relate negative slopes to the change in *y* as *x* increases.	*illuminations.nctm.org/ActivityDetail.aspx?ID=144.* NCTM. (3-8 Activity: Chairs)
Sample Formative Assessment Tasks	
Skill-Based Task Points A, D, B, and E are collinear. Show that $\overline{AB}$ and $\overline{DE}$ have the same slope. 	**Problem Task** How is it possible to have similar triangles that do not yield the same slope?

Core Content

Cluster Title: Analyze and solve linear equations and pairs of simultaneous linear equations.
Standard: Solve linear equations in one variable. a) Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b) Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
Concepts and Skills to Master
• Identify and provide examples of equations that have one solution, infinitely many solutions or no solutions. • Solve multistep linear equations with rational coefficients and variables on both sides.

Supports for Teachers

Critical Background Knowledge	
• Solve one- and two-step equations (7.EE.4a). • Use properties of algebra to simplify algebraic expressions.	
Academic Vocabulary	
solve, variable, order of operations, solution, like terms, distributive property	
Suggested Instructional Strategies	**Resources**
• Build on the equations solved in seventh grade and move toward increased fluency and procedural skill in solving more complex linear equations. • Examine solutions in the context of the original equation. • Consider teaching unique solutions, no solutions, and infinitely many solutions with 8.EE.8.	
Sample Formative Assessment Tasks	
Skill-Based Task Solve the following equations and identify the number of solutions: • $3(x+7)=10$ • $2(x-5)=\frac{1}{2}(4x+6)$ • $2(x+3)=2x+6$	**Problem Task** Create equations that would result in one solution, no solutions, or infinitely many solutions. What is it about the structure of the original equation that reveals the number of solutions?

Core Content

Cluster Title: Analyze and solve linear equations and pairs of simultaneous linear equations.
Standard: Analyze and solve pairs of simultaneous linear equations. a) Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b) Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.* c) Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.
Concepts and Skills to Master
• Identify and provide examples of systems of equations that have one solution, infinitely many solutions or no solutions. • Solve a system of equations algebraically. • Estimate solutions by graphing systems of equations. • Create and utilize systems of linear equation to model real-world situations.

Supports for Teachers

Critical Background Knowledge	
• Graph linear equations. • Solve a one variable equation. • Solve for a specified variable in an equation.	
Academic Vocabulary	
elimination, substitution, solution, intersection, solve, system of linear equations	
Suggested Instructional Strategies	**Resources**
• Compare cell phone plans, bike rental rates, or repair costs using equations. • Explore cases where one solution strategy is more efficient than another.	

Sample Formative Assessment Tasks	
Skill-Based Task Solve the system of equation. $2x + 3y = 4$ $-x + 4y = -13$	**Problem Task** You have been hired by a cell phone company to create two rate plans for customers, one that benefits customers with low usage and one that benefits customers with high usage. At 500 minutes, both plans should be within $5 of each other. Design a presentation showing two plans that will meet these requirements, including graphs and equations.

Core Content

Cluster Title: Define, evaluate, and compare functions.
Standard: Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
Concepts and Skills to Master
• Understand that functions describe relationships where one variable determines a unique value of the other. • Recognize a graph of a function as the set of ordered pairs consisting of an input and corresponding output.

Supports for Teachers

Critical Background Knowledge	
• Graph ordered pairs on the coordinate plane. • Evaluate expressions for a given value.	
Academic Vocabulary	
function, input, output, dependent, independent	
Suggested Instructional Strategies	**Resources**
• Explore functions that arise from real-life relationships where one variable determines a unique value of another. • Use a variety of representations to have students identify functions and relations that are not functions.	*illuminations.nctm.org/ActivityDetail.aspx?ID=215* NCTM. (9-12 Activity: Function Matching) Lloyd, Gwendolyn, et al. *Developing Essential Understanding of Expressions, Equations, and Functions Grades 6-8*. NCTM, 2011.

Sample Formative Assessment Tasks	
Skill-Based Task • Could the set of ordered pairs, (2,5), (3,5), (4,6), (2,8), (6,7) describe the number of seconds since you left home and the number of meters you've walked? Is this a function? Justify your answers. • Does the set of students in the classroom and their birthdays represent a function? Justify your answer.	**Problem Task** Find three examples of relationships in the real world that can be represented by functions and three relationships that are not functions.

