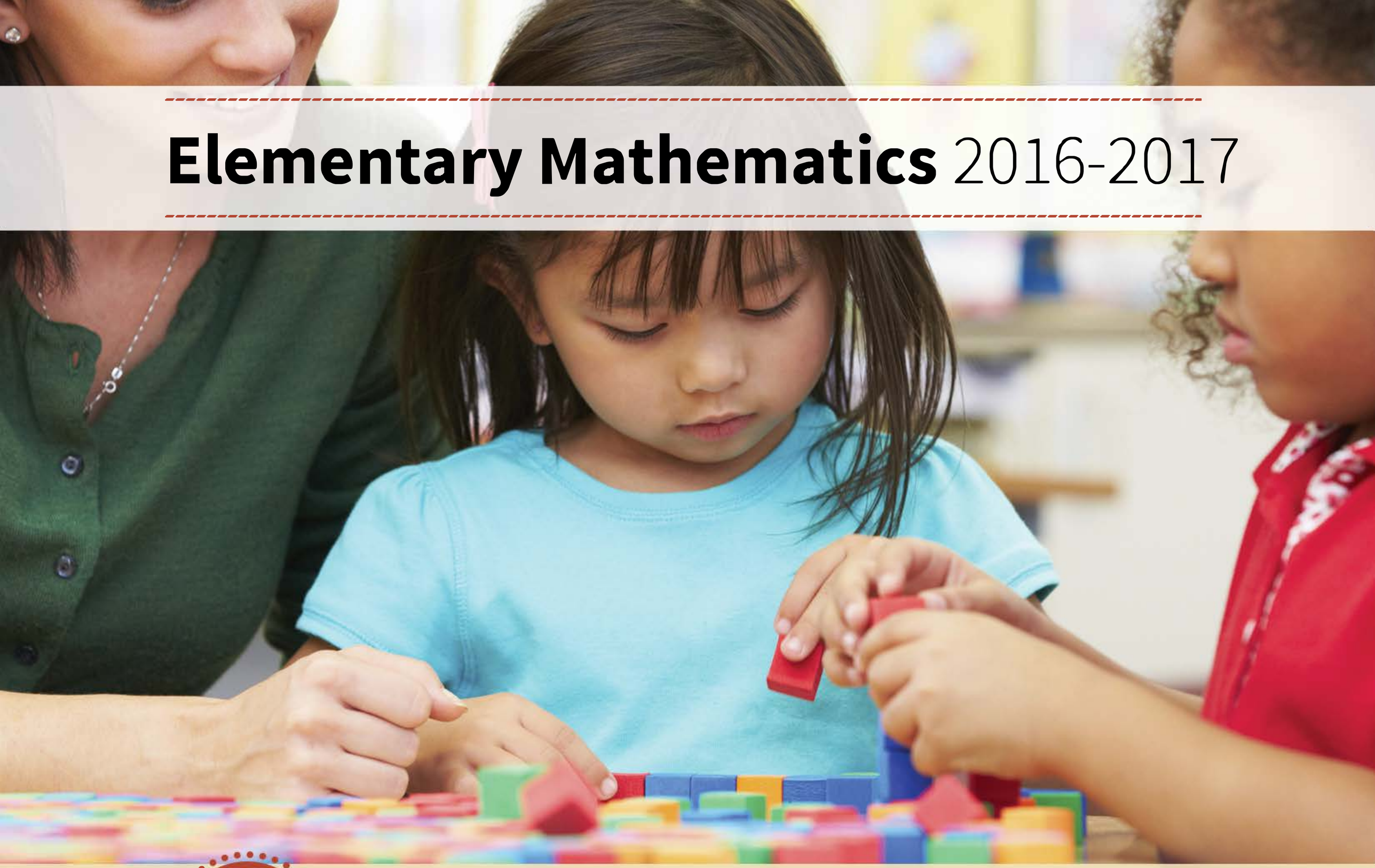
Elementary Mathematics 2016-2017



5th Grade

CANYONS
School District

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ENVISION MATH CURRICULUM MAP
CANYONS SCHOOL DISTRICT
2016-2017

Curriculum Mapping Purpose

Canyons School District's curriculum math maps are standards-based maps driven by the Utah Core State Standards for Mathematics and implemented using Pearson enVisionMATH 2.0. Student achievement is increased when both teachers and students know where they are going, why they are going there, and what is required of them to get there. The additional instructional days were intentionally built into the map to allow teachers to go into more depth on concepts and allow flexible pacing based on student need. Supporting resources for these additional days can be found in the General Information section.

Curriculum Maps are a tool for:

- **ALIGNMENT:** Provides support and coordination between concepts, skills, standards, curriculum, and assessments
- **COMMUNICATION:** Articulates expectations and learning goals for students
- **PLANNING:** Focuses instruction and targets critical information
- **COLLABORATION:** Promotes professionalism and fosters dialogue between colleagues about best practices in both instruction and assessment.
- **SCAFFOLDED INSTRUCTION AND GROUPING STRUCTURES:** The organization of a scaffolded classroom includes whole group, small group (e.g., teacher-led skill-based, cooperative learning), partner, and independent work where students are provided support towards mastery. As students assume more responsibility for the learning, gradual support is decreased in order to shift the responsibility for learning from the teacher to the students.

General Information

Pacing

This curriculum map provides guidance for intertwining the Utah Core Math Standards and the enVision 2.0 curriculum. Following the map will allow students to access all core standards by the end of the year. To support students' mastery of the standards, targeted standard clusters have been identified. Attending to these targeted standards will allow teachers to focus instruction for the given topic and better assess students' understanding of each standard.

Intentional Planning

For each domain, the map specifies both procedural checks and application tasks. These tasks represent what students should know and be able to do after instruction. Understanding these tasks will assist with designing instruction around targeted standards and critical areas.

- **Procedural Check***:* The purpose of the procedural check is to identify if students have the basic procedural understanding of the mathematical concept being highlighted.
- **Application Task**: The purpose of the application task is to assess student ability to understand and apply the skill with a heightened level of depth and complexity.

Critical Areas for Conceptual Understanding

In addition to targeted standards, critical areas have been identified and are highlighted in blue within the scope and sequence of the map. Students are expected to demonstrate a conceptual understanding of these critical areas in order to be prepared for future grades. Additional instructional days have been scheduled into the scope and sequence to provide additional time for increasing conceptual understanding of the standards. Conceptual understanding requires a focus of depth and complexity which may go beyond the enVision lessons. The following resources may be useful for extending instruction to address depth of knowledge demands of the standards.

Online:

Illustrative Mathematics: Mathematical tasks aligned to the standards https://www.illustrativemathematics.org
Inside Mathematics: More mathematical tasks aligned to the standards
http://www.insidemathematics.org/index.php/tools-for-teachers
Illuminations: Lessons, interactives, and web links to support math instruction. http://illuminations.nctm.org

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Print Resources:

Elementary and Middle School Mathematics: Teaching Developmentally by John A. Van De Walle

Assessment

Throughout the enVision 2.0 curriculum there are many opportunities to check for understanding with items such as the Quick Check, Do You Understand? Show Me, and Guided Practice. In addition, each topic ends with a Topic Assessment that can be given digitally or paper/pencil as well as a Performance Assessment.

Focused Review

It is critical to provide an ongoing review of previously taught concepts and skills. Teacher-directed, interactive reviews daily are ideal to assess student learning and inform instruction. Daily Common Core Review is provide daily within the enVisionMATH 2.0 program and may be used to provide a cumulative review. The math block allocates 5-10 minutes for a daily, focused review.

Homework

The struggle to develop new concepts should occur while the teacher is available to support and scaffold the learning and correct students' errors in thinking. Work that is sent home for students to complete should consist of concepts that have already been taught in class, been practiced, and the student can already do independently. Math homework should be used to build automaticity of skills already acquired and not for development of new skills without instruction. Practicing concepts incorrectly at home can reinforce errors in thinking and cause frustration for students and families. Practicing the skill to automaticity with homework assignments is appropriate after students have acquired the skill. *Reflex Math* is available for students in grades 2-5 and can be accessed at home as well as at school. *Reflex* Math *helps* students develop fluency with their basic facts in addition, subtraction multiplication and division and could be assigned as homework to support students' automaticity.

Online Supports for Unpacking the Core

For additional information about teaching math standards, please visit the following websites:

USOE Curriculum Guides *http://csdmathematics.weebly.com/usoe-elementary-curriculum-guides.html*

North Carolina http://www.ncpublicschools.org/acre/standards/common-core-tools/#unpacking

Howard County Public Schools https://grade4commoncoremath.wikispaces.hcpss.org (Change grade number to match yours—grade_commoncoremath.wikispaces.hcpss.org)

Delware—Under assessment examples http://www.doe.k12.de.us/aab/Mathematics/assessment_tools.shtml

EngageNY—Mathematics Modules--http://www.engageny.org/mathematics

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Utah SAGE Elementary Mathematics Blueprints

Grade 3		
45 Operational Items		
Domain	**Min.**	**Max.**
Operations and Algebraic Thinking (OA)	29%	38%
Number and Operations in Base Ten (NBT)	18%	22%
Number and Operations-Fractions (NF)	27%	31%
Measurement and Data and Geometry (MD/G)	18%	22%
DOK 1	18%	31%
DOK 2	38%	58%
DOK 3	9%	20%

Grade 4		
50 Operational Items		
Domain	**Min.**	**Max.**
Operations and Algebraic Thinking (OA)	18%	22%
Number and Operations in Base Ten (NBT)	28%	32%
Number and Operations-Fractions (NF)	28%	32%
Measurement and Data and Geometry (MD/G)	16%	22%
DOK 1	22%	44%
DOK 2	44%	58%
DOK 3	12%	22%

Grade 5		
50 Operational Items		
Domain	**Min.**	**Max.**
Operations and Algebraic Thinking (OA)	16%	20%
Number and Operations in Base Ten (NBT)	30%	36%
Number and Operations-Fractions (NF)	28%	34%
Measurement and Data and Geometry (MD/G)	18%	22%
DOK 1	16%	28%
DOK 2	50%	64%
DOK 3	10%	24%

Grade 6		
50 Operational Items		
Domain	**Min.**	**Max.**
Ratios and Proportional Relationships (RP)	28%	32%
The Number System (NS)	18%	22%
Expressions and Equations (EE)	28%	34%
Geometry/Statistics and Probability (G/SP)	16%	20%
DOK 1	18%	32%
DOK 2	46%	62%
DOK 3	8%	20%

Note: The percentages shown represent target aggregate values; individual student experiences will vary based on the adaptive algorithm.

Disclosure: Depth of Knowledge (DOK) and Elements of Rigor are essential components of the Utah Mathematics Core Standards. As such, DOK and Elements of Rigor are integrated into the Student Assessment of Growth and Excellence (SAGE) assessment items. All students will see a variety of DOK and Elements of Rigor on the SAGE summative assessment. For more information about DOK and Elements of Rigor please see: http://www.schools.utah.gov/assessment/Criterion-Referenced-Tests/Math.aspx

Systematic Vocabulary Routine- Math

Acquisition	**Introduction Phase** 1. Teacher writes/says the word. 2. Students repeat the word. 3. Multisyllabic breakdown 4. Teacher gives a student friendly definition, incorporating synonyms as appropriate. 5. Students restate definition with teacher guidance. 6. Teacher identifies any prefixes, suffixes, base/root words, origin, etc.	**Teacher/Student Responsibilities** T: The word is polygon. What word? S: polygon T: Let's clap/tap "polygon" into syllables. T & S: "pol" "y" "gon". T: How many syllables? S: 3 syllables T: A closed plane figure with three or more sides that is made up of line segments that do not cross. T & **S**: A closed plane figure with three or more sides that is made up of line segments that do not cross is called a _____. T: The prefix "poly" means much or many. So a polygon has not just one side, but many sides.
Building Automaticity	**Demonstration Phase** 7. Illustrate with examples/non-examples a) Concrete examples (*realia*) b) Visual representations—video, pictures, diagrams, etc. c) Physical gesture d) Verbal Examples e) Sentence Frames (ex. If I had to survive cold weather, I would need _____. 8. Check for students' understanding by discerning between examples and non-examples (repeat as necessary)	T: Look at the figures on this picture. This figure is a polygon because it is closed figure, it is made of line segments that do not cross. These figures are not polygons because they have curved lines, they are open, and some have crossed lines. T: (Example) Draw a polygon on the board? Ones tell your partner if this is a polygon and explain why or why not. S1: The figure is a polygon because it has line segments that are closed and they do not cross. T: (Non-example) Draw a figure that is not a polygon on the board. Twos tell your partner if this is a polygon and explain why or why not. S2: The figure is not a polygon because it is made of curved lines and it is also not closed.
Application	**Application Phase** 9. Deepen students' understanding by applying the word in a new context a) Teacher asks a deep processing question b) Students respond via a quick write and/or orally with a partner or in a small group or whole group setting.	• Students use the word in a sentence. The sentence must be at least five words long. • Number 2's will say the sentence while number 1's count the words in the sentence and makes sure the sentence is a true statement. They switch and follow the same procedure.

Evidence-Based Instructional Priorities
Applied to Math Instruction

Explicit Instruction

I Do - We Do - Y'all Do - You Do
Model - Guide Practice – Partner - Independent

Systematic

- ❑ Focused on critical content
- ❑ Vocabulary routine
- ❑ Skills, strategies, and concepts are sequenced logically
- ❑ Break down complex skills
- ❑ Lessons are organized and focused
- ❑ Instructional routines are used
- ❑ Examples and non-examples
- ❑ Step-by-step demonstrations
- ❑ C-R-A Model

Relentless

- ❑ Adequate initial practice
  NOTE: Students who struggle may require 10-30 more times as many practice opportunities than their peers.
- ❑ Distributed practice--frequent exposure to content/skill over time
- ❑ Daily focused review
- ❑ Daily focus on number sense and problem solving
- ❑ Teach to mastery
- ❑ Cumulative review periodically

Engaging

- ❑ Classroom Positive Behavioral Interventions and Supports (PBIS)
- ❑ Feedback Cycle
- ❑ Scaffolded Instruction & Grouping Structures
- ❑ Acquisition, Automaticity, Application (AAA)
- ❑ Maximizing Opportunities to Respond (OTR)
- ❑ Create various contexts for problem solving that students can relate to
- ❑ Pacing

Increasing Opportunities to Respond

Saying, Writing, Doing

- ❑ **Choral Responses:** give think time, use a signal for response, repeat if all students don't respond
- ❑ **Partner Sharing**: Look-Lean-Whisper; Think-Pair-Share; Study-Tell-Help-Check
- ❑ **Individual Responses:** give wait time, individual shares after partner discussion, Cold Call, random calling pattern
- ❑ **Math Journals:** Quick Writes, vocabulary practice, draw visuals of math concepts
- ❑ **Individual White Boards**: use a signal for displaying, establish a routine, provide feedback
- ❑ **Manipulatives**: establish a routine, explain expectations, all students interact with materials, provide visual bridge to concept
- ❑ **Response Cards**: red/green, yes/no; odd/even; +/-; </>/=; etc.
- ❑ **Action Responses**: thumbs up/down; modeling operations, angles, or other math concepts, act it out, hand signals

Systematic Vocabulary Instruction Routine

- ❑ **Introduce the word**
  - Teacher says the word and posts the word
  - All students repeat the word
  - Teacher gives a child-friendly definition
  - All students repeat the definition (with teacher guidance)
  - Repeat above steps as necessary
- ❑ **Demonstrate**
  - Provide an example
  - Provide a non-example
  - Repeat above steps as necessary
- ❑ **Apply**
  - Students turn to a partner and use the word in a sentence
  - Teacher shares a sentence using the word
- ❑ **Vocabulary Cards:** Grade-level vocabulary cards available on the CSD math website; utilized during instruction and posted on Word Wall

Feedback Cycle

- ❑ Corrective and Affirmative
- ❑ Timely and Frequent
- ❑ Specific and Reinforcing

Scaffolded instruction and Grouping

- ❑ Whole group, Small groups, Partners
- ❑ Fluid and flexible
- ❑ Skill-Based Small Group Instruction for identified skill gaps or extension

Acquisition – Automaticity – Application

- ❑ Learn (acquire) the skill
- ❑ Build the skill to automaticity
- ❑ Attend to fluency standards in the core
- ❑ Apply the skill

Classroom PBIS

- ❑ Forming clear behavior expectations
- ❑ Explicitly teaching expectations to students
- ❑ Reinforcing expectations with students
- ❑ Correcting of problem behaviors in a systematic manner

Fifth Grade Overview

Mathematical Practices (5.MP)
The eight mathematical habits of mind that teachers seek to develop in their students.

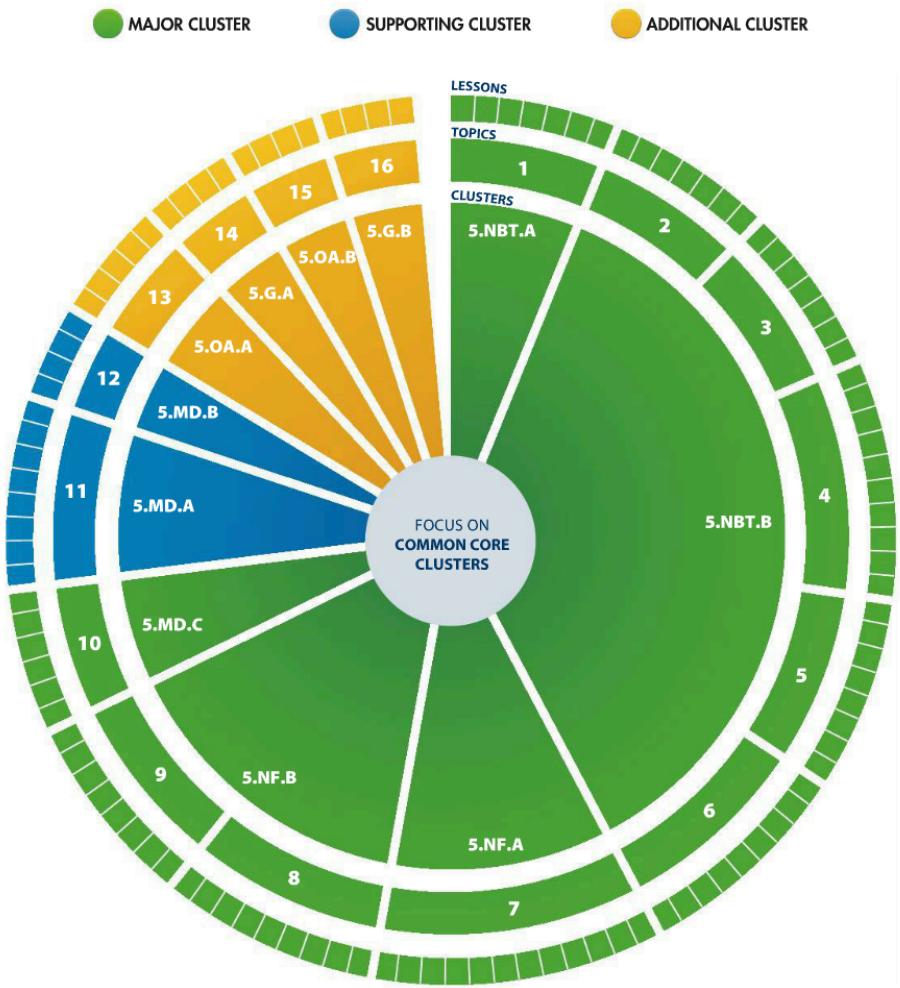
Operations and Algebraic Thinking (5.OA)
- Write and interpret numerical expressions
- Analyze patterns and relationships

Number and Operations in Base Ten (5.NBT)
- Understand the place value system
- Perform operations with multi-digit whole numbers and with decimals to hundredths

Number and Operations - Fractions (5.NF)
- Use equivalent fractions as a strategy to add and subtract fractions
- Apply and extend previous understanding of multiplication and division to multiply and divide fractions

Measurement and Data (5.MD)
- Convert like measurement units within a given measurement system
- Represent and interpret data
- Geometric measurement: understand concepts of volume and relate to multiplication and to addition

Geometry (5.G)
- Graph points on the coordinate plane to solve real-world and mathematical problems
- Classify two-dimensional figures into categories based on their properties



Mathematics | Grade 5

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to two-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

(1) Students will apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They will develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students will also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: This is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

(2) Students will develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They will finalize fluency with multi-digit addition, subtraction, multiplication, and division. They will apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They will develop fluency in these computations, and make reasonable estimates of their results. Students will use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (e.g., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They will compute products and quotients of decimals to hundredths efficiently and accurately.

(3) Students will recognize volume as an attribute of three-dimensional space. They will understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They will understand that a one-unit by one-unit by one-unit cube is the standard unit for measuring volume. They will select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They will decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They will measure necessary attributes of shapes in order to determine volumes to solve real-world and mathematical problems.

Strand: MATHEMATICAL PRACTICES (5.MP)

The Standards for Mathematical Practice in Fifth Grade describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes.

- **Standard 5.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. When a solution pathway does not make sense, look for another pathway that does. Explain connections between various solution strategies and representations. Upon finding a solution, look back at the problem to determine whether the solution is reasonable and accurate, often checking answers to problems using a different method or approach.

- **Standard 5.MP.2 Reason abstractly and quantitatively.** Make sense of quantities and their relationships in problem situations. Contextualize quantities and operations by using images or stories. Decontextualize a given situation and represent it symbolically. Interpret symbols as having meaning, not just as directions to carry out a procedure. Know and flexibly use different properties of operations, numbers, and geometric objects.

- **Standard 5.MP.3 Construct viable arguments and critique the reasoning of others.** Use stated assumptions, definitions, and previously established results to construct arguments. Explain and justify the mathematical reasoning underlying a strategy, solution, or conjecture by using concrete referents such as objects, drawings, diagrams, and actions. Listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and build on those arguments.

- **Standard 5.MP.4 Model with mathematics.** Identify the mathematical elements of a situation and create a mathematical model that shows the relationships among them. Identify important quantities in a contextual situation, use mathematical models to show the relationships of those quantities, analyze the relationships, and draw conclusions. Models may be verbal, contextual, visual, symbolic, or physical.

- **Standard 5.MP.5 Use appropriate tools strategically.** Consider the tools that are available when solving a mathematical problem, whether in a real-world or mathematical context. Choose tools that are relevant and useful to the problem at hand, such as drawings, diagrams, technologies, and physical objects and tools, as well as mathematical tools such as estimation or a particular strategy or algorithm.

- **Standard 5.MP.6 Attend to precision.** Communicate precisely to others by crafting careful explanations that communicate mathematical reasoning by referring specifically to each important mathematical element, describing the relationships among them, and connecting their words clearly to representations. Calculate accurately and efficiently, and use clear and concise notation to record work.

- **Standard 5.MP.7 Look for and make use of structure.** Recognize and apply the structures of mathematics such as patterns, place value, the properties of operations, or the flexibility of numbers. See complicated things as single objects or as being composed of several objects.

- **Standard 5.MP.8 Look for and express regularity in repeated reasoning.** Notice repetitions in mathematics when solving multiple related problems. Use observations and reasoning to find shortcuts or generalizations. Evaluate the reasonableness of intermediate results.

Strand: OPERATIONS AND ALGEBRAIC THINKING (5.OA)

Write and interpret numerical expressions **(Standards 5.OA.1–2),** and analyze patterns and relationships **(Standard 5.OA.3).**

- **Standard 5.OA.1** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

- **Standard 5.OA.2** Write and interpret simple numerical expressions.

  a. Write simple expressions that record calculations with numbers. *For example, use 2 x (8+7) to express the calculation "add 8 and 7, then multiply by 2."*

  b. Interpret numerical expressions without evaluating them. *For example, use conceptual understanding of multiplication to interpret 3 x (18939 + 921) as being three times as large as 18932 + 921 without calculating the indicated sum or product.*

- **Standard 5.OA.3** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "add 3" and the starting number 0, and given the rule "add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Strand: NUMBER AND OPERATIONS IN BASE TEN (5.NBT)

Understand the place value system **(Standards 5.NBT.1–4).** Perform operations with multi-digit whole numbers and with decimals to hundredths **(Standards 5.NBT.5–7).**

- **Standard 5.NBT.1** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

- **Standard 5.NBT.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

- **Standard 5.NBT.3** Read, write, and compare decimals to thousandths.
  - **a.** Read and write decimals to thousandths using base-ten numerals, number names, and expanded form. *For example, 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000).*
  - **b.** Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
- **Standard 5.NBT.4** Use place value understanding to round decimals to any place.
- **Standard 5.NBT.5** Fluently multiply multi-digit whole numbers using the standard algorithm.
- **Standard 5.NBT.6** Find whole-number quotients of whole numbers with up to four-digit dividends and two- digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
- **Standard 5.NBT.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. In this standard, dividing decimals is limited to a whole number dividend with a decimal divisor or a decimal dividend with a whole number divisor. Compare the value of the quotient on the basis of the values of the dividend and divisor.

Strand: NUMBER AND OPERATIONS—FRACTIONS (5.NF)

Use equivalent fractions as a strategy to add and subtract fractions **(Standards 5.NF.1–2).** Apply and extend previous understandings of multiplication and division to multiply and divide fractions **(Standards 5.NF.3–7).**

- **Standard 5.NF.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general,* a/b + c/d = (ad + bc)/bd.*)*
- **Standard 5.NF.2** Solve real-world problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by, *for example, using visual fraction models or equations to represent the problem*. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize 2/5 + 1/2 = 3/7 as an incorrect result, by observing that 3/7 < 1/2.*
- **Standard 5.NF.3** Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve real-world problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, through the use of visual fraction

models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing three by four, noting that 3/4 multiplied by four equals three, and that when three wholes are shared equally among four people each person has a share of size 3/4. If nine people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

■ **Standard 5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product (*a/b*) x *q* as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations *a* x *q* ÷ *b* using a visual fraction model. *For example, use a fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation. Do the same with (2/3) x (4/5) = 8/15. (In general,* (a/b) *x* (c/d) = ac/bd.*)*

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

■ **Standard 5.NF.5** Interpret multiplication as scaling.

a. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. *For example, the products of expressions such as 5 x 3 or ½ x 3 can be interpreted in terms of a quantity, three, and a scaling factor, five or ½. Thus in addition to knowing that 5 x 3 = 15, they can also say that 5 x 3 is five times as big as three, without evaluating the product. Likewise they see ½ x 3 as half the size of three.*

b. Explain why multiplying a given number by a fraction greater than one results in a product greater than the given number (recognizing multiplication by whole numbers greater than one as a familiar case); explain why multiplying a given number by a fraction less than one results in a product smaller than the given number; and relate the principle of fraction equivalence. *For example, 6/10 = (2x3)/(2x5). In general,* a/b = *(*n *x* a*)/(*n *x* b*) has the effect of multiplying* a/b *by one*.

■ **Standard 5.NF.6** Solve real-world problems involving multiplication of fractions and mixed numbers, f*or example, by using visual fraction models or equations to represent the problem*.

■ **Standard 5.NF.7** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. Use strategies to divide fractions by reasoning about the relationship between multiplication and division. Division of a fraction by a fraction is not a requirement at this grade.

a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) x 4 = 1/3.*

b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 x (1/5) = 4.*

c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, *for example, by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if three people share 1/2 lb. of chocolate equally? How many 1/3-cup servings are in two cups of raisins?*

Strand: MEASUREMENT AND DATA (5.MD)

Convert like measurement units within a given measurement system **(Standard 5.MD.1).** Represent and interpret data **(Standard 5.MD.2).** Understand concepts of geometric measurement and volume, as well as how multiplication and addition relate to volume **(Standard 5.MD.3).**

- **Standard 5.MD.1** Convert among different-sized standard measurement units within a given measurement system *(for example, convert 5 cm to 0.05 m)*; use these conversions in solving multi-step, real-world problems.

- **Standard 5.MD.2** Make a line plot to display a data set of measurements in fractions of a unit (halves, quarters, eighths). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given graduated cylinders with different measures of liquid in each, find the amount of liquid each cylinder would contain if the total amount in all the cylinders were redistributed equally.*

- **Standard 5.MD.3** Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

  a. A cube with side length one unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

  b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

- **Standard 5.MD.4** Measure volumes by counting unit cubes, using cubic cm, cubic in., cubic ft., and improvised units.

- **Standard 5.MD.5** Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.

  a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, *for example, to represent the associative property of multiplication*.

b. Apply the formulas $V = l \times w \times h$ *and* $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

Strand: GEOMETRY (5.G)

Graph points on the coordinate plane to solve real-world and mathematical problems in quadrant one **(Standards 5.G.1–2).** Classify two-dimensional figures into categories based on their properties. **(Standards 5.G.3–4).**

■ **Standard 5.G.1** Compose and understand the coordinate plane.

a. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the zero on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates.

b. Using quadrant one on the coordinate plane, understand that the first number in a coordinate pair indicates how far to travel from the origin in the direction of the horizontal axis, and the second number indicates how far to travel in the direction of the vertical axis, with the convention that the names of the two axes and the coordinates correspond (*x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).

■ **Standard 5.G.2** Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

■ **Standard 5.G.3** Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and all squares are rectangles, so all squares have four right angles.*

■ **Standard 5.G.4** Classify two-dimensional figures in a hierarchy based on properties.

Previous	2016/2017
Mathematical Practices 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	**Strand: Mathematical Practices (5.MP)** The Standards for Mathematical Practice in Fifth Grade describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes. **Standard 5.MP.1 Make sense of problems and persevere in solving them.** Explain the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. When a solution pathway does not make sense, look for another pathway that does. Explain connections between various solution strategies and representations. Upon finding a solution, look back at the problem to determine if the solution is reasonable and accurate, often checking answers to problems using a different method or approach. **Standard 5.MP.2 Reason abstractly and quantitatively.** Make sense of quantities and their relationships in problem situations. Contextualize quantities and operations by using images or stories. Decontextualize a given situation and represent it symbolically. Interpret symbols as having meaning, not just as directions to carry out a procedure. Know and flexibly use different properties of operations, numbers, and geometric objects. **Standard 5.MP.3 Construct viable arguments and critique the reasoning of others.** Use stated assumptions, definitions, and previously established results to construct arguments. Explain and justify the mathematical reasoning underlying a strategy, solution, or conjecture by using concrete referents such as objects, drawings, diagrams, and actions. Listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and build on those arguments. **Standard 5.MP.4 Model with mathematics.** Identify the mathematical elements of a situation and create a mathematical model that shows the relationships among them. Identify important quantities in a contextual situation, use mathematical models to show the relationships of those quantities, analyze the relationships, and draw conclusions. Models may be verbal, contextual, visual, symbolic, or physical. **Standard 5.MP.5 Use appropriate tools strategically.** Consider the tools that are available when solving a mathematical problem, whether in a real-world or mathematical context. Choose tools that are relevant and useful to the problem at hand, such as drawings, diagrams, technologies, and physical objects and tools, as well as mathematical tools such as estimation or a particular strategy or algorithm.

	Standard 5.MP.6 Attend to precision. Communicate precisely to others by crafting careful explanations that communicate mathematical reasoning by referring specifically to each important mathematical element, describing the relationships among them, and connecting their words clearly to their representations. Calculate accurately and efficiently, and use clear and concise notation to record work. **Standard 5.MP.7 Look for and make use of structure.** Recognize and apply the structures of mathematics such as patterns, place value, the properties of operations, or the flexibility of numbers. See complicated things as single objects or as being composed of several objects. **Standard 5.MP.8 Look for and express regularity in repeated reasoning.** Notice repetitions in mathematics when solving multiple related problems. Use observations and reasoning to find shortcuts or generalizations. Evaluate the reasonableness of intermediate results.

OPERATIONS AND ALGEBRAIC THINKING (5.OA)

Previous	2016/2017
Operations and Algebraic Thinking **5.OA** **Write and interpret numerical expressions. 5.*OA.A*** 1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. 2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.* **Analyze patterns and relationships 5.*OA.B*** 3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and*	**Strand: OPERATIONS AND ALGEBRAIC THINKING (5.OA)** Write and interpret numerical expressions **(Standards 5.OA.1-2)**, and analyze patterns and relationships **(Standard 5.OA.3).** **Standard 5.OA.1** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. **Standard 5.OA.2** Write and interpret simple numerical expressions. a. Write simple expressions that record calculations with numbers. *For example, 2 x (8+7) to express the calculation "add 8 and 7, then multiply by 2***."** b. Interpret numerical expressions without evaluating them. *For example use their conceptual understanding of multiplication to interpret 3 x (18939 + 921) as being three times as large as18932 + 921 without calculating the indicated sum or product.*

the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.	**Standard 5.OA.3** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "add 3" and the starting number 0, and given the rule "add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*
NUMBERS AND OPERATIONS IN BASE TEN (5.NBT)	
Previous	**2016/2017**

Number and Operations in Base Ten
5.NBT
 	
Understand the place value system. *5. NBT.A*
 	
1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
3. Read, write, and compare decimals to thousandths.
  1. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).
  2. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
4. Use place value understanding to round decimals to any place.
 	