Core Content

Cluster Title: Define, evaluate, and compare functions.
Standard: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.
Concepts and Skills to Master
• Compare two linear functions each represented a different way and describe similarities and differences in slopes, *y*-intercepts, and values.

Supports for Teachers

Critical Background Knowledge	
• Determine slopes and *y*-Intercepts.	
Academic Vocabulary	
slope, intercept, rate of change, function, linear, non-linear	
Suggested Instructional Strategies	**Resources**
• Given one representation of a function, create the others. • Put students in small groups. Give groups scenarios and ask each group to create a different representation of the scenario (table, equation, graph). • Identify attributes (slope, *y*-intercept, values) of a function in its equation, graph, or a table.	Lloyd, Gwendolyn, et al. *Developing Essential Understanding of Expressions, Equations, and Functions Grades 6-8*. NCTM, 2011.
Sample Formative Assessment Tasks	
Skill-Based Task Is $y=2(x+5)$ the same as the function described as "twice a quantity plus 5"?	**Problem Task** Billy argues that the equation $y=4x+5$ is equivalent to the equation of the line that goes through (2,6) and (3,10). How did he arrive at this conclusion? Is he correct? Justify your answer.

Core Content

Cluster Title: Define, evaluate, and compare functions.

Standard: Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

Concepts and Skills to Master

- Distinguish between linear and non-linear functions given their algebraic expression, a table, or a graph.
- Recognize functions written in the form $y = mx + b$ are linear and that every linear function can be written in the form $y = mx + b$.

Supports for Teachers

Critical Background Knowledge

- Generate and plot ordered pairs from an equation.
- Understand linear slope as a constant rate of change.

Academic Vocabulary

collinear, linear, nonlinear

Suggested Instructional Strategies

- Examine constant and non-constant rates of change in tables of values.
- Explore growing patterns generated from a variety of contexts to explore linear and nonlinear relationships.

Resources

Sample Formative Assessment Tasks

Skill-Based Task

Determine which of the following equations are linear:

$y = x^2 + 5x + 6$ $\quad y = x(2 + x)$

$y = x^3$ $\quad y = 7x + 6$

$y = \frac{1}{x}$ $\quad y = \frac{x}{2}$

Problem Task

Hermione argues that the table below represents a linear function. Is she correct? How do you know?

x	2	4	8	16
y	1	3	5	7

Core Content

Cluster Title: Use functions to model relationships between quantities.

Standard: Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Concepts and Skills to Master

- Determine and interpret the initial value and rate of change given two points, a graph, a table of values, a geometric representation, or a story problem (verbal description) of a linear relationship.
- Write the equation of a line given two points, a graph, a table of values, a geometric representation, or a story problem (verbal description) of a linear relationship.

Supports for Teachers

Critical Background Knowledge

- Understand the meaning of slope and *y*-intercept.
- Write an equation as $y = mx + b$ given a graph.

Academic Vocabulary

linear relationship, *y*-intercept, slope

Suggested Instructional Strategies	Resources
• Use a real-world application to generate a table of values. Use the table to construct a function that models the relationship. • Connect to other standards in the Expressions and Equations Domain.	*illuminations.nctm.org.* NCTM.

Sample Formative Assessment Tasks

Skill-Based Task

- The student council is planning a ski trip to Sundance. There is a $220 deposit for the lodge and the tickets will cost $70 per student. Construct a function, build a table, and graph the data showing how much it will cost for the students' trip.
- Find the equation of the line that goes through (3,5) and (-5,7).

Problem Task

Wally created the table below for a function he knows to be linear. He thinks something must be wrong with his table because he can't find the original function from the table. Find the error and the original function. Explain your strategy for finding the error.

3.2	6.4	9.6	12.8	16	19.2	22.4	25,6
17.8	30.6	43.4	56.2	66	81.8	94.6	107.4

Core Content

Cluster Title: Use functions to model relationships between quantities.
Standard: Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
Concepts and Skills to Master
• Describe attributes of a function by analyzing a graph. • Create a graphical representation given the description of the relationship between two quantities.