Perform operations with multi-digit whole numbers and with
	Strand: NUMBER AND OPERATIONS IN BASE TEN (5.NBT)
	
	Understand the place value system **(Standards 5.NBT.1-4)**. Perform operations with multi-digit whole numbers and with decimals to hundredths **(Standards 5.NBT.5-7).**
	
	Standard 5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
	
	Standard 5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
	
	Standard 5.NBT.3 Read, write, and compare decimals to thousandths.
	
	a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form. *For example, 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000).*

decimals to hundredths. ***5.NBT.B*** 5. Fluently multiply multi-digit whole numbers using the standard algorithm. 6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **Standard 5.NBT.4** Use place value understanding to round decimals to any place. **Standard 5.NBT.5** Fluently multiply multi-digit whole numbers using the standard algorithm. **Standard 5.NBT.6** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **Standard 5.NBT.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. In this standard, dividing decimals is limited to a whole number dividend with a decimal divisor or a decimal dividend with a whole number divisor. Compare the value of the quotient on the basis of the values of the dividend and divisor.

NUMBERS AND OPERATIONS—FRACTIONS (5.NF)	
Previous	**2016/2017**
Number and Operations – Fractions **5.NF** **Use equivalent fractions as a strategy to add and subtract fractions.** ***5.NF. A*** 1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions	**Strand: NUMBER AND OPERATIONS—FRACTIONS (5.NF)** Use equivalent fractions as a strategy to add and subtract fractions **(Standards 5.NF.1-2)**. Apply and extend previous understandings of multiplication and division to multiply and divide fractions **(Standards 5.NF.3-7)**.

with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Apply and extend previous understandings of multiplication and division to multiply and divide fractions. *5.NF.B*

3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*
4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
   a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*
   b. Find the area of a rectangle with fractional side lengths

Standard 5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general,* a/b + c/d = (ad + bc)/bd.*)*

Standard 5.NF.2 Solve real-world problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by, *for example, using visual fraction models or equations to represent the problem.* Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize 2/5 + 1/2 = 3/7 as incorrect result, by observing that 3/7 < 1/2.*

Standard 5.NF.3 Interpret a fraction as division of the numerator by the denominator *(a/b = a ÷ b)*. Solve real-world problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, through the use of visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing three by four, noting that 3/4 multiplied by four equals three, and that when three wholes are shared equally among four people each person has a share of size 3/4. If nine people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

Standard 5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product *(a/b) x q* as *a* parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations *a x q ÷ b* using a visual fraction model. *For example, use a fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation. Do the same with (2/3) x (4/5) = 8/15. (In general,* (a/b) *x* (c/d) = ac/bd.*)*

by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5. Interpret multiplication as scaling (resizing), by:
   a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
   b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
7. Apply and extend previous understanding of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

   a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*
   b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between*

   b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Standard 5.NF.5 Interpret multiplication as scaling.

a. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. *For example, the products of expressions such as 5 x 3 or 1/2 x 3 can be interpreted in terms of a quantity, three, and a scaling factor, five or 1/2. Thus in addition to knowing that 5 x 3 = 15, they can also say that 5 x 3 is five times as big as three, without evaluating the product. Likewise they see 1/2 x 3 as half the size of three.*

b. Explain why multiplying a given number by a fraction greater than one results in a product greater than the given number (recognizing multiplication by whole numbers greater than one as a familiar case); explain why multiplying a given number by a fraction less than one results in a product smaller than the given number; and relate the principle of fraction equivalence. *For example, 6/10 = (2x3)/(2x5).* In general, a/b = (n x a)/(n x b) *has the effect of multiplying* a/b by one.

Standard 5.NF.6 Solve real-world problems involving multiplication of fractions and mixed numbers, *for example, by using visual fraction models or equations to represent the problem.*

Standard 5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. Use strategies to divide fractions by reasoning about the relationship between multiplication and division. Division of a fraction by a fraction is not a requirement at this grade.

a. Interpret division of a unit fraction by a non-zero whole number,

multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4. c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.	and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) x 4 = 1/3.* b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ *(1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 x (1/5) = 4.* c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, *for example, by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if three people share ½ lb. of chocolate equally? How may 1/3-cup servings are in two cups of raisins?*

MEASUREMENT AND DATA (5.MD)	
Previous	**2016/2017**
Measurement and Data **5.MD** **Convert like measurement units within a given measurement system. *5.MD.A*** 1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems. **Represent and interpret data. *5.MD.B*** 2. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in	**Strand: MEASUREMENT AND DATA (5.MD)** Convert like measurement units within a given measurement system **(Standard 5.MD.1).** Represent and interpret data **(Standard 5.MD.2)**. Understand concepts of geometric measurement and volume, as well as how multiplication and addition relate to volume **(Standard 5.MD.3)**. **Standard 5.MD.1** Convert among different-sized standard measurement units within a given measurement system (*for example, convert 5 cm to 0.05 m*); use these conversions in solving multi-step, real-world problems. **Standard 5.MD.2** Make a line plot to display a data set of measurements in fractions of a unit (halves, quarters, eighths). Use operations on fractions for this grade to solve problems involving

line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition. *5.MD.C*

1. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
    a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
    b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.
2. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
3. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
    a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
    b. Apply the formulas $V=l \times w \times h$ and $V=b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole- number edge lengths in the context of solving real world and mathematical problems.
    c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-

information presented in line plots. *For example, given graduated cylinders with different measures of liquid in each, find the amount of liquid each cylinder would contain if the total amount in all the cylinders were redistributed equally.*

Standard 5.MD. 3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length one unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

Standard 5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in., cubic ft., and improvised units.

Standard 5.MD.5 Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, *for example, to represent the associative property of multiplication*.

b. Apply the formulas $V = l \times w \times h$ *and* $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

overlapping parts, applying this technique to solve real world problems.	
GEOMETRY (5.G)	
Previous	**2016/2017**
Geometry **5.G** **Graph points on the coordinate plane to solve real-world and mathematical problems.** ***5.G.A*** 1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate). 2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. **Classify two-dimensional figures into categories based on their properties.** ***5.G.B*** 3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have 4 right angles.* 4. Classify two-dimensional figures in a hierarchy based on properties.	**Strand: GEOMETRY (5.G)** Graph points on the coordinate plane to solve real-world and mathematical problems in quadrant one **(Standards 5.G.1-2)**. Classify two-dimensional figures into categories based on their properties **(Standards 5.G.3-4)**. **Standard 5.G.1** Compose and understand the coordinate plane. a. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the zero on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. b. Using quadrant one on the coordinate plane, understand that the first number in a coordinate pair indicates how far to travel from the origin in the direction of the horizontal axis, and the second number indicates how far to travel in the direction of the vertical axis, with the convention that the names of the two axes and the coordinates correspond (*x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate). **Standard 5.G.2** Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. **Standard 5.G.3** Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and all squares are rectangles, so all squares have four right angles.* **Standard 5.G.4** Classify two-dimensional figures in a hierarchy based on properties.

Utah Core Standards for Mathematics Progressions

	Kindergarten	1st Grade
Counting and Cardinality	• Count to 100 by ones and tens • Represent and write numbers for 0 - 20 • Count to tell the number of objects • Compare numbers; greater than, less than, equal • Compare written numerals between 1 and 10	

Operations and Algebraic Thinking	• Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from
	  o Represent addition and subtraction
	  o Solve addition and subtraction word problems within 10
	  o Decompose numbers less than or equal to 10
	  o For any number from 1 to 9, find the number that makes 10 when add to the given number
	  o Fluently add and subtract within 5
		• Represent and solve problems involving addition and subtraction within 20
		• Understand and apply properties of operations and the relationship between addition and subtraction
		  o Understand subtraction as an unknown-addend problem
		• Relate addition and subtraction with 20 to counting
		• Add and subtract within 20
		• Understand the meaning of the equal sign
		• Work with addition and subtraction equations

Numbers and Operations in Base Ten	• Work with numbers 11-19 to gain foundation for place value
	  o Compose and decompose numbers
		• Read, write, count and represent to 120
		• Understand place value of tens and ones
		• Compare two-digit numbers based on tens and ones
		• Use place value understanding and properties of operations to add and subtract
		  o Add within 100
		  o Mentally find 10 more or 10 less with two-digit numbers
		  o Subtract multiples of 10 in the range of 10 -90 from multiples of 10 in the range of 10-90

Measurement and Data	• Describe and compare measureable attributes such as length and weight • Directly compare two objects with the same measurable attribute in common and describe the difference • Classify objects and count the numbers of objects in categories	• Measure lengths indirectly and by iterating lengths units • Tell and write time in hours and half-hours using analog and digital clocks • Organize, represent and interpret data up to three categories • Identify and compare the values of pennies, nickels, dimes and quarters

Geometry	• Identify, name and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres)
	• Identify shapes as two-dimensional or three-dimensional
	• Analyze, compare, create and compose shapes
		• Reason with shapes and their attributes
		  o Distinguish between defining vs. non-defining attributes
		  o Compose two-dimensional or three-dimensional shapes to compose and create shapes
		  o Partition circles and rectangles into two and four equal shares

Utah Core Standards for Mathematics Progressions

	2nd Grade	3rd Grade
Operations and Algebraic Thinking	• Represent and solve one- and two-step word problems involving addition and subtraction within 100 • Fluently add and subtract within 20 using mental strategies • Work with equal groups of objects to gain foundations for multiplication • Use addition to find the total number of objects in rectangular arrays with up to 5 rows and up to 5 columns	• Represent and solve problems involving multiplication and division within 100 • Understand properties of multiplication and the relationship between multiplication and division • Multiply and divide within 100 • Solve two-step word problems involving the four operations and identify and explain patterns in arithmetic

Numbers and Operations in Base Ten	• Use place value understanding and properties of operations to add and subtract within 100
	  o Count, read and write within 1000
	  o Compare three-digit numbers using symbols
		• Use place value understanding and properties of operations to perform multi-digit arithmetic
		  o Round whole numbers to nearest 10 or 100
		  o Fluently add and subtract within 1000
		  o Multiply one-digit whole numbers by multiples of 10 in range 10-90

Numbers and Operations- Fractions		• Develop understanding of fractions as numbers with denominators 2, 3, 4, 6, 8 using number lines • Explain equivalence of fractions and compare by reasoning about their size
Measurement and Data	• Measure lengths of an object by selecting and using appropriate tools in standard units. • Measure and estimate lengths using units of inches, feet centimeters and meters • Measure to determine how much longer • Relate addition and subtraction to length within 100 • Represent whole numbers as distance from 0 on the number line • Work with time on digital and analog clocks to the nearest 5 minutes • Solve word problems involving money • Represent and interpret data by measuring objects and making repeated measurements of the same object • Represent and interpret data by drawing a picture graph and a bar graph to represent a data set up to four categories	• Solve problems involving measurement and estimation of intervals of time to the nearest minute • Solve problems involving measurement and estimation of liquid volumes and masses of objects using grams, kilograms and liters • Represent and interpret data using scaled picture and bar graphs • Generate measurement data by measuring lengths to halves and fourths • Geometric measurement: Understand concepts of area and relate area to multiplication and to addition • Geometric measurement: Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.
Geometry	• Recognize and draw shapes having specified attributes • Partition a rectangle into rows and columns • Partition circles and rectangles into two, three, or four equal shares	• Understand that shapes in different categories may share attributes • Partition shapes into parts with equal areas

Utah Core Standards for Mathematics Progressions

	4th Grade	5th Grade
Operations and Algebraic Thinking	• Use the four operations with whole numbers to solve word problems o Interpret a multiplication equation as a comparison o Involve multiplicative comparisons o Solve multistep word problems using whole numbers with whole number answers • Gain familiarity with factors and multiples in the range 1-100 • Generate and analyze patterns that follow a given rule	• Write and interpret numerical expressions o Use parenthesis, brackets, or braces in numerical expressions and evaluate expression with these symbols o Write simple expressions and interpret numerical expressions without evaluating them • Analyze patterns and relationships o Generate two numerical patterns using two given rules o Form ordered pairs
Numbers and Operations in Base Ten	• Generalize place value understanding for multi-digit whole numbers o Read, write, compare and expand multi-digit whole numbers o Round multi-digit numbers to any place • Fluently add and subtract multi-digit whole numbers using the • Use place value understanding and properties of operations to perform multi-digit multiplication o Multiply up to four digits by a one-digit number o Multiply two two-digit numbers using strategies and properties (illustrate and explain the calculations using equations, rectangular arrays and area models)	• Understand the place value system o Recognize a multi-digit number in the one place represents 10 times as much as it represents in the place to its right and 1/10 to its left o Explain patterns when multiplying by zero and explain patterns when a decimal is multiplied or divided o Use whole-number exponents to denote powers of 10 o Read, write and compare decimals to thousandths o Round to any place o Fluently multiply multi-digit whole numbers • Perform operations with multi-digit whole numbers and with decimal to hundredths o Fluently multiply multi-digit whole numbers o Find whole-number quotients of whole numbers up to four-digit dividends (illustrate and explain the calculations using equations, rectangular arrays and area models) o Add, subtract, multiply, and divide decimals to hundredths
Numbers and Operations-Fractions	• Extend understanding of fraction equivalence and ordering with denominators 2,3,4,5,6,8,10,12,10 o Explain and generate equivalent fractions using visual models o Compare with justification two fractions with different denominators and numerators and use the symbols >, =, <. • Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers o Understand addition and subtraction of fractions as joining and separating parts referring to the same whole o Decompose a fraction into a sum of fractions with same denominator o Add and subtract mixed numbers with like denominators o Solve word problems involving addition and subtraction of fractions having like denominators o Understand a fraction a/b as a multiple of 1/b and use this	• Use equivalent fractions as a strategy to add and subtract fractions o Add and subtract fractions with unlike denominators o Solve word problems involving addition and subtraction of fractions with unlike denominators • Apply and extend previous understandings of multiplication and division to multiply and divide fractions o Interpret a fraction as division of the numerator by the denominator o Solve word problems involving division of whole numbers o Find the are of a rectangle with fractional side lengths by tiling it with unit squares o Multiply fractional side lengths to find area of rectangle to get a rectangular areas o Interpret multiplication as scaling o Solve real world problems involving multiplication of

	understanding to multiply a fraction by a whole number o Solve word problems involving multiplication of a fraction by a whole number • Understand decimal notation for fractions and compare decimal fractions o Express a fraction with denominator 10 as an equivalent fraction with denominator 100 o Use decimal notation for fractions with denominators 10 or 100 o Compare two decimals to hundredths by reasoning about their size	fractions and mixed numbers o Divide a unit fraction by a whole number and whole numbers by unit fractions
Measurement and Data	• Solve problems involving measurement and conversion of measurements form a larger unit to a smaller unit o Know relative sizes of measurement units within one system of units including km, m, cm; kg, g, oz; l, ml; hr, min, sec. and express measurement equivalents in terms of a smaller unit, recording measurement in a two-column table o Use the four operations to solve problems involving distances, intervals of time, liquid volumes, masses of objects, and money including problems involving simple fractions or decimals o Represent measurement quantities using diagrams such as number line diagrams such as number line diagrams that feature a measurement scale o Apply the area and perimeter formulas in real world problems o Make a line plot to display data set of measurements in fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$) • Represent and interpret data by making a line plot to display data set of measurements in fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$) • Understand concepts of angle and measure angles o As angle is measured with reference to a circle o An angle that turns through *n* one-degree is said to have an angle measure of *n* degrees o Measure and sketch angles in whole-number degrees using a protractor o Recognize angles measures as additive o Solve addition and subtraction problems to find unknown angles	• Convert like measurement units within a given measurement system • Represent and Interpret data o Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$) o Use operations on fractions for this grade to solve problems from information on the line plot • Recognize volume as an attribute of solid figures and understand concepts of volume measurement. o Measure volume by counting unit cubes • Relate volume to the operations of multiplication and addition and solve real world problems involving volume o Find the volume of a right triangle by packing it with unit cubes o Apply formulas V=l x w x h and V= b x h o Recognize volume as additive o Find volume of solid figures composed of two non-overlapping right rectangular prisms
Geometry	• Draw points, lines, line segments, ray, angles (right, acute, obtuse), and perpendicular and parallel lines in two-dimensional figures • Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. • Recognize right triangles as a category and identify right triangles • Recognize a line of symmetry for a two-dimensional figure and identify line-symmetric figures and draw lines of symmetry	• Graph points on the coordinate plane to solve real-world and mathematical problems in the first quadrant • Classify two-dimensional figures into categories based on their properties o Understand that attributes belonging to a category of two-dimensional figures belong to all subcategories o Classify two-dimensional figures in a hierarchy based on properties

CCSS
WHERE TO FOCUS
MATHEMATICS

An important subset of the major work in grades K–8 is the progression that leads toward middle school algebra.

K	1	2	3	4	5	6	7	8
Know number names and the count sequence Count to tell the number of objects Compare numbers Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from Work with numbers 11-19 to gain foundations for place value	Represent and solve problems involving addition and subtraction Understand and apply properties of operations and the relationship between addition and subtraction Add and subtract within 20 Work with addition and subtraction equations Extend the counting sequence Understand place value Use place value understanding and properties of operations to add and subtract Measure lengths indirectly and by iterating length units	Represent and solve problems involving addition and subtraction Add and subtract within 20 Understand place value Use place value understanding and properties of operations to add and subtract Measure and estimate lengths in standard units Relate addition and subtraction to length	Represent & solve problems involving multiplication and division Understand properties of multiplication and the relationship between multiplication and division Multiply & divide within 100 Solve problems involving the four operations, and identify & explain patterns in arithmetic Develop understanding of fractions as numbers Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects Geometric measurement: understand concepts of area and relate area to multiplication and to addition	Use the four operations with whole numbers to solve problems Generalize place value understanding for multi-digit whole numbers Use place value understanding and properties of operations to perform multidigit arithmetic Extend understanding of fraction equivalence and ordering Build fractions from unit fractions by applying and extending previous understandings of operations Understand decimal notation for fractions, and compare decimal fractions	Understand the place value system Perform operations with multi-digit whole numbers and decimals to hundredths Use equivalent fractions as a strategy to add and subtract fractions Apply and extend previous understandings of multiplication and division to multiply and divide fractions Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition Graph points in the coordinate plane to solve real-world and mathematical problems*	Apply and extend previous understandings of multiplication and division to divide fractions by fractions Apply and extend previous understandings of numbers to the system of rational numbers Understand ratio concepts and use ratio reasoning to solve problems Apply and extend previous understandings of arithmetic to algebraic expressions Reason about and solve one-variable equations and inequalities Represent and analyze quantitative relationships between dependent and independent variables	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers Analyze proportional relationships and use them to solve real-world and mathematical problems Use properties of operations to generate equivalent expressions Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Work with radical and integer exponents Understand the connections between proportional relationships, lines, and linear equations** Analyze and solve linear equations and pairs of simultaneous linear equations Define, evaluate, and compare functions Use functions to model relationships between quantities

* Indicates a cluster that is well thought of as a part of a student's progress to algebra, but that is currently not designated as major by the assessment consortia in their draft materials. Apart from the one asterisked exception, the clusters listed here are a subset of those designated as major in the assessment consortia's draft documents.
** Depends on similarity ideas from geometry to show that slope can be defined and then used to show that a linear equation has a graph which is a straight line and conversely.

The Utah Core Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important processes and proficiencies with longstanding importance in mathematics education.

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

"The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices" (CCSS, 2010).

Common Core State Standards
Standards for Mathematical Practice
Questions for Teachers to Ask

Make sense of problems and persevere in solving them	Reason abstractly and quantitatively	Construct viable arguments and critique the reasoning of others	Model with mathematics
Teachers ask: • What is this problem asking? • How could you start this problem? • How could you make this problem easier to solve? • How is ___'s way of solving the problem like/different from yours? • Does your plan make sense? Why or why not? • What tools/manipulatives might help you? • What are you having trouble with? • How can you check this?	*Teachers ask:* • What does the number ____ represent in the problem? • How can you represent the problem with symbols and numbers? • Create a representation of the problem.	*Teachers ask:* • How is your answer different than _____'s? • How can you prove that your answer is correct? • What math language will help you prove your answer? • What examples could prove or disprove your argument? • What do you think about _____'s argument • What is wrong with ____'s thinking? • What questions do you have for ____? **it is important that the teacher poses tasks that involve arguments or critiques*	*Teachers ask:* • Write a number sentence to describe this situation • What do you already know about solving this problem? • What connections do you see? • Why do the results make sense? • Is this working or do you need to change your model? **It is important that the teacher poses tasks that involve real world situations*
Use appropriate tools strategically	**Attend to precision**	**Look for and make use of structure**	**Look for and express regularity in repeated reasoning**
Teachers ask: • How could you use manipulatives or a drawing to show your thinking? • Which tool/manipulative would be best for this problem? • What other resources could help you solve this problem?	*Teachers ask:* • What does the word ____ mean? • Explain what you did to solve the problem. • Compare your answer to _____'s answer • What labels could you use? • How do you know your answer is accurate? • Did you use the most efficient way to solve the problem?	*Teachers ask:* • Why does this happen? • How is ____ related to ____? • Why is this important to the problem? • What do you know about ____ that you can apply to this situation? • How can you use what you know to explain why this works? • What patterns do you see? **deductive reasoning (moving from general to specific)*	*Teachers ask:* • What generalizations can you make? • Can you find a shortcut to solve the problem? How would your shortcut make the problem easier? • How could this problem help you solve another problem? **inductive reasoning (moving from specific to general)*

Grades 1-5 CSD Math Block 90 Minutes Daily

Numeracy Component	Range of Time	Focus of Instruction		Instructional Materials	
				Hard Copy	**Digital**

Review	5-10 minutes	• Focused Review
		  ○ Identified skill deficit that have been identified through formative assessment to review (DWSBB, exit ticket, whiteboards, etc.)
		  ○ Cumulative review of previously taught skills and standards
			Check for Understanding (Formative Assessment)
			Monitor progress towards mastery of grade-level core standard
				• Daily Common Core Review
				• Today's Challenge
				• Review What you Know
					• Today's Challenge

Vocabulary	3-5 minutes	• Teach Appropriate Vocabulary using the Systematic Vocabulary Routine		• Systematic Vocabulary Routine • Vocabulary Review Activity • My Word Cards	
Lesson Objectives	1-3 Minutes	• Content Objectives- What are students going to learn? • Language Objectives- How will students demonstrate learning through reading, writing, speaking, or listening?		• Lesson objectives are posted and referred to throughout the lesson • Objectives include both content and math practice standards	

Concept/Skill Development (Acquisition, Automaticity & Application)	30-45 minutes	Develop the Concept:
		• Acquisition: Students develop understanding of skills through the CRA Model
		  ○ Concrete: Hands-on (manipulatives)
		  ○ Representational: Visual (pictures or video)
		  ○ Abstract: Symbolic (numbers or algorithm)
		• Automaticity: Students perform skills flexibly, accurately, and efficiently
		• Application: Students apply skills to solve problems in new contexts
				• Problem-Based Interactive Learning
				• Visual Learning Bridge
				  ○ (K-2) Do You Understand? Show Me!
				  ○ 3-5) Convince Me!
				  ○ Guided Practice
				  ○ Independent Practice (Quick Check)
					• Solve and Share (Problem Based Learning)
					• Visual Learning Animation Plus
					• Convince Me! (3-5)
					• Do You Understand? (K-2)
					• Student and Teacher eTexts
					• Listen and Look Videos (teacher)
					•

Skill-Based Instruction: Pre-teach, Review, Reinforce & Extend	30-45 minutes	• Pre-teach upcoming concepts to groups and individual students that need support/scaffolding • Students practice concepts independently as appropriate • Reteach with skill-based groups who need extra support/scaffolding • Provide extension opportunities for students who have shown mastery of the concept/skill • Build Fluency with math facts and computation		• Intervention Activity • ON-level and Advanced Activity Centers • Reteach • Leveled Assignment • Differentiated Center materials • Close/Assess and Differentiate	• Practice Buddy • Reflex (grades 2-5)

Skill-Based Instruction: Assisting All Students to Succeed in Mathematics

Skill-Based Instruction is additional support given to students during the math block by the teacher aimed at building targeted math skills. This is in addition to core instruction given to entire class.

enVision 2.0 supports skill-based instruction with the following resources:

- **Intervention Activity** (Assess and Differentiate section at the end of each lesson) Students needing intervention get focused instruction from the teacher.
- **Math Diagnosis and Intervention System 2.0 (**MDIS**)** Provides additional lessons to focus intervention for students.
- **Item Analysis for Diagnosis and Intervention (**RtI**)** Provided with assessments to support analyzing gaps in mastery of standards
- **Reteaching** Problem sets at the end of each topic that connect to the math standards

Skill-based instruction is explicit & systematic *(I do, we do, y'all do, and you do)*	***Examples***
Provide additional **concrete models** to build understanding **with accompanying teacher think-alouds**	• Use manipulatives such as place value blocks, Unifix cubes, and fraction circles. • Use visual representations such as number lines, arrays, and bar diagrams. • Teacher Think-Aloud: *"When I have fourteen cubes, I can create one ten stick and I have four cubes left over to make 14."*
Provide students opportunities to understand the **relationship between the abstract symbols and visual representations.**	• The = sign means that we have the same amount on both sides of the equal sign. ✪ ✪ ✪ = ✪ ✪ ✪
Provide numerous **examples with accompanying teacher think-alouds**	Skill: Addition of Fractions Examples: • ½ + ¼ = • ¼ + ¼ = Teacher Think-Aloud: *"We know that when we add fractions with common denominators the denominator will stay the same because we still have the same size piece. So when I add ¼ + ¼ I have* $^2/_4$ *because I have 2, ¼ pieces."*
Provide **students** with opportunities to **solve problems** in a group **and communicate problem-solving strategies.**	• Students effectively communicate their strategies to *one another* using appropriate mathematical vocabulary. • Students effectively communicate their strategies to the *teacher* using appropriate mathematical vocabulary.
Provide students ongoing, **specific feedback** that clarifies what students did correctly or what they need to improve.	• Student correctly answers that 5 + 3 = 8. Teacher says, "Yes, that is correct. The total of five and three is eight." • Student incorrectly identifies that 5 + 3 = 7. Teacher says, "Five plus three is not seven. Pull out your unifix cubes and show me the problem with your cubes." *Student counts the cubes and answers that 5 + 3 = 8.* "That is correct. The total of five and three is eight. Thank you for trying again."
Provide **frequent cumulative review** to ensure that knowledge is maintained over time.	Skill: Adding Decimals • Teacher quickly reviews multi-digit addition with an emphasis on place value.
Provide opportunity for **students to apply the skill in word problems.**	Skill: Area - finding the area of a rectangle given the side lengths. • Students create word problems using the area of squares for example a student creates the following problem, *"Bobbie is tiling the kitchen floor with square foot tiles. The floor has side lengths of 10 feet and*

	12 feet. How many tiles are needed to cover the floor?"

During skill-based instruction, students not with the teacher could engage in the following math center activities:

Center Options	Description
Center Activities from enVision 2.0	• At the end of each enVision2.0 lesson in the Assess and Differentiate section are the On-Level and Advanced Center Activities which include: Center Games, Problem-Solving Reading Mat, Math and Science Activity
Digital Centers from enVision 2.0	• The following digital components from enVision 2.0 could be utilized by students during math centers: Today's Challenge, Game from the Game Center, Digital Math Tool Activities, Another Look video, Bounce Pages, Practice Buddy (grades 3-5)
Technology	• Reflex- Students work independently in grades 2-5 to build fluency of basic math facts • Students use appropriate technology to deepen their understanding of math.
Fluency	• Fluency is built on any skill that has been taught throughout the year (*e.g., previous instruction focused on fact families and pairs of students work together and to create fact families using number cards, including numbers 0-9. The student created fact families would be recorded on a piece of paper or graphic organizer.)*
Four-Square Math	• Students are given a four square graphic organizer with a previously learned vocabulary word or concept in the middle of the graphic. The four areas to write could include any of the following: three words or pictures that help you remember the word, characteristics, non-example, example, a statement that is true about the word, three words related to the word, or a conclusion statement. • Students write a math practice standard in the middle of the four square and could add any of the following to the squares: characteristics of the MP, list what students do when they engage in the MP, write questions that you would ask your partner when you are focusing on the MP, six word summary of the MP, etc.
Literature in Math	• Students read or look at a book that relates to the current or past math concept. The teacher provides questions or sentence starters for the group at the center to support discussion after reading.
Manipulatives	• Students manipulate math tools to complete a grade level task.
Math Journals	• Students write or draw in math journals to summarize their learning. • Students review their notes and star key ideas.
Problem-Solving using DOK 3	• Students in small groups are presented with an application problem that requires reasoning, problem solving, and justification of their thought process by using words, pictures or equations. • Tasks are available at the following websites: http://www.insidemathematics.org https://www.illustrativemathematics.org http://illuminations.nctm.org
Vocabulary	• Students match previously taught vocabulary words with illustrations. After finding a match the student would define the word. • Students do a word sort with the enVision vocabulary cards. • Students find similarities and differences in words using a Venn Diagram.

5th Grade
Year-at-a-Glance 2016-2017

Flexible Pacing	Strands/Standards	enVision 2.0 Math Topic Titles	TOPICS	District Assessment Dates
Aug 24 - Nov 11 52 Days	**Mathematical Practices: 3, 4, 7** Number and Operations in Base Ten: Standards 1-3 (5.NBT.A) Number and Operations in Base Ten: Standards 5-7 (5.NBT.B)	• **Understand Place Value (7 Lessons)**	Topic 1	Due by November 11 District-Wide Standards-Based Benchmark #1
		• **Add and Subtract Decimals to Hundredths (7 Lessons)**	Topic 2	
		• **Fluently Multiply Multi-Digit Whole Numbers (7 Lessons)**	Topic 3	
		• **Use Models and Strategies to Multiply Decimals (10 lessons)**	Topic 4	
Nov 14 - Mar 3 64 Days	**Mathematical Practices: 1, 2, 4** Number and Operations in Base Ten: Standards 5-7 (5.NBT.B) Number and Operations in Fractions: Standards 1- 2 (5.NF.A) Number and Operations in Fractions: Standards 3-7 (5.NF.B)	• **Use Models and Strategies to Divide Whole Numbers (8 Lessons)**	Topic 5	Due by March 3 District-Wide Standards-Based Benchmark #2
		• **Use Models and Strategies to Divide Decimals (6 Lessons)**	Topic 6	
		• **Use Equivalent Fractions to Add and Subtract Fractions (12 Lessons)**	Topic 7	
		• **Apply Understanding of Multiplication to Multiply Fractions (9 Lessons)**	Topic 8	
Mar 6 – Apr 28 35 Days	**Mathematical Practices: 3, 5, 6, 8** Number and Operations in Fractions: Standards 3-7 (5.NF.B) Measurement and Data Standards: 3-5 (5.MD.C) Measurement and Data Standard: 1 (5.MD.A) Measurement and Data Standard: 2 (5.MD.B)	• **Apply Understanding Division to Divide Fractions (8 Lessons)**	Topic 9	Due by April 28 District-Wide Standards-Based Benchmark #3
		• **Understand Volume Concepts (6 Lessons)**	Topic 10	
		• **Convert Measurements (8 Lessons)**	Topic 11	
		• **Represent and Interpret Data (4 Lessons)**	Topic 12	
May 1 – June 6 25 Days	**Mathematical Practices: 1, 2, 3** Operations and Algebraic Thinking: Standards 1-2 (5.OA.A) Geometry: Standards 1-2 (5.G.A) Operations and Algebraic Thinking: Standard 3 (5.OA.B) Geometry: Standards 3-4 (5.G.B	• **Algebra: Write and Interpret Numerical Expressions (5 Lessons)**	Topic 13	Due by June 6 District-Wide Standards-Based Benchmark #4
		• **Graph Points on the Coordinate Plane (4 Lessons)**	Topic 14	
		• **Algebra: Analyze Patterns and Relationships (4 Lessons)**	Topic 15	
		• **Geometric Measurement: Classify Two-Dimensional Figures (4 Lessons)**	Topic 16	

DWSBB 1: August 24-November 11 **Flexible Pacing: 52 days**

NUMBER AND OPERATIONS IN BASE TEN (NBT)
Topic 1 - Understand Place Value

Report Card Learning Targets
I can….
- Explain patterns in powers of ten
- Understand place value in the decimal system

TOPIC 1

Coherence		**pp. 1C-1D**
Look back: **Grade 4-** • Whole-Number Place Value • Decimal Place Value	**Topic 1:** • Exponents and Expanded Form • Place-Value Relationships in Whole Numbers and Decimals • Compare and Order Decimals • Round Whole Numbers and Decimals	**Look ahead:** **Later in Grade 5-** • Operations with Whole numbers and Decimals • Convert Measurements **Grade 6-** • Exponents • Whole-Number and Decimal Computation

Rigor		**p. 1E**
Conceptual Understanding: • Understand Exponents • Understand the Relationship Between Adjacent Place-Value Positions • Make Sense of Comparing Numbers • Make Sense of Round Numbers	**Procedural Skill and Fluency:** • Use Conceptual Understanding to Write Whole Numbers in Expanded Form • Use Conceptual Understanding to Read and Write Decimals • Use Conceptual Understanding to Compare and Round Decimals	**Applications:** • Situations Involving Whole Numbers and Decimals

Focus	**Strand: Mathematical Practice Standard #7** **p. 1F**
5.MP.7	**Look for and make use of structure.** Recognize and apply the structures of mathematics such as patterns, place value, the properties of operations, or the flexibility of numbers. See complicated things as single objects or as being composed of several objects. ***Fifth grade students use structure when they apply place-value relationships to read and write numbers.*** I can recognize, describe, and use patterns in numbers.