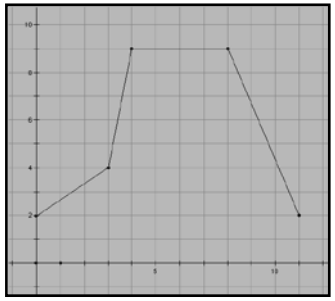
Supports for Teachers

Critical Background Knowledge	
• Understand basic graphing conventions. • This standard is a good place to begin exploring function.	
Academic Vocabulary	
increasing and decreasing rates of change, linear, nonlinear, initial value	
Suggested Instructional Strategies	**Resources**
• Tell a story based on a graph. • Given a story, draw a graph.	Friel, Susan, et. al. *Navigating through Algebra.* NCTM, 2001.

Sample Formative Assessment Tasks

Skill-Based Task

The following is a graph of the heart rate of a man running on a treadmill. When is his heart rate changing at the greatest rate? What is happening when the graph is horizontal? Does the decreasing graph show a constant rate of change?



Problem Task

Graph your distance from the school over time for the hours from 6 a.m. to 9 a.m.

Core Content

Cluster Title: Understand congruence and similarity using physical models, transparencies, or geometry software.
Standard: Verify experimentally the properties of rotations, reflections, and translations: a) Lines are taken to lines, and line segments to line segments of the same length. b) Angles are taken to angles of the same measure. c) Parallel lines are taken to parallel lines.
Concepts and Skills to Master
• Verify that congruence of line segments and angles is maintained through rotation, reflection, and translation. • Verify that lines remain lines through rotation, reflection, and translation. • Verify that when parallel lines are rotated, reflected, or translated, each in the same way, they remain parallel lines.

Supports for Teachers

Critical Background Knowledge	
• Know definitions and properties of angles, segments, lines, and parallel lines. • Measure angles and line segments.	
Academic Vocabulary	
line, angle, segment, parallel line, rigid motion, congruent, center of rotation, line of reflection, rotation, reflection, translation, transformation	
Suggested Instructional Strategies	**Resources**
• Use dynamic geometry software or Excel to explore properties of rotations, reflections, and translations. • Use a coordinate grid and apply rules such as (-*x, y*) or (*x, y*+7) to the coordinates of a given figure. Compare the resulting image to the original.	*illuminations.nctm.org/LessonDetail.aspx?ID=U139*. NCTM. (9-12 Lessons: Symmetries II)
Sample Formative Assessment Tasks	
Skill-Based Task Verify that a triangle, when rotated, remains a triangle.	**Problem Task** Create a tessellation using rotations, reflections, and translations.

Core Content

Cluster Title: Understand congruence and similarity using physical models, transparencies, or geometry software.
Standard: Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
Concepts and Skills to Master
• Understand that the congruency of two dimensional figures is maintained while undergoing rigid transformations. • Describe the transformation of a figure as a rotation, reflection, translation or a combination of transformations.

Supports for Teachers

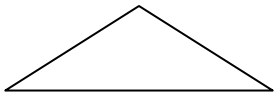
Critical Background Knowledge	
• Identify rotations, reflections, and translations with lines, segments and angles.	
Academic Vocabulary	
rotation, reflection, translation, congruent, center of rotation, line of reflection, angle of rotation	
Suggested Instructional Strategies	**Resources**
• Use geometry software to explore rotations, reflections and translations of two-dimensional figures. • Use digital photographs with at least two congruent shapes and discuss the needed transformations to map one to the other.	*http://illuminations.nctm.org/LessonDetail.aspx?ID=U139.* NCTM. (9-12 Lessons: Symmetries II)

Sample Formative Assessment Tasks

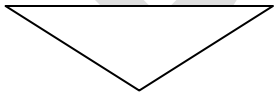
Skill-Based Task

Triangle *x* was transformed to *x*'. Describe the sequence of transformations that was used to show that Triangle *x* is congruent to Triangle *x*'.

Triangle *x*

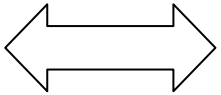


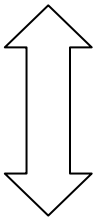
Triangle *x*'



Problem Task

Find at least two different ways to describe the transformation(s) that map the first figure onto the second.





Core Content

Cluster Title: Understand congruence and similarity using physical models, transparencies, or geometry software.
Standard: Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
Concepts and Skills to Master
• Understand how to dilate, translate, rotate, and reflect two-dimensional figures on the coordinate plane. • Describe the effects of dilations, translations, rotations, and reflections using coordinate notation. • Given an image and its transformed image, use coordinate notation to describe the transformation.