	I can understand, identify, and use equivalent representations of numbers. I can describe how numbers and expressions are organized and put together as parts and wholes.		
Focus	**Standards**	**Curriculum Supports – envision 2.0**	**Vocabulary**
5.NBT.1 **5.NBT.2** **5.NBT.3** **5.NBT.4** **(5.NBT.A)**	**Strand: Number and Operations in Base Ten** Fifth grade students will understand the place value system. **Standard 5.NBT.1** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. **Standard 5.NBT.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. **Standard 5.NBT.3** Read, write, and compare decimals to thousandths. **a.** Read and write decimals to thousandths using base-ten numerals, number names, and expanded form. *For example, 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000).* **b.** Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **Standard 5.NBT.4** Use place value understanding to round decimals to any place.	**Topic 1: Understand Place Value** *(pp. 1I-1K)* **1-1** Patterns with Exponents and Powers of 10 *(pp. 5-10)* **1-2** Understand Whole-Number Place Value *(pp. 11-16)* **1-3** Decimals to Thousandths *(pp. 17-22)* **1-4** Understand Decimal Place Value *(pp. 23-28)* **1-5** Compare Decimals *(pp. 29-34)* **1-6** Round Decimals *(pp. 35-40)* **1-7 Math Practices and Problem Solving: Look For and Use Structure** *(pp. 41-46)*	**Topic 1:** • exponent • power • base • value • expanded form • thousandths • equivalent decimals
	Assessment Options:	**Topic 1 Assessment -** Understand Place Value *(print or online)* *(pp. 51-52)* **Topic 1 Performance Assessment -** Understand Place Value *(pp. 53-54)*	

Assessment Tasks – Topic 1		
	Procedural Check	**Application Task**
5NBT.1	Write the number that is 10 times smaller than 100. (1) Write the number that is 10 times smaller than 40. (4) Write the number that is 10 times larger than 40. (.04) These aren't all decimals. Does it matter? (DOK 1)	Tell if the statement below is true. Then explain your reasoning to justify your answer. **0.500 is ten times larger than 0.50** (DOK 3)
5.NBT.2	Complete the number sentences below. 1. _____ x 10^2 = 2,500 2. _____ ÷ 10^3 = 0.016 3. 3.3 x _____ = 33,000 (DOK 1) Write the missing power of ten. 5. 7.8 x _____ = 78,000 6. 0.34 x _____ = 340 7. 512 ÷ _____ = 0.512 (DOK 1)	Compare the result of multiplying a whole number by a power of 10 to the result of multiplying a decimal by a power of 10. Is the pattern the same? Use an example to justify your answer. (DOK 3)
5.NBT.3	Write 562.376 in expanded form. (DOK 1) Given 1.02, 1.2 and 100.2, place the numbers in order from least to greatest. (DOK 1)	Find four numbers that are between 0.11 and 0.12 and put all six numbers in order from least to greatest. (DOK 2) Carlos and Rita are trying to decide who has read more of their book. Carlos has read 95/100 of his book and Rita has read 9/10 of the same book. Who has read more? Use words or pictures to explain your answer. (DOK3) Using the following list of numbers, decide which number is closest to 4.5. Use a number line to justify your answers. (DOK 3) **4.7 4.35 4.025 4.9 4.24 4.473**

5 NBT.4	Round 1.069 to the nearest tenth, hundredth and one. (DOK 1)	If the rounded number is 9.6, what could be the original number have been? Use words and drawings to justify your answer. (DOK 3) When Thomas saw his race result posted as 4.85 seconds, he was confused. He said that his time 4.847 seconds was actually faster than the posted time. Was he correct? Use words or a number line to explain your answer. (DOK 3)

DWSBB 1: August 24-November 11 **Flexible Pacing: 52 days**

NUMBERS AND OPERATIONS IN BASE TEN (NBT)
Topic 2 - Add and Subtract Decimals to Hundredths
Topic 3 - Fluently Multiply Multi-Digit Whole Numbers
Topic 4 - Use Models and Strategies to Multiply Decimals

Report Card Learning Targets
I can….
- Explain patterns in powers of ten
- Understand place value in the decimal system
- Fluently multiply multi-digit whole numbers
- Model and calculate decimals to the hundredths place using the four operations

TOPICS 2, 3, and 4

Coherence		**pp. 55C-55D**
Look back: **Grade 4-** • Whole-Number Operations • Fraction Multiplication • Decimal Notation **Earlier in Grade 5-** • Decimal Place Value	**Topics 2,3, and 4:** • Multiplication with Whole Numbers and with Decimals • Division with Whole Numbers and with Decimals • Use Place-Value Concepts	**Look ahead:** **Later in Grade 5-** • Convert Measurements **Grade 6-** • Evaluate Expressions • Fluency with Whole Numbers and Decimals

Rigor		**p. 55E**
Conceptual Understanding: • Use Place Value and Properties to Add and Subtract Decimals • Use Place Value and Properties to Multiply Decimals • Use Place Value and Properties to Divide Multi-digit Whole Numbers and Decimals	**Procedural Skill and Fluency:** • Develop Fluency with Multiplying Multi-digit Whole Numbers • Divide Multi-digit Whole Numbers • Perform Computations with Decimals	**Applications:** • Solve Real-World Problems

Focus	**Strand: Mathematical Practice Standard #4 and #3**	**p. 55F**
5.MP.4 **5.MP.3**	**4. Model with mathematics.** (Topics 2 and 4) Identify the mathematical elements of a situation and create a mathematical model that shows the relationships among them.	

	Identify important quantities in a contextual situation, use mathematical models to show the relationships of those quantities, analyze the relationships, and draw conclusions. Models may be verbal, contextual, visual, symbolic, or physical. ***Fifth grade students model with math when they use bar diagrams and equations to represent problems involving whole numbers and decimals.*** I can identify the correct prior knowledge that needs to be applied to solve a problem. I can identify the hidden question(s) in multiple-step problems. I can use numbers, symbols, and words to solve problems. I can identify the operation(s) needed to solve a problem. I can use estimation as appropriate. **3. Construct viable arguments and critique the reasoning of others.** (Topic 3) Use stated assumptions, definitions, and previously established results to construct arguments. Explain and justify the mathematical reasoning underlying a strategy, solution, or conjecture by using concrete referents such as objects, drawings, diagrams, and actions. Listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and build on those arguments. ***Fifth grade students construct and critique their own and others' arguments to justify solutions to problems and computations involving whole numbers and decimals.*** I can ask questions to understand other people's thinking. I can identify mistakes in other people's thinking. I can provide suggestions for improving other people's thinking.

Focus	Standards	Curriculum Supports – envision 2.0	Vocabulary
5.NBT.5 **5.NBT.6** **5.NBT.7** **(5.NBT.B)**	**Strand: Number and Operations in Base Ten** Fifth grade students will perform operations with multi-digit whole numbers and with decimals to hundredths. **Standard 5.NBT.5** Fluently multiply multi-digit whole numbers using the standard algorithm. **Standard 5.NBT.6** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the	**Topic 2: Add and Subtract Decimals to Hundredths** *(pp. 55I-55K)* **2-1** Mental Math *(pp. 59-64)* **2-2** Estimate Sums and Differences *(pp. 65-70)* **2-3** Use Models to Add and Subtract Decimals *(pp. 71-76)* **2-4** Add Decimals *(pp. 77-82)* **2-5** Subtract Decimals *(pp. 83-88)* 2-**6** Add and Subtract Decimals *(pp. 89-94)* **2-7 Math Practices and Problem Solving: Model with Math** *(pp. 95-100)*	**Topic 2:** • compatible numbers • associative property of addition • commutative property of addition • compensation

	relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **Standard 5.NBT.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. In this standard, dividing decimals is limited to a whole number dividend with a decimal divisor or a decimal dividend with a whole number divisor. Compare the value of the quotient on the basis of the values of the dividend and divisor.	**Topic 3: Fluently Multiply Multi-Digit Whole Numbers** *(pp. 109A-109C)* **3-1** Multiply Greater Numbers by Powers of 10 *(pp. 113-118)* **3-2** Estimate Products *(pp. 119-124)* **3-3** Multiply 3-Digit by 2-Digit Numbers *(pp.125-130)* **3-4** Multiply Whole Numbers with Zeros *(pp.131-136)* **3-5** Multiply Multi-Digit Numbers *(pp. 137-142)* **3-6** Solve Word Problems Using Multiplication *(pp. 143-148)* **3-7 Math Practices and Problem Solving: Critique Reasoning** *(pp. 149-154)* **Topic 4: Use Models and Strategies to Multiply Decimals** *(pp. 163A-163D)* **4-1** Multiply Decimals by Power of 10 *(pp. 165-170)* **4-2** Estimate the Product of a Decimal and a Whole Number *(pp. 171-176)* **4-3** Use Models to Multiply a Decimal and a Whole Number *(pp. 177-182)* **4-4** Multiply a Decimal by a Whole Number *(pp. 183-188)* **4-5** Use Models to Multiply a Decimal and a Decimal *(pp. 189-194)* **4-6** Multiply Decimals Using Partial Products *(pp. 195-200)* **4-7** Use Properties to Multiply Decimals *(pp.201-206)* **4-8** Use Number Sense to Multiply Decimals *(pp. 207-212)*	**Topic 3:** • underestimate • overestimate • partial products • variable **Topic 4:** No new vocabulary words Review as needed

		4-9 Multiply Decimals *(pp. 213-218)* **4-10 Math Practices and Problem Solving: Model with Math** *(pp. 219-224)*	
	Assessment Options: **Topic 2 Assessment -** Add and Subtract Decimals to Hundredths *(print or online)* *(pp. 105-106)* **Topic 2 Performance Assessment -** Add and Subtract Decimals to Hundredths *(pp. 107-108)*	**Topic 3 Assessment -** Fluently Multiply Multi-Digit Whole Numbers *(print or online)* ***(pp. 159-160))*** **Topic 3 Performance Assessment -** Fluently Multiply Multi-Digit Whole Numbers ***(pp. 161-162)*** **Topic 4 Assessment -** Use Models and Strategies to Multiply Decimals *(print or online)* ***(pp. 231-234)*** **Topic 4 Performance Assessment -** Use Models and Strategies to Multiply Decimals ***(pp. 235-236)***	
District-Wide Standards-Based Benchmark #1 due by November 11			

Assessment Tasks – Topics 2, 3, and 4		
	Procedural Check	**Application Task**
5.NBT.5	Multiply 524 X 639 and show your work. (DOK 1)	A bakery has 245 dozen cupcakes. How many individual cupcakes are there? (DOK 1) How would you arrange the following numbers to create the largest possible product? Explain how you decided to place the numbers and how you know you created the largest product. (DOK 3) 9 2 0 8 4 ? ? ? x ? ?
5.NBT.6	Divide the following and show your work 588 ÷ 14. (DOK 1) Samantha wants to split a collection of stickers into groups of 48. Samantha has 1,008 stickers. How many groups will be created? Show two ways to find the answer. (DOK 1)	Write a division problem using a 4-digit dividend and a 2-digit divisor that results in an even quotient. Explain your thinking. (DOK 3)
5.NBT.7	Calculate the following: (DOK 1) 1. 3.4 + 6.2 2. 7.7 – 4-1 3. 5.6 X 2.4 4. 8.4 | 2.1	I divided 6.12 by 3 and got the quotient 2.4. What did I do wrong? Write to give evidence for your answer. Write a similar problem where you might make the same error. (DOK 3) How many different ways can you make your calculator show a number with a particular decimal, such as 12.34, without pressing the decimal point? (DOK 2) I added 3 decimals together and got exactly 4. What might those 3 decimals numbers be? Justify your answer with words. (DOK 3)

DWSBB 2: November 14 – March 3 **Flexible Pacing: 64 days**

NUMBERS AND OPERATIONS IN BASE TEN (NBT)
Topic 5 - Use Models and Strategies to Divide Whole Numbers
Topic 6 - Use Models and Strategies to Divide Decimals

Report Card Learning Targets
I can….
- Explain patterns in powers of ten
- Understand place value in the decimal system
- Model and divide multi-digit whole numbers
- Model and calculate decimals to the hundredths place using the four operations

TOPICS 5 and 6

Coherence		**pp. 55C-55D**
Look back: **Grade 4-** • Whole-Number Operations • Fraction Multiplication • Decimal Notation **Earlier in Grade 5-** • Decimal Place Value	**Topics 5 and 6:** • Multiplication with Whole Numbers and with Decimals • Division with Whole Numbers and with Decimals • Use Place-Value Concepts	**Look ahead:** **Later in Grade 5-** • Convert Measurements **Grade 6-** • Evaluate Expressions • Fluency with Whole Numbers and Decimals
Rigor		**p. 55E**
Conceptual Understanding: • Use Place Value and Properties to Add and Subtract Decimals • Use Place Value and Properties to Multiply Decimals • Use Place Value and Properties to Divide Multi-digit Whole Numbers and Decimals	**Procedural Skill and Fluency:** • Develop Fluency with Multiplying Multi-digit Whole Numbers • Divide Multi-digit Whole Numbers • Perform Computations with Decimals	**Applications:** • Solve Real-World Problems
Focus	**Strand: Mathematical Practice Standards #1 and #2**	**p. 55F**

5.MP.1 **5.MP.2**	**1. Make sense of problems and persevere in solving them.** (Topic 5) Explain the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. When a solution pathway does not make sense, look for another pathway that does. Explain connections between various solution strategies and representations. Upon finding a solution, look back at the problem to determine whether the solution is reasonable and accurate, often checking answers to problems using a different method or approach. ***Fifth grade students make sense of problems involving operations with whole numbers and decimals, plan how to solve them, and determine if their solutions make sense.*** I can give a good explanation of the problem. I can think about a plan before jumping into the solution. I can think of similar problems or use a simpler form. I can, if needed, organize data or use representations. I can make sure that the work being done and answer make sense. **2. Reason abstractly and quantitatively.** (Topic 6) Make sense of quantities and their relationships in problem situations. Contextualize quantities and operations by using images or stories. Decontextualize a given situation and represent it symbolically. Interpret symbols as having meaning, not just as directions to carry out a procedure. Know and flexibly use different properties of operations, numbers, and geometric objects. ***Fifth grade students use quantitative reasoning to estimate and perform mental math involving multi-digit whole numbers and decimals.*** I can identify and understand the quantities in the problem. I can show and explain how quantities are related (e.g., bar diagram). I can translate real-world contexts correctly to numbers, expressions, equations, or concrete or pictorial representations. I can connect numbers, expressions, equations, or concrete or pictorial representations back to real-world contexts.

Focus	Standards	Curriculum Supports – envision 2.0	Vocabulary
5.NBT.5 **5.NBT.6** **5.NBT.7** **(5.NBT.B)**	**Strand: Number and Operations in Base Ten** Fifth grade students will perform operations with multi-digit whole numbers and with decimals to hundredths. **Standard 5.NBT.5** Fluently multiply multi-digit whole numbers using the standard algorithm. **Standard 5.NBT.6** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place	**Topic 5: Use Models and Strategies to Divide Whole Numbers** *(pp. 237A-237C)* **5-1** Use Patterns and Mental Math to Divide *(pp. 239-244)* **5-2** Estimate Quotients with 2-Digit Divisors *(pp. 245-250)* **5-3** Use Models to Divide with 2-Digit Divisors *(pp. 251-256)* **5-4** Use Partial Quotients to Divide *(pp. 257-262)* **5-5** Divide by Multiples of 10 *(pp. 263-268)* **5-6** Use Estimation to Place the First Digit of the	**Topic 5:** No new vocabulary words Review as needed

	value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **Standard 5.NBT.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. In this standard, dividing decimals is limited to a whole number dividend with a decimal divisor or a decimal dividend with a whole number divisor. Compare the value of the quotient on the basis of the values of the dividend and divisor.	Quotient *(pp. 269-274)* **5-7** Divide by 2-Digit Divisors *(pp. 275-280)* **5-8 Math Practices and Problem Solving: Make Sense and Persevere** *(pp. 281-286)* **Topic 6: Use Models and Strategies to Divide Decimals** *(pp. 299A-299C)* **6-1** Patterns for Dividing with Decimals *(pp. 301-306)* **6-2** Estimate Decimal Quotients *(pp.307-312)* **6-3** Use Models to Divide by a 1-Digit Whole Number *(pp. 313-318)* **6-4** Divide by a 1-Digit Whole Number *(pp. 319-324)* **6-5** Divide by a 2-Digit Whole Number *(pp. 325-330)* **6-6** Use Number Sense to Divide Decimals *(pp. 331-336)* **6-7** Divide by a Decimal *(pp. 337-342)* **6-8** Continue to Divide with Decimals *(pp. 343-348)* **6-9 Math Practices and Problem Solving: Reasoning** *(pp. 349-354)*	**Topic 6:** No new vocabulary words Review as needed
	Assessment Options: **Topic 5 Assessment -** Use Models and Strategies to Divide Whole Numbers *(print or online)* ***(pp. 293-296)*** **Topic 5 Performance Assessment -** Use Models and Strategies to Divide Whole Numbers ***(pp. 297-298)***	**Topic 6 Assessment -** Use Models and Strategies to Divide Decimals *(print or online)* ***(pp. 361-364)*** **Topic 6 Performance Assessment -** Use Models and Strategies to Divide Decimals ***(pp. 365-366)***	

Assessment Tasks – Topics 5 and 6		
	Procedural Check	**Application Task**
5.NBT.5	Multiply 524 X 639 and show your work. (DOK 1)	A bakery has 245 dozen cupcakes. How many individual cupcakes are there? (DOK 1) How would you arrange the following numbers to create the largest possible product? Explain how you decided to place the numbers and how you know you created the largest product. (DOK 3)

		9 2 0 8 4 ? ? ? x ? ?
5 NBT.6	Solve 9984 | 64 using an area model. (DOK 2)	There are 1,716 students participating in Field Day. They are put into teams of 16 for the competition. How many teams get created? If you have left over students, what do you do with them? (DOK 2) Using mental computation, will the quotient of 812 ÷ 48 be over or under 16? Explain how you know. (DOK 3)
5.NBT.7	Solve 3.5 x 8 = ___ (DOK 1)	Jeanne makes hair bows to sell at the craft fair. Each bow requires 1.5 yards of ribbon. a) At the fabric store, ribbon is sold by the foot. If Jeanne wants to make 84 bows, how many feet of ribbon must she buy? Show all your work. (DOK 2) b) If the ribbon costs 10¢ per foot, what is the total cost of the ribbon in dollars? Explain your reasoning, including how you decided where to place the decimal. (DOK 3) Kyle can earn $6.75 an hour doing yard work. He can earn $7.25 per hour washing cars. If he washes cars for 6 hours, will he earn more money than doing yard work for 7 hours? Use words and a chart to justify your answer. How many hours of each activity would he have to do to earn the same amount of money? (DOK 3)

DWSBB 2: November 14 – March 3 **Flexible Pacing: 64 days**

NUMBERS AND OPERATIONS—FRACTIONS (NF)
Topic 7 - Use Equivalent Fractions to Add and Subtract Fractions

Report Card Learning Targets

I can….

- Add and subtract fractions with unlike denominators
- Solve word problems with fractions using multiplication

TOPIC 7

Coherence		**pp. 367C-367D**
Look back: **Grade 4-** • Factors and Multiples • Equivalent Fractions • Add and Subtract Fractions and Mixed Numbers with Like Denominators	**Topic 7:** • Find Common Denominators • Add and Subtract Fractions with Unlike Denominators • Use Estimation • Problems Involving Fractions and Mixed Numbers	**Look ahead:** **Later in Grade 5-** • Solve Problems Involving Measurements • Solve Problems Involving Data **Grade 6-** • Expressions and Equations with Fractions

Rigor		**p. 367E**
Conceptual Understanding: • Estimate Sums and Differences of Fractions and Mixed Numbers • Conceptual Development of Addition and Subtraction	**Procedural Skill and Fluency:** • Add and Subtract Fractions and Mixed Numbers with Unlike Denominators	**Applications:** • Addition and Subtraction Situations

Focus	**Strand: Mathematical Practice Standard #4**	**p. 367F**
5.MP.4	**Model with mathematics.** Identify the mathematical elements of a situation and create a mathematical model that shows the relationships among them. Identify important quantities in a contextual situation, use mathematical models to show the relationships of those quantities, analyze the relationships, and draw conclusions. Models may be verbal, contextual, visual, symbolic, or physical. ***Fifth grade students apply the math they know to use bar diagrams and equations to represent addition and subtraction of fractions with unlike denominators.*** I can identify the correct prior knowledge that needs to be applied to solve a problem. I can identify the hidden question(s) in multiple-step problems.	

	I can use numbers, symbols, and words to solve problems. I can use estimation as appropriate.		
Focus	**Standards**	**Curriculum Supports – envision 2.0**	**Vocabulary**
5.NF.1 **5.NF.2** **(5.NF.A)**	**Strand: Number and Operations—Fractions** Fifth grade students will use equivalent fractions as a strategy to add and subtract fractions. **Standard 5.NF.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* **Standard 5.NF.2** Solve real-world problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by, *for example, using visual fraction models or equations to represent the problem.* Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize 2/5 + 1/2 = 3/7 as incorrect result, by observing that 3/7 < 1/2.*	**Topic 7: Use Equivalent Fractions to Add and Subtract Fractions** *(pp. 367I-367L)* **7-1** Estimate Sums and Differences of Fractions *(pp. 371-376)* **7-2** Find Common Denominators *(pp. 377-382)* **7-3** Add Fractions with Unlike Denominators *(pp. 383-388)* **7-4** Subtract Fractions with Unlike Denominators *(pp. 389-394)* **7-5** Add and Subtract Fractions *(pp. 395-400)* **7-6** Estimate Sums and Differences of Mixed Numbers *(pp. 401-406)* **7-7** Use Models to Add Mixed Numbers *(pp. 407-412)* **7-8** Add Mixed Numbers *(pp. 413-418)* **7-9** Use Models to Subtract Mixed Numbers *(pp. 419-424)* **7-10** Subtract Mixed Numbers *(pp. 425-430)* **7-11** Add and Subtract Mixed Numbers *(pp. 431-436)* **7-12 Math Practices and Problem Solving: Model with Math** *(pp. 437-442)*	**Topic 7:** • benchmark fraction • equivalent fractions • common denominator • mixed number
	Assessment Options:	**Topic 7 Assessment -** Use Equivalent Fractions to Add and Subtract Fractions *(print or online)* ***(pp. 449-452)*** **Topic 7 Performance Assessment -** Use Equivalent Fractions to Add and Subtract Fractions ***(pp. 453-454)***	

Assessment Tasks – Topic 7		
	Procedural Check	**Application Task**
5.NF.1	**Solve:** $\frac{1}{4} + \frac{2}{3} =$ _____ $+\frac{8}{12} = \frac{11}{12}$ $\frac{2}{4} + \frac{1}{6} =$ _____ + _____ **= 1 whole** (DOK 1)	1/4 + 5/6 Robert added the fraction above and got an answer of 6/10. Use what you know about addition of fraction to explain why Robert's answer is incorrect. Formulate an original problem that would have the answer 6/10. (DOK 3)
5.NF.2	A recipe calls for $\frac{3}{4}$ cup breadcrumbs, $2\frac{1}{4}$ cups tomato sauce, and $\frac{1}{2}$ cup tomato paste. How many cups of ingredients are used altogether in the recipe? (DOK 1)	A Road Construction Company is paving a road. During the first week the company paved $15\frac{3}{8}$ miles. During the second week, the company paved $22\frac{1}{3}$ miles. How much of the road was paved during the first two weeks? The company will have to finish the 45-mile road during the third week of paving. How many miles will the company have to pave during the third week? Use what you know about fractions to explain why your answer is correct. (DOK 3)

DWSBB 2: November 14 – March 3

Flexible Pacing: 64 days

NUMBERS AND OPERATIONS—FRACTIONS (NF)
Topic 8 - Apply Understanding of Multiplication to Multiply Fractions

Report Card Learning Targets
I can….
- Multiply fractions
- Solve word problems with fractions using

TOPIC 8

Coherence		**pp. 455C-455D**
Look back: **Grade 4-** • Adding and Subtracting Fractions with Like Denominators • Multiplying Whole Numbers and Fractions **Earlier in Grade 5-** • Add and Subtract Fractions with Unlike Denominators	**Topic 8:** **Grade 4-** • Multiplication of Fractions • Multiplication of Mixed Numbers • Fractions and Division • Word Problems Involving Fractions	**Look ahead:** **Later in Grade 5-** • Fractions in Measurement • Fractions in Data **Grade 6-** • Fractions and Numerical Expressions • Fractions and Equations • Divide Fractions

Rigor		**p. 455E**
Conceptual Understanding: • Meanings of Multiplication • Understand Area Models of Multiplication • Meanings of Division	**Procedural Skill and Fluency:** • Fraction Multiplication	**Applications:** • Multiplication Situations • Division Situations

Focus	**Strand: Mathematical Practice Standard #1**	**p. 455F**
5.MP.1	**Make sense of problems and persevere in solving them.** Explain the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. When a solution pathway does not make sense, look for another pathway that does. Upon finding a solution, look back at the problem to determine whether the solution is reasonable and accurate, often checking answers to problems using a different method or approach. ***Fifth grade students persevere as they try to understand problems involving fractions, plan how to solve them, and determine if their solution makes sense.***	

	I can choose a strategy or strategies to use to solve problems. I can identify the quantities in a problem, the data given, and, if present, the question to be answered. I can think of similar problems or use a simpler form of the problem. I can, if needed, organize data or use representations to help make sense of a problem. I can identify likely strategies for solving the problem. I can pause when solving problems to make sure that the work being done makes sense. I can make sure the answer makes sense before stopping work.		
Focus	**Standards**	**Curriculum Supports – envision 2.0**	**Vocabulary**
5.NF.3 **5.NF.4** **5.NF.5** **5.NF.6** **(5.NF.B)**	**Strand: Number and Operations—Fractions** Fifth grade students will apply and extend previous understandings of multiplication and division to multiply and divide fractions. **Standard 5.NF.3** Interpret a fraction as division of the numerator by the denominator *(a/b = a ÷ b)*. Solve real-world problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, through the use of visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing three by four, noting that 3/4 multiplied by four equals three, and that when three wholes are shared equally among four people each person has a share of size 3/4. If nine people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* **Standard 5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product *(a/b) x q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a x q ÷ b* using a *visual fraction model. For example, use a fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation. Do the same with (2/3) x (4/5) = 8/15. (In general, (a/b) x (c/d) = ac/bd.)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side	**Topic 8: Apply Understanding of Multiplication to Multiply Fractions** *(pp. 455I-455K)* **8-1:** Use Models to Multiply a Whole Number by a Fraction (pp. 457-462) **8-2:** Use Models to Multiply a Fraction by a Whole Number *(pp. 463-468)* **8-3:** Multiply Fractions and Whole Numbers *(pp. 469-474)* **8-4:** Use Models to Multiply Two Fractions *(pp. 475-480)* **8-5:** Multiply Two Fractions *(pp. 481-486)* **8-6:** Area of a Rectangle *(pp. 487-492)* **8-7:** Multiply Mixed Numbers *(pp. 493-498)* **8-8:** Multiplication as Scaling *(pp. 499-504)* **8-9: Math Practices and Problem Solving: Make Sense and Persevere** *(pp. 505-510)*	**Topic 8:** No new vocabulary words Review as needed

	lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. **Standard 5.NF.5** Interpret multiplication as scaling. a. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. *For example, the products of expressions such as 5 x 3 or 1/2 x 3 can be interpreted in terms of a quantity, three, and a scaling factor, 5 or 1/2. Thus in addition to knowing that 5 x 3 = 15, they can also say that 5 x 3 is five times as big as three, without evaluating the product. Likewise they see 1.2 x 3 as half the size of three.* b. Explain why multiplying a given number by a fraction greater than one results in a product greater than the given number (recognizing multiplication by whole numbers greater than one as a familiar case); explain why multiplying a given number by a fraction less than one results in a product smaller than the given number; and relate the principle of fraction equivalence. *For example,6/10 = (2x3)/(2x5). In general, a/b = (n x a)/(n x b) has the effect of multiplying a/b by one.* **Standard 5.NF.6** Solve real-world problems involving multiplication of fractions and mixed numbers, *for example, by using visual fraction models or equations to represent the problem.*		
	Assessment Options:	**Topic 8 Assessment -** Apply Understanding of Multiplication to Multiply Fractions *(print or online) (pp. 517-520)* **Topic 8 Performance Assessment -** Apply Understanding of Multiplication to Multiply Fractions *(pp. 521-522)*	

District-Wide Standards-Based Benchmark #2 due by March 3

Assessment Tasks		
	Procedural Check	**Application Task**
5.NF.3	Write a word problem to show that 3/4 a division problem. Draw a model to illustrate the story problem. Write a word problem with a fraction less than 1 used as a division problem. Draw a model to illustrate the story problem. Write a word problem with a fraction greater than 1 used as a division problem. Draw a model to illustrate the story problem. (DOK 2)	Mrs. Baker's class is working on a Science project. There are 5 students in a project group. Mrs. Baker has 3 packs of construction paper for everyone to share. What fraction of a pack will each group get to use? Draw a representation to show your thinking. (DOK 2) Mario's family is going on a hike and he wants to have trail mix for a snack. Each person will get ½ cup of trail mix. The trail mix package says it contains 3 cups and Mario has 5 people. Will there be enough trail mix for each person to have at least ½ cup of trail mix? Explain using pictures, numbers and/or words. (DOK 3)
5.NF.4	Interpret the product with a visual model. 1/3 x 5 = 2 ¼ x 3 = 1/3 x 7/8 = (DOK 1) Find the area of a rug that is 3-1/2 feet by 2-1/2 feet. (DOK 1)	Maurice has ¾ yard of webbing. He only needs ½ of the piece for his scout project. What fraction of the original length of webbing does he need? Which of the following equations correctly represents the problem? Explain your choice. Solve. Draw a representation of the equation to justify your answer. ½ x ¾ = ? ¾ of ½ = ? (DOK 3)
5.NF.5	Without multiplying, which product is larger and why? 12 x 12 or 12 x 48 (DOK 2) Without multiplying, which product is larger and why? 12 x 1/5 or 6 x 1/5 (DOK 2)	Write an expression that will have a product less than both factors. Explain how you know your expression is correct? Write an expression that will have a product greater than both factors. Solve. Draw a representation to show your work. (DOK 3)

5.NF.6	**Solve:** $\frac{1}{4}$ **of** $\frac{2}{3}$**=** $\frac{3}{4}$ **of** $\frac{1}{2}$ **=** $\frac{3}{4}$ **of** $3\frac{1}{2}$**=** $\frac{1}{5}$ **of 3=** (DOK 1)	John uses ¾ of a gallon of paint for one room. How much paint does he need to do three rooms? Use words, numbers, and/or pictures to justify your answer. (DOK 3) The following is a recipe for vanilla cupcakes: **2 cups flour** **1/2 teaspoon salt** **2 teaspoons baking powder** **1/2 cup butter, softened** **3/4 cup sugar** **2 eggs** **1 cup milk** **1/2 teaspoon vanilla** **Yield: 24 cupcakes** Bella wants to bake a dozen cupcakes. How much vanilla would she need to use? Use words, pictures, number lines, etc. to justify your answer. (DOK 3)

DWSBB 3: March 6 – April 28 **Flexible Pacing: 35 days**

NUMBERS AND OPERATIONS—FRACTIONS (NF)
Topic 9 - Apply Understanding of Division to Divide Fractions

Report Card Learning Targets
I can….
- Divide fractions

TOPIC 9

Coherence		**pp. 455C-455D**
Look back: **Grade 4-** • Adding and Subtracting Fractions with Like Denominators • Multiplying Whole Numbers and Fractions **Earlier in Grade 5-** • Add and Subtract Fractions with Unlike Denominators	**Topic 9:** **Grade 4-** • Multiplication of Fractions • Multiplication of Mixed Numbers • Fractions and Division • Word Problems Involving Fractions	**Look ahead:** **Later in Grade 5-** • Fractions in Measurement • Fractions in Data **Grade 6-** • Fractions and Numerical Expressions • Fractions and Equations • Divide Fractions

Rigor		**p. 455E**
Conceptual Understanding: • Meanings of Multiplication • Understand Area Models of Multiplication • Meanings of Division	**Procedural Skill and Fluency:** • Fraction Multiplication	**Applications:** • Multiplication Situations • Division Situations

Focus	**Strand: Mathematical Practice Standard #8** **p. 455F**
5.MP.8	**Look for and express regularity in repeated reasoning.** Notice repetitions in mathematics when solving multiple related problems. Use observations and reasoning to find shortcuts or generalizations. Evaluate the reasonableness of intermediate results. ***Fifth grade students use repeated reasoning when they generalize about fraction operations.*** I can notice and describe when certain calculations or steps in a procedure are repeated. I can generalize from examples or repeated observations. I can recognize and understand appropriate shortcuts.