Supports for Teachers

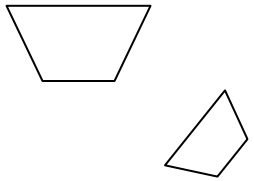
Critical Background Knowledge	
• Plot or identify points on the coordinate plane.	
Academic Vocabulary	
transformation, coordinate, dilation, rotation, reflection, translation, image, center of rotation, line of reflection, angle of rotation	
Suggested Instructional Strategies	**Resources**
• Have students take pictures of transformations they see in the world around them, then overlay the picture with the coordinate plane and describe the transformation using coordinate notation. • Use a coordinate grid and apply rules such as (-*x*, *y*) or (*x*, *y*+7) to the coordinates of a given figure. Compare the resulting image to the original.	*shodor.org.* Shodor. The Transmographer

Sample Formative Assessment Tasks	
Skill-Based Task Given a triangle with vertices at (5, 2), (-7, 8) and (0, 4), find the new vertices of the triangle after undergoing the transformation described as follows: $(x, y) \rightarrow (x+6, y-3)$	**Problem Task** Given an original shape and its image on a coordinate plane, determine the rule or rules that translated the original to the resulting image. The vertices of Triangle A are (1,0), (1,1), (0,0) and Triangle A' are (2,1), (2,2), (3,1). Describe the series of transformations performed on Triangle A that result in Triangle A'.

Core Content

Cluster Title: Understand congruence and similarity using physical models, transparencies, or geometry software.
Standard: Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
Concepts and Skills to Master
• Understand that any combination of transformation will result in similar figures. • Describe the sequence of transformations needed to show how one figure is similar to another. • Make dilations of figures by a given scale factor.

Supports for Teachers

Critical Background Knowledge	
• Perform rotations, translations, reflections, and dilations. • Understand proportions.	
Academic Vocabulary	
similar, similarity, dilation, rotation, reflection, translation, transformation	
Suggested Instructional Strategies	**Resources**
• Use Patty Paper transformations. • Use dynamic geometry software to make transformations and compare transformations.	
Sample Formative Assessment Tasks	
Skill-Based Task. Which of the following transformations will result in a similar figure? A) $(3x, 2y)$ B) $(-x+2, y-2)$ C) $(5x, y+5)$ D) $(3x, x+y)$	**Problem Task** List the sequence of transformations that verifies the similarity of the two figures. 

Core Content

Cluster Title: Understand congruence and similarity using physical models, transparencies, or geometry software.
Standard: Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.
Concepts and Skills to Master
• Understand that the measure of an exterior angle of a triangle is equal to the sum of the measures of the non-adjacent angles. • Know that the sum of the angles of a triangle equals 180^{o}. • Determine the relationship between corresponding angles, alternate interior angles, alternate exterior angles, vertical pairs, and supplementary pairs when parallel lines are cut by a transversal. • Recognize that if two triangles have two congruent angles, then they are similar triangles (angle-angle).

Supports for Teachers

Critical Background Knowledge	
• Understand the definition of similar figures. • Know how to measure angles.	
Academic Vocabulary	
corresponding angles, alternate interior angles, alternate exterior angles, consecutive interior angles, supplementary pairs, vertical pairs, transversal, adjacent, non-adjacent, exterior angle of a triangle, remote interior angles of a triangle	
Suggested Instructional Strategies	**Resources**
• Use a series of transformations of triangles to produce parallel lines and examine the properties of resultant angles and triangles. 	

Sample Formative Assessment Tasks	
Skill-Based Task Identify and name sets of angles of parallel lines cut by a transversal and tell which are congruent.	**Problem Task** The streets 400 E and 900 E run north and south. Euclid Drive cuts both of these streets at an angle from SE to NW. Pythagoras Way passes through all three streets SW to NE. Are all possible triangles created by the intersection of the streets similar? Justify.

Core Content

Cluster Title: Understand and apply the Pythagorean Theorem.
Standard: Explain a proof of the Pythagorean Theorem and its converse.
Concepts and Skills to Master
• Know that in a right triangle $a^2 + b^2 = c^2$ (the Pythagorean Theorem). • Understand and explain a proof of the Pythagorean Theorem. • Understand and explain a proof of the converse of the Pythagorean Theorem.

Supports for Teachers

Critical Background Knowledge	
• Understand the relationship between a and a^2, b and b^2, and c and c^2. • Understand the relationship between squares and square roots.	
Academic Vocabulary	
right triangle, leg, hypotenuse, square, Pythagorean Theorem	
Suggested Instructional Strategies	**Resources**
• Consider introducing this with an application regarding distance. • Explore various proofs of the Pythagorean Theorem and discuss the logic within each.	
Sample Formative Assessment Tasks	
Skill-Based Task Explain the logical reasoning behind a proof of the Pythagorean Theorem.	**Problem Task** Investigate the historical context of one of the proofs of the Pythagorean Theorem and present the proof in context to the class.

Core Content

Cluster Title: Understand and apply the Pythagorean Theorem.
Standard: Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
Concepts and Skills to Master
• Use the Pythagorean Theorem to solve for a missing side of a right triangle given the other two sides. • Use the Pythagorean Theorem to solve problems in real-world contexts, including three-dimensional contexts.

Supports for Teachers

Critical Background Knowledge	
• Solve an equation using squares and square roots. • Use rational approximations of irrational numbers to express answers.	
Academic Vocabulary	
right triangle, leg, hypotenuse, Pythagorean Theorem, square, square root, $\sqrt{\ }$	
Suggested Instructional Strategies	**Resources**
• Find and solve right triangles in career situations such as construction.	

Sample Formative Assessment Tasks	
Skill-Based Task If the height of a cone is 10 meters and the radius is 6 meters, what is the slant height?	**Problem Task** TV's are measured along their diagonal to find their dimension. How does a 52-inch HD (wide-screen) TV compare to a traditional 52-inch (full screen) TV?

Core Content

Cluster Title: Understand and apply the Pythagorean Theorem.
Standard: Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
Concepts and Skills to Master
• Calculate the distance between two points in a coordinate system using the Pythagorean Theorem.

Supports for Teachers

Critical Background Knowledge	
• Use the Pythagorean Theorem to solve for the hypotenuse of a right triangle.	
Academic Vocabulary	
right triangle, distance formula, leg, hypotenuse, Pythagorean Theorem, square, square root	
Suggested Instructional Strategies	**Resources**
• Overlap a map with a coordinate grid and use the Pythagorean Theorem to find the distance between two locations. • Investigate the relationship between the Pythagorean Theorem and the distance formula. • Use the Pythagorean Theorem to explore and categorize triangles and quadrilaterals on a coordinate system.	
Sample Formative Assessment Tasks	
Skill-Based Task Using the Pythagorean Theorem, find the distance between (4,2) and (7,10).	**Problem Task** List 3 coordinate pairs that are 5 units away from the origin in the first quadrant. Describe how to find the points and justify your reasoning. (Note: Points on the axes are not in the quadrant.)

Core Content

Cluster Title: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.
Standard: Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
Concepts and Skills to Master
• Understand when and how to use formulas for volume of cones, cylinders, and spheres. • Apply volume formulas to real-world problems.

Supports for Teachers

Critical Background Knowledge	
• Represent rational approximations of irrational numbers such as pi. • Solve equations involving square roots and cube roots. • Understand that volume is measured in cubic units.	
Academic Vocabulary	
pi, π, radius, slant height, height, volume, hemisphere, diameter	
Suggested Instructional Strategies	**Resources**
• Use physical models to compare volumes of cones and cylinders using water or rice. • Derive the formula for cylinders using physical models. • Explore questions such as "why is the volume of a cone 1/3 the volume of a cylinder of the same base?" • Compare and contrast the formulas for cones, cylinders, and spheres.	

Sample Formative Assessment Tasks	
Skill-Based Task A silo has 1500 ft^3 of grain. The grain fills the silo to 20 ft. in height. What is the radius of the silo? What is the relationship between the volume of a cylinder and a cone with the same radius and height?	**Problem Task** What does the height of the cone need to be so that one spherical scoop of ice cream with the same radius as the cone won't overflow if it all melts? A Christmas tree is 7 feet tall with a 5-foot diameter at the base, with one foot between the floor and the lowest branch. How far up the tree should your first of two strings of lights end so that you will have enough lights to evenly fill the Christmas tree?

Core Content

Cluster Title: Investigate patterns of association in bivariate data.

Standard: Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

Concepts and Skills to Master

- Collect, record, and construct a set of bivariate data using a scatter plot.
- Determine whether the relationship between bivariate data is approximately linear or nonlinear by examination of a scatter plot.
- Interpret patterns on a scatter plot such as clustering, outliers, and positive, negative, or no association.