	I can evaluate the reasonableness of intermediate results.		
Focus	**Standards**	**Curriculum Supports – envision 2.0**	**Vocabulary**
5.NF.7 (5.NF.B)	**Strand: Number and Operations—Fractions** Fifth grade students will apply and extend previous understandings of multiplication and division to multiply and divide fractions. **Standard 5.NF.7** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. Use strategies to divide fractions by reasoning about the relationship between multiplication and division. Division of a fraction by a fraction is not a requirement at this grade. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient.* Use the relationship between multiplication and division to explain that *(1/3) ÷ 4 = 1/12* because *(1/12) x 4 = 1/3*.	**Topic 9: Apply Understanding of Division to Divide Fractions** *(pp. 523A-523C)* **9-1:** Fractions and Division *(pp. 527-532)* **9-2:** Fractions and Mixed Numbers as Quotients *(pp. 533-538)* **9-3:** Use Multiplication to Divide *(pp. 539-544)* **9-4:** Divide Whole Numbers by Unit Fractions *(pp. 545-550)* **9-5:** Divide Unit Fractions by Non-Zero Whole Numbers *(pp. 551-556)* **9-6:** Divide Whole Numbers and Unit Fractions *(pp. 537-562)* **9-7:** Solve Problems Using Division *(pp. 563-568)* **9-8: Math Practices and Problem Solving: Repeated Reasoning** *(pp. 569-574)*	**Topic 9:** • unit fractions
	Assessment Options:	**Topic 9 Assessment -** Apply Understanding of Division to Divide Fractions *(print or online)* ***(pp. 579-580)*** **Topic 9 Performance Assessment -** Apply Understanding of Division to Divide Fractions ***(pp. 581-582)***	

Assessment Tasks – Topic 9		
	Procedural Check	**Application Task**
5.NF.7	1/9 ÷ 3 = n 12 ÷ ¼ = n (DOK 1)	Create a story context for (1/4) ÷ 6, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/4) ÷ 6= 1/12 because (1/12) × 4 = 1/3. (DOK 3) Students are running in a relay race. Each team will run a total of 3 miles. Each member of a team may only run 1/4 of a mile. How many students will each team need to complete the 3-mile race? Draw a picture to justify your answer. (DOK 3)

DWSBB 3: March 6 – April 28 **Flexible Pacing: 35 days**

MEASUREMENT AND DATA (MD)
Topic 10 - Understand Volume Concepts

Report Card Learning Targets
I can….
- Understand and measure volume

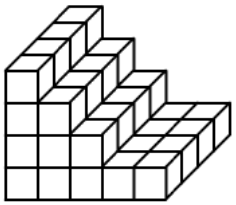
TOPIC 10

Coherence		**pp. 583C-583D**
Look back: **Grade 4-** • Apply Formulas to Solve Perimeter and Area Problems **Earlier in Grade 5-** • Fluently Multiply Whole Numbers	**Topic 10:** • Model Volume and Develop Formulas • Solve Problems Involving Volume	**Look ahead:** **Later in Grade 5-** • Relate Volume and Capacity **Grade 6-** • Solve Volume Problems

Rigor		**p. 583E**
Conceptual Understanding: • Connect Volume and a Formula for Finding the Number of Cubes • Relate Volume to Addition	**Procedural Skill and Fluency:** • Use a Formula to Find the Volume of a Rectangular Prism	**Applications:** • Find the Volume of Objects • Use Volume to Find the Dimensions of a Right Rectangular Prism • Use the Additive Property of Volume

Focus	**Strand: Mathematical Practice Standard #5**	**p. 583F**
5.MP.5	**Use appropriate tools strategically.** Consider the tools that are available when solving a mathematical problem, whether in a real-world or mathematical context. Choose tools that are relevant and useful to the problem at hand, such as drawings, diagrams, technologies, and physical objects and tools, as well as mathematical tools such as estimation or a particular strategy or algorithm. ***Fifth grade students select from tools such as cubes, grids, and geometry software to solve problems involving volume.*** I can identify available tools. I can use tools correctly and accurately.	

	I know when to use a particular tool. I can consider options before selecting a particular tool. I can decide if the results obtained using a tool make sense. I can think about using tools to explore and solve problems, without prompting from the teacher.		
Focus	**Standards**	**Curriculum Supports – envision 2.0**	**Vocabulary**
5.MD.3 **5.MD.4** **5.MD.5** **(5.MD.C)**	**Strand: Measurement and Data** Fifth grade students will understand concepts of geometric measurement and volume, as well as how multiplication and addition relate to volume. **Standard 5.MD. 3** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length one unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. **Standard 5.MD.4** Measure volumes by counting unit cubes, using cubic cm, cubic in., cubic ft., and improvised units. **Standard 5.MD.5** Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. *a.* Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, *for example, to represent the associative property of multiplication.*	**Topic 10: Understand Volume Concepts** *(pp. 583I-583J)* **10-1:** Model Volume *(pp. 587-592)* **10-2:** Develop a Volume Formula *(pp. 593-598)* **10-3:** Volume of Prisms *(pp. 599-604)* **10-4:** Combine Volumes of Prisms *(pp. 605-610)* **10-5:** Solve Word Problems Using Volume *(pp. 611-616)* **10-6: Math Practices and Problem Solving: Use Appropriate Tools** *(pp. 617-622)*	**Topic 10:** • volume • cubic unit • cube • rectangular prism • unit cube • formula

	Assessment Options: **Topic 10 Assessment -** Understand Volume Concepts *(print or online) (pp. 627-628)*	**Topic 10 Performance Assessment -** Understand Volume Concepts *(pp. 629-630)*	
Assessment Tasks – Topic 10			
	Procedural Check	**Application Task**	
5.MD.3	Form a group of solids, identify a cube and recognize that each is a cubic unit used to measure volume. (DOK 1) How many cubic cm would it take to fill a rectangular prism with the height of 2 cm, length 3 cm, and width 5 cm? (DOK 1)	Ask students to design a science station for the ocean floor that is composed of several rooms that are right rectangular prisms and that meet a set criterion specifying the total volume of the station. They draw their station and justify how their design meets the criterion. (DOK 3) Give students a net and ask them to predict the number of cubes required to fill the container formed by the net. Have them fold the net to make the shape so they can see how many rectangles fit together to determine the number of layers. Have them find the volume and write to justify their answer and how they found the volume. (DOK 3)	
5.MD.4	Identify cubic measures and transfer to volume using correct unit measurement. (DOK 2) Provide students with three-dimensional drawings that represent cubic units, then find the volume and correctly label. (DOK 1)	Give students 24 cubes and have them make as many rectangles as possible with a volume of 24 cubic units. Students build the prisms and record possible dimensions. (DOK 2) Length / Width / Height 1 / 2 / 12 2 / 2 / 6 4 / 2 / 3 8 / 3 / 1	
5.MD.5	What is the volume of this stack of blocks? Answer: 44 cubic units  (DOK 1)	Give the students three or four rectangular prisms (tissue box, cereal box, etc.). Ask them to predict which box has the greatest volume and explain their choice. Then ask students to find the volume of each solid compare their prediction to the actual box that has the greatest volume. Have them justify their work. (DOK 3)	

DWSBB 3: March 6 – April 28 **Flexible Pacing: 35 days**

MEASUREMENT AND DATA (MD)
Topic 11 - Convert Measurements

Report Card Learning Targets
I can….
- Solve problems using measurement conversions

TOPIC 11

Coherence		**pp. 631C-631D**
Look back: **Grade 4-** • Find Equivalence in Units of Measure **Earlier in Grade 5-** • Identify Patterns in Place Value • Multiply and Divide with Fractions and Mixed Numbers	**Topic 11:** • Use Multiplication to Convert from a Larger Unit to a Smaller Unit • Use Division to Convert from a Smaller Unit to a Larger Unit • Place Value and Metric Conversion	**Look ahead:** **Later in Grade 5-** • Analyze Patterns and Relationships **Grade 6-** • Convert Measurements

Rigor		**p. 631E**
Conceptual Understanding: • Understand Attributes • Customary Units of Measure • Metric Units of Measure	**Procedural Skill and Fluency:** • Use Multiplication or Division to Convert Units	**Applications:** • Measurement Problems

Focus	**Strand: Mathematical Practice Standard #6**	**p. 631F**
5.MP.6	**Attend to precision.** Communicate precisely to others by crafting careful explanations that communicate mathematical reasoning by referring specifically to each important mathematical element, describing the relationships among them, and connecting their words clearly to their representations. Calculate accurately and efficiently, and use clear and concise notation to record work. ***Fifth grade students attend to precision when they consider the symbols and units as they calculate conversions.*** I can compute accurately. I can use symbols appropriately. I can accurately use problem-solving strategies. I can specify and use units of measure appropriately. I can decide whether an exact answer or estimate is needed.	

	I can calculate efficiently, accurately, and fluently.		
Focus	**Standards**	**Curriculum Supports – envision 2.0**	**Vocabulary**
5.MD.1 (5.MD.A)	**Strand: Measurement and Area** Fifth grade students will convert like measurement units within a given measurement system. **Standard 5.MD.1** Convert among different-sized standard measurement units within a given measurement system (*for example, convert 5 cm to 0.05 m*); use these conversions in solving multi-step, real-world problems.	**Topic 11: Convert Measurements** *(pp. 631-631K)* **11-1:** Convert Customary Units of Length *(pp. 639-644)* **11-2:** Convert Customary Units of Capacity *(pp. 645-650)* **11-3:** Convert Customary Units of Weight *(pp. 651-656)* **11-4:** Convert Metric Units of Length *(pp. 657- 662)* **11-5:** Convert Metric Units of Capacity *(pp. 663-668)* **11-6:** Convert Metric Units of Mass *(pp. 669-674)* **11-7:** Solve Word Problems Using Measurement Conversions *(pp. 675-680)* **11-8:** **Math Practices and Problem Solving: Precision** *(pp. 681-686)*	**Topic 11:** • foot (ft) • inch (in.) • yard (yd) • mile (mi) • capacity • gallon (gal) • quart (qt) • pint (pt) • cup (c) • fluid ounce (fl oz) • weight • ton (T) • pound (lb) • ounce (oz) • kilometer (km) • meter (m) • centimeter (cm) • millimeter (mm) • liter (L) • milliliter (mL) • mass • milligram (mg) • gram (g) • kilogram (kg)
	Assessment Options:	**Topic 11 Assessment -** Convert Measurements *(print or online)* ***(pp. 691-692)*** **Topic 11 Performance Assessment -** Convert Measurements ***(pp. 693-694)***	

Assessment Tasks – Topic 11		
	Procedural Check	**Application Task**
5.MD.1	Convert the following: • Meters to kilometers • Yards to miles • Grams to kilograms • Ounces to pound • Cups to gallons (DOK 1)	A fifth grade class is running a 5k race. The class will begin practicing to increase their endurance, starting with 1,500 meters and adding 500 meters each week. The coach wants them to be able to run at least 5k before the actual race. How many weeks will it take for them to run farther than 5 kilometers? Create a table to justify your answer. (DOK 3) A fifth grade class is running a three-mile race. The class will begin practicing to increase their endurance, starting with 880 yards and adding 440 yards per week. The coach wants them to be able to run at least three miles before the actual race. How many weeks will it take for them to run farther than three miles? Create a table to justify your answer. (DOK 3) Zuri, the baby elephant was born August 10, 2009 at Hogle Zoo. The calf weighed 251 lbs. at birth. If the baby elephant gains 48 ounces a day, how much will she weigh at the end of 7 days? If Zuri continues to gain at this same rate, predict her weight at one month. Create a table and an expression to justify your answer. (DOK 3) Nutritionists recommend drinking eight glasses of water each day. If a glass holds 250 mL, does the recommended daily water intake exceed 1L? Use pictures to justify your answer. (DOK 3)

DWSBB 3: March 6 – April 28 **Flexible Pacing: 35 days**

MEASUREMENT AND DATA (MD)
Topic 12 - Represent and Interpret Data

Report Card Learning Targets
I can….
- Solve problems in all operations using line plots

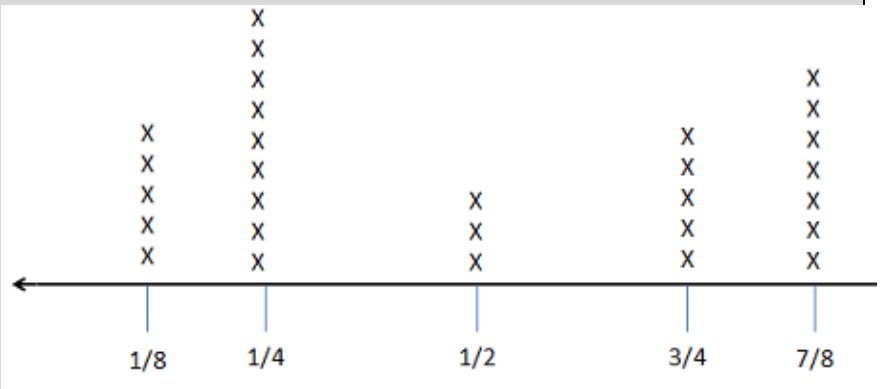
TOPIC 12

Coherence		**pp. 695C-695D**
Look back: **Grade 4-** • Line plots **Earlier in Grade 5-** • Add and Subtract Fractions • Multiply and Divide Fractions	**Topic 12:** • Analyze and Make Line Plots • Use Data Represented in a Line Plot to Solve Problems • Use Fractions Operations in Data Problems	**Look ahead:** **Grade 6-** • Display and Summarize Data • Histograms

Rigor		**p. 695E**
Conceptual Understanding: • Number Lines and Line Plots • Frequency Tables and Line Plots • Outlier in a Line Plot	**Procedural Skill and Fluency:** • Read and Analyze a Line Plot • Make a Line Plot	**Applications:** • Real-World Data • Use Fraction Operations to Solve Problems Involving Data in Line Plots

Focus	**Strand: Mathematical Practice Standard #3**	**p. 695F**
5.MP.3	**Construct viable arguments and critique the reasoning of others.** Use stated assumptions, definitions, and previously established results to construct arguments. Explain and justify the mathematical reasoning underlying a strategy, solution, or conjecture by using concrete referents such as objects, drawings, diagrams, and actions. Listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and build on those arguments.	

	Fifth grade students critique others' interpretations of data represented on a line plot. I can ask questions to understand other people's thinking. I can identify mistakes in other people's thinking. I can provide suggestions for improving other people's thinking.		
Focus	**Standards**	**Curriculum Supports – envision 2.0**	**Vocabulary**
5.MD.2 (5.MD.B)	**Strand: Measurement and Data** Fifth grade students will represent and interpret data. **Standard 5.MD.2** Make a line plot to display a data set of measurements in fractions of a unit (halves, quarters, eighths). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given graduated cylinders with different measures of liquid in each, find the amount of liquid each cylinder would contain if the total amount in all the cylinders were redistributed equally.*	**Topic 12: Represent and Interpret Data** *(pp. 695I-695J)* **12-1:** Analyze Line Plots *(pp. 699-704)* **12-2:** Make Line Plots *(pp. 705-710)* **12-3:** Solve Word Problems Using Measurement Data *(pp. 711-716)* **12-4: Math Practices and Problem Solving: Critique Reasoning** *(pp. 717-722)*	**Topic 12:** • data • line plot • outlier
	Assessment Options:	**Topic 12 Assessment –** Represent and Interpret Data *(print or online)* ***(pp. 727-728)*** **Topic 12 Performance Assessment –** Represent and Interpret Data ***(pp. 729-730)***	
District-Wide Standards-Based Benchmark #3 due by April 28			

Assessment Tasks - Topic 12		
	Procedural Check	**Application Task**
5.MD.2	With a given set of data, students will create a line plot (DOK 1)	Ask students to select objects from their desk that they think are less than one inch and then, without measuring, organize the objects into groups of ¼, ½, ¾ inch. They should have at least 10 objects. Then ask the students to measure the objects to the nearest 1/8 inch, and record their measurements on a line plot as shown below. Which length had the greatest, fewest number of objects? Ask students to compare their own line plot with other students and record their comparisons. (DOK 2)  Give the line plot above a title. Create a story problem to describe the data on the line plot. (DOK 3)

DWSBB 4: May 1- June 6 **Flexible Pacing: 25 days**

OPERATIONS AND ALGEBRAIC THINKING (OA)
Topic 13 – Write and Interpret Numerical Expressions

Report Card Learning Targets
I can….
- Solve numerical expressions using parentheses

TOPIC 13

Coherence		**pp. 731C-731D**
Look back: **Grade 4-** • Use the Distributive Property • Solve Multi-Step Problems **Earlier in Grade 5-** • Use Operations to Solve Problems	**Topic 13:** • Use the Order of Operations	**Look ahead:** **Later in Grade 5-** • Describe Relationships **Grade 6-** • Exponents and the Order of Operations • Understand Algebraic Expressions

Rigor		**p. 731E**
Conceptual Understanding: • Recognize the Need for the Order of Operations • Interpret Numerical Expressions Without Calculating	**Procedural Skill and Fluency:** • Use the Order of Operations to Evaluate a Numerical Expression	**Applications:** • Write Numerical Expressions to Solve Problems • Interpret Numerical Expressions Without Evaluating Them

Focus	**Strand: Mathematical Practice Standard #2** **p. 731F**
5.MP.2	**Reason abstractly and quantitatively.** Make sense of quantities and their relationships in problem situations. Contextualize quantities and operations by using images or stories. Decontextualize a given situation and represent it symbolically. Interpret symbols as having meaning, not just as directions to carry out a procedure. Know and flexibly use different properties of operations, numbers, and geometric objects. ***Fifth grade students use reasoning when they use properties of operations to write equivalent numerical expressions.*** I can identify and understand the quantities in the problem. I can show and explain how quantities are related (e.g., bar diagram).