Supports for Teachers

Critical Background Knowledge

- Basic understanding of graphing and linear relationships.

Academic Vocabulary

bivariate data, scatter plot, outlier, clustering, positive association, negative association, linear, nonlinear

Suggested Instructional Strategies	**Resources**
• Use data from multiple sources to construct a scatter plot. • Compare and contrast scatter plots with various degrees of association.	

Sample Formative Assessment Tasks

Skill-Based Task

Construct a scatter plot and describe any association you observe.

Height	Hand span
70 in	10 in
72 in	9.5 in
61 in	8 in
62 in	9.5 in
68 in	9 in

Problem Task

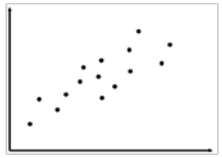
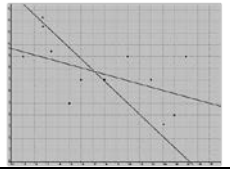
Compare class test scores to hours of television watched.

- Predict whether there is a positive, a negative or no association.
- Collect data and make a scatter plot.
- Compare your prediction to the scatter plot result.
- Describe any association you observe.
- Interpret your findings and explain your reasoning.

Core Content

Cluster Title: Investigate patterns of association in bivariate data.
Standard: Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
Concepts and Skills to Master
• Recognize that straight lines can be used on scatter plots to model the relationship between two quantitative variables. • Place a straight line on a scatter plot that closely fits the data points. • Judge how well the trend line fits the data by looking at the closeness of the data points.

Supports for Teachers

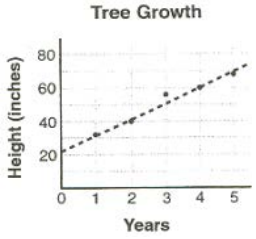
Critical Background Knowledge	
• Graph a linear relationship.	
Academic Vocabulary	
linear association, scatter plot, trend line	
Suggested Instructional Strategies	**Resources**
• Given a scatter plot and an uncooked spaghetti noodle, have students place the noodle on the scatter plot to create the trend line. • Use technology to create scatter plots with a trend line, and then observe changes to the trend line as data points are deleted or added.	*gapminder.org.* Gapminder Foundation. *nlvm.usu.edu.* Utah State University, 2010. (scatterplot)
Sample Formative Assessment Tasks	
Skill-Based Task Draw a trend line and describe the closeness of the fit: 	**Problem Task** Which line is the best fit for the data? Justify your answer. 

Core Content

Cluster Title: Investigate patterns of association in bivariate data.
Standard: Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.
Concepts and Skills to Master
• Use the equation of a linear model to solve problems. • Interpret the meaning of the slope as a rate of change and the meaning of the *y*-intercept in context given bivariate data.

Supports for Teachers

Critical Background Knowledge	
• Graphing linear equations	
Academic Vocabulary	
rate of change, slope, intercept	
Suggested Instructional Strategies	**Resources**
• Find linear models of approximately linear data in newspapers, magazines, or on the Internet and discuss the meaning of the slope and intercepts in context. • Create linear models using data from other disciplines such as science, social studies, or careers and describe the meaning of their slopes and intercepts in context.	

Sample Formative Assessment Tasks	
Skill-Based Task Find and interpret the slope and *y*-intercept of the trend line. Create an equation and use it to predict how much a tree will grow in three years.  	**Problem Task** Create a story problem that uses a line with a slope of 2/5 and *y*-intercept of 3. Describe the meaning of the slope and *y*-intercept in the context of the problem.

Core Content

Cluster Title: Investigate patterns of association in bivariate data.

Standard: Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

Concepts and Skills to Master

- Construct a two-way frequency table of categorical data.
- Interpret and describe relative frequencies for possible associations from a two-way table.

Supports for Teachers

Critical Background Knowledge

None

Academic Vocabulary

relative frequency, categorical data, frequency, two-way table, associations

Suggested Instructional Strategies	Resources
• Explore questions such as: o Are honor students more likely to wear athletic shoes? o Is gender related to video console ownership? o Are eighth graders more or less likely to have a cell phone based on birth order (youngest, middle, or oldest)?	

Sample Formative Assessment Tasks

Skill-Based Task

Are boys or girls more likely to be in band?