	I can translate real-world contexts correctly to numbers, expressions, equations, or concrete or pictorial representations. I can connect numbers, expressions, equations, or concrete or pictorial representations back to real-world contexts.		
Focus	**Standards**	**Curriculum Supports – envision 2.0**	**Vocabulary**
5.OA.1 **5.OA.2** **(5.OA.A)**	**Strand: Operations and Algebraic Thinking** Fifth Grade students write and interpret numerical expressions. **Standard 5.OA.1** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. **Standard 5.OA.2** Write and interpret simple numerical expressions. ***a.*** Write simple expressions that record calculations with numbers. *For example, use 2 x (8+7) to express the calculation "add 8 and 7, then multiply by 2.*" **b.** Interpret numerical expressions without evaluating them. *For example use conceptual understanding of multiplication to interpret 3 x (18939 + 921) as being three times as large as18932 + 921 without calculating the indicated sum or product.*	**Topic 13: Write and Interpret Numerical Expressions** *(pp. 731I-731J)* **13-1:** Order of Operations *(pp. 735-740)* **13-2:** Evaluate Expressions *(pp. 741-746)* **13-3:** Write Numerical Expressions *(pp. 747-752)* **13-4:** Interpret Numerical Expressions *(pp. 753-758)* **13-5: Math Practices and Problem Solving: Reasoning** *(pp. 759-764)*	**Topic 13:** • numerical expression • evaluate • order of operations • parentheses • brackets • braces
	Assessment Options:	**Topic 13 Assessment -** Write and Interpret Numerical Expressions *(print or online)* *(pp. 769-770)* **Topic 13 Performance Assessment -** Write and Interpret Numerical Expressions *(pp. 771-772)*	

Assessment Tasks – Topic 13		
	Procedural Check	**Application Task**
5.OA.1	What is the value of the expression? 50 ÷ (2 + 8) – 3 (DOK 1)	Monique went to the store to buy groceries for her party. She bought 5 bananas for 50 cents each. She also bought 4 cartons of ice cream for $3.00 each. At checkout, she was given 10 cents off the bananas. Write an expression that represents the problem. You may use models if you choose to do so. Then solve the problem to determine how much Monique spent in all. Explain your reasoning to justify your work. (DOK 3) Answer: [(5 X .50) - .10)] + (4 X 3.00) = $14.40
5.OA.2	Alex and Chet both collect cards. Write an algebraic equation to show that Alex has twice as many cards as Chet. Let c represent the number of cards Chet has. (DOK 2)	Adam is twice the height of 4 increased by 9. Wendell is three times the height of 5 decreased by 2. Who is taller? Use words and pictures to justify your answer. (DOK 3)

DWSBB 4: May 1- June 6 **Flexible Pacing: 25 days**

GEOMETRY
Topic 14 - Graph Points on the Coordinate Plane

Report Card Learning Targets
I can….
- Solve problems using points on a coordinate plane

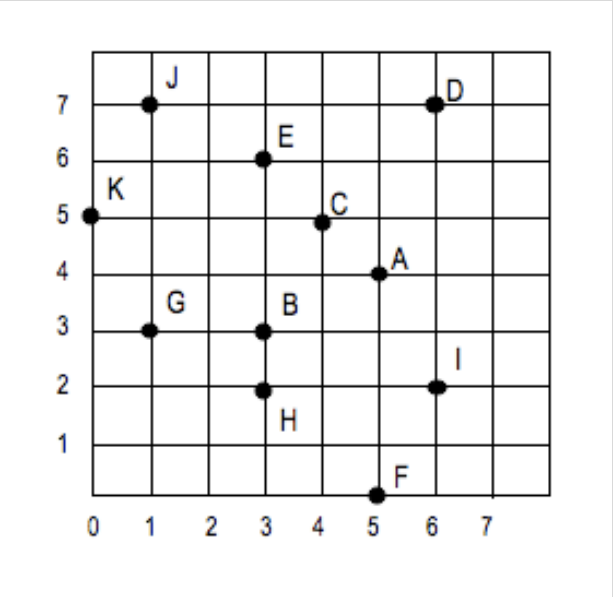
TOPIC 14

Coherence		**pp. 773C-773D**
Look back: **Grade 4-** • Line Plots **Earlier in Grade 5-** • Analyze Line Plots •	**Topic 14:** • Graph Points • Solve Problems in Context	**Look ahead:** **Later in Grade 5-** • Analyze and Graph Relationships **Grade 6-** • Graph in All Four Quadrants

Rigor		**p. 773E**
Conceptual Understanding: • Understand the Coordinate Grid • Understand Ordered Pairs	**Procedural Skill and Fluency:** • Graph Points	**Applications:** • Real-World Contexts

Focus	**Strand: Mathematical Practice Standard #2**	**p. 773F**
5.MP.2	**Reason abstractly and quantitatively.** Make sense of quantities and their relationships in problem situations. Contextualize quantities and operations by using images or stories. Decontextualize a given situation and represent it symbolically. Interpret symbols as having meaning, not just as directions to carry out a procedure. Know and flexibly use different properties of operations, numbers, and geometric objects. ***Fifth grade students use a graph to recognize relationships between quantities.*** I can identify and understand the quantities within a problem. I can show and explain how quantities are related (e.g., bar diagram). I can translate real-world contexts correctly to numbers, expressions, equations, or concrete or pictorial representations. I can connect numbers, expressions, equations, or concrete or pictorial representations back to real-world contexts.	

Focus	Standards	Curriculum Supports – envision 2.0	Vocabulary
5.G.1 **5.G.2** **(5.G.A)**	**Strand: Geometry** Fifth grade students will graph points on the coordinate plane to solve real-world and mathematical problems in quadrant one. **Standard 5.G.1** Compose and understand the coordinate plane. a. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the zero on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. b. Using quadrant one on the coordinate plane, understand that the first number in a coordinate pair indicates how far to travel from the origin in the direction of the horizontal axis, and the second number indicates how far to travel in the direction of the vertical axis, with the convention that the names of the two axes and the coordinates correspond (*x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate). **Standard 5.G.2** Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	**Topic 14: Graph Points On the Coordinate Plane** *(pp. 773I-773J)* **14-1:** The Coordinate System *(pp. 777-782)* **14-2:** Graph Data Using Ordered Pairs *(pp. 783-788)* **14-3:** Solve Problems Using Ordered Pairs *(pp. 789-794)* **14-4: Math Practices and Problem Solving: Reasoning** *(pp. 795-800)*	**Topic 14:** • coordinate grid • ordered pair • X-axis • Y-axis • origin • X-coordinate • Y-coordinate
	Assessment Options:	**Topic 14 Assessment -** Graph Points on the Coordinate Plane *(print or online)* *(pp. 805-806)* **Topic 14 Performance Assessment -** Graph Points on the Coordinate Plane *(pp. 807-808)*	

Assessment Tasks – Topic 14		
	Procedural Check	**Application Task**
5.G.1	On a coordinate grid, have students identify a specific point (e.g., What are the coordinates of the point where the bird is located?) (DOK 1) Give the students ordered pairs that they must match to points on the plane within the first quadrant. (DOK 1)	Give students a map of the school on a coordinate grid and ask them to identify where certain places in the school are found. Then have students tell what is located at a particular given coordinate pair. (DOK 1) Or Ask students to create a coordinate grid of the classroom using actual measurements. Then ask them to locate objects on the grid and determine the distances between the objects. They could create question sets to challenge a partner or other group to identify an object. For example, I am thinking of an object that is located at ((5, 8). (DOK 3) Points G, B, and H are 3 corners of a rectangle. What is the ordered pair of the fourth corner?  _____

		Once you have found the ordered pair of the fourth corner, connect the corners to create the rectangle. Are there other sets of 3 points that form corners of rectangles? Identify at least two other sets of 3 points. Compare the areas of the rectangles. (DOK 3)
5.G.2	Give students a map that has the coordinate grid overlaid. Ask students to find the coordinates of specifics landmarks and have them find landmarks at given ordered pairs. (DOK 1)	The local department store has recently released its information regarding video game sales over the last nine months. In the first month they sold 75 games. In the next month they sold 72. In the third month they sold 60, and in the fourth they sold 42. In the fifth month they sold 45, in the sixth they sold 38, in the seventh they sold 56, in the eighth they sold 62, and in the ninth they sold 79. Organize this data in a chart and then plot it on a coordinate grid to help the store understand their video games sales. Can you predict future sales based on this data? Use the chart to justify your answer. (DOK 3)

DWSBB 4: May 1- June 6 **Flexible Pacing: 25 days**

OPERATIONS AND ALGEBRAIC THINKING (OA)
Topic 15 - Algebra: Analyze Patterns and Relationships

Report Card Learning Targets
I can….
- Create patterns and analyze their relationships

TOPIC 15

Coherence		**pp. 809C-809D**
Look back: **Grade 4-** • Generate and Analyze Patterns **Earlier in Grade 5-** • Analyze Patterns • Coordinate Graphs	**Topic 15:** • Look for Relationships • Coordinate Graphs	**Look ahead:** **Grade 6-** • Dependent and Independent Variables
Rigor		**p. 809E**
Conceptual Understanding: • Understand Relationships	**Procedural Skill and Fluency:** • Extend patterns • Graph Relationships	**Applications:** • Real-World Patterns

Focus	**Strand: Mathematical Practice Standard #1** **p. 809F**
5.MP.1	**Make sense of problems and persevere in solving them.** Explain the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. When a solution pathway does not make sense, look for another pathway that does. Explain connections between various solution strategies and representations. Upon finding a solution, look back at the problem to determine whether the solution is reasonable and accurate, often checking answers to problems using a different method or approach. ***Fifth grade students persevere as they generate and analyze patterns, determine rules, and then graph rules to solve problems.*** I can give a good explanation of the problem. I can think about a plan before jumping into the solution. I can think of similar problems, try special cases, or use a simpler form of the problem. I can, if needed, organize data or use representations to help make sense of the problem. I can identify likely strategies for solving the problem. I can pause when solving problems to make sure that the work being done makes sense.

	I can make sure that the answer makes sense before stopping work.		
Focus	**Standards**	**Curriculum Supports – envision 2.0**	**Vocabulary**
5.OA.3 (5.OA.B)	**Strand: Operations and Algebraic Thinking** Fifth Grade students will classify two-dimensional figures into categories based on their properties. **Standard 5.OA.3** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "add 3" and the starting number 0, and given the rule "add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*	**Topic 15: Algebra: Analyze Patterns and Relationships** *(pp. 809I-809J)* **15-1:** Numerical Patterns *(pp. 813-818)* **15-2:** More Numerical Patterns *(pp. 819-824)* **15-3:** Analyze and Graph Relationships *(pp. 825-830)* **15-4: Math Practices and Problem Solving: Make Sense and Persevere** *(pp.831-836)*	**Topic 15:** • corresponding terms • number sequence
	Assessment Options: **Topic 15 Assessment -** Algebra: Analyze Patterns and Relationships *(print or online)* ***(pp. 841-842)***	**Topic 15 Performance Assessment -** Algebra: Analyze Patterns and Relationships ***(pp. 843-844)***	

Assessment Tasks		
	Procedural Check	**Application Task**
5.OA.3	Generate two numerical patterns Example: "add 2" and "add 4" (DOK 1) Answer: Add 2: 2, 4, 6, 8, 1, 12 Answer: Add 4: 4, 8, 12, 16, 20 Have students graph ordered pairs on a coordinate grid. (DOK 1)	Terri catches 4 fish each day, and Sam catches 2 fish, the amount of Terri's fish is always greater. Terri's fish is also always twice as much as Sam's fish. Today, both Sam and Terri have no fish. They both go fishing each day. Sam catches 2 fish each day. Terri catches 4 fish each day. How many fish do they have after each of the five days? Make a table and create a graph of the number of fish. Write to justify your answer. (DOK 3)

DWSBB 4: May 1- June 6 **Flexible Pacing: 25 days**

GEOMETRY (G)

Topic 16 - Geometric Measurement: Classify Two-Dimensional Figures

Report Card Learning Targets
I can….
- Classify 2D shapes by their properties

TOPIC 16

Coherence		**pp. 845C-845D**
Look back: **Grade 4-** • Understand Angles • Classify Shapes **Earlier in Grade 5-** • Graph Polygons	**Topic 16:** • Classify by Attributes • Hierarchy of Quadrilaterals	**Look ahead:** **Grade 6-** • Solve Area Problems • Solve Surface Area Problems
Rigor		**p. 845E**
Conceptual Understanding: • Hierarchy of Quadrilaterals	**Procedural Skill and Fluency:** • Classify Shapes	**Applications:** • Analyze Shapes in the Real World

Focus	**Strand: Mathematical Practice Standard #3**	**p. 845F**
5.MP.3	**Construct viable arguments and critique the reasoning of others.** Use stated assumptions, definitions, and previously established results to construct arguments. Explain and justify the mathematical reasoning underlying a strategy, solution, or conjecture by using concrete referents such as objects, drawings, diagrams, and actions. Listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and build on those arguments. ***Fifth grade students construct arguments to justify or rebut conjectures about classifying shapes.*** I can provide complete and clear explanations of my thinking and work. I can decide if other students' explanations make sense; clarify or improve other students' arguments. I can use counterexamples when appropriate.	

Focus	**Standards**	**Curriculum Supports – envision 2.0**	**Vocabulary**
5.G.3 **5.G.4** **(5.G.B)**	**Strand: Geometry** Fifth Grade students will classify two-dimensional figures into categories based on their properties.	**Topic 16: Geometric Measurement: Classify Two-Dimensional Figures** *(pp. 845I-845J)* **16-1:** Classify Triangles *(pp. 851-856)*	**Topic 16:** • equilateral triangle • isosceles

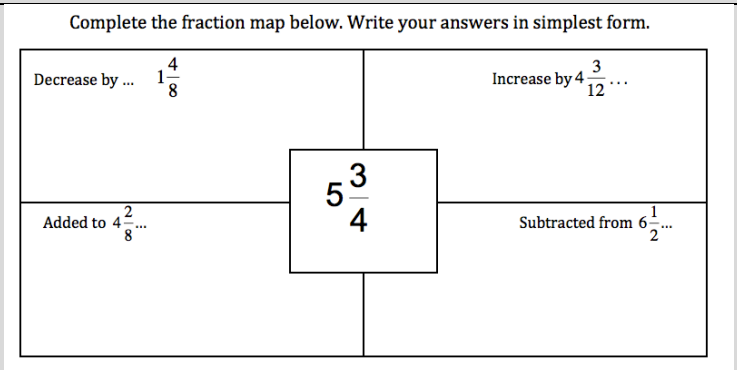
	Standard 5.G.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and all squares are rectangles, so all squares have four right angles.* **Standard 5.G.4** Classify two-dimensional figures in a hierarchy based on properties.	**16-2:** Classify Quadrilaterals *(pp. 857-862)* **16-3:** Continue to Classify Quadrilaterals *(pp. 863-868)* **16-4:** **Math Practices and Problem Solving: Construct Arguments** *(pp. 869-874)*	triangle • scalene