	Band	No Band	Total
Girls	10	7	17
Boys	9	2	11
Total	19	9	28

Problem Task

Construct a two-way table to display data from two or more categories. Explain why you believe there is or is not an association between the two variables.

Honors Core Content

Cluster Title: Fair Division & Apportionment
Standard: Understand the concepts and applications of fairness and apportionment.
Concepts and Skills to Master
• Understand the concept of fairness as equal division of like or unlike objects and apportionment as proportional division of like objects. • Understand that fair division may be discrete or continuous while apportionment must be discrete. • Determine mathematical criteria for fairness, using both quantitative and qualitative measures. • Determine mathematical criteria for apportionment. • Understand the use of apportionment in the U.S. political system.

Guiding Instructional Questions
• If everyone gets the same thing, is that always fair? • How is the number of representatives a state gets determined? • How can 17 students fairly divide 3 pizzas, each with 8 slices, without cutting additional pieces? • In which situations is apportionment preferable to fair division? • How is apportionment used in U.S. Presidential elections?
Instructional Strategies
• Conduct a mock negotiation using principles of fairness and well posed arguments. • Explore various methods of apportionment • Compare different methods of apportionment such as Hamilton's Method vs. Jefferson's Method. • Fairly divide a budget between 5 clubs at your school when each has need for more than 1/5 of the budget. • Give small groups of students something (candy, objects, etc.) to divide fairly with no guidelines to develop the concept of fair division. • Find two different ways to fairly divide a scalene right triangle into 3 equal parts. • Study apportionment in connection with current elections.

Honors Core Content

Cluster Title: Voting Theory
Standard: Examine different methods of voting.
Concepts and Skills to Master
• Understand different voting systems. • Determine the results of an election using various voting systems.

Guiding Instructional Questions
• What issues does "majority rule" raise? • Why might different voting systems produce different outcomes? • How might knowing the voting method change the way a voter would vote? • How might knowing the voting method change the way a candidate would run a campaign? • Is there a better way to conduct an election for president than the Electoral College?
Instructional Strategies
• Compare and contrast the results of various voting methods (e.g. plurality, approval, ranking, run-off). • Conduct class votes on various topic(s) and compute the results using various methods. • Research how different countries conduct elections and count votes and compare them to the way the U.S. votes. • Write a letter to the principal arguing for a method of voting on class elections. • Consider covering this topic in conjunction with current or most recent state/national election results.

Honors Core Content

Cluster Title: Sets
Standard: Understand sets and use set notation to communicate mathematical ideas.
Concepts and Skills to Master
• Understand the definition and classifications of sets and subsets. • Use the element set notation for union $\cup$, element $\in$, intersection $\cap$, non-element $\notin$, null set Ø. • Find unions and intersections of sets given a description, set notation, or a graph.

Guiding Instructional Questions
• How can set notation be used to communicate mathematical ideas? • Are Ø, 0, and "no solution" equivalent? • Do R (all real numbers) and "infinitely many solutions" mean the same thing?
Instructional Strategies
• Use set notation for Real [R], Integer [Z], Natural [N], Whole [W], Prime [P], Rational [Q], Irrational [I] numbers. • Create Venn Diagrams to indicate sets, subsets, intersections, and unions. • Explore the concept of closure in sets. • Compare and contrast intersections of sets and unions of sets.

Honors Core Content

Cluster Title: 3-D Graphing & Graph Theory
Standard: Use graphing techniques to model situations that extend beyond the coordinate plane.
Concepts and Skills to Master
• Graph points and line segments in a three-dimensional coordinate system. • Use Euler Circuits to describe and solve problems involving network paths. • Use Hamiltonian Circuits to describe and solve problems involving network paths.

Guiding Instructional Questions
• How do computer programmers draw 3-D spaces on a 2-D screen? • How can a delivery truck drive an efficient route? • How can a snowplow drive an efficient route? • How does the efficient route for a delivery truck differ from that of a snowplow? • What is operations research?
Instructional Strategy
• Use technology to construct 3-D graphs. • Compare historical computer game graphics to modern computer game graphics • Use puzzles and classic mathematics problems such as the Konigsberg Bridge to explore Euler Circuits and continuous paths. • Create circuits that describe contextual problems. • Explore sociological behavior using networks. • Compare and contrast Euler and Hamiltonian Circuits for the same networks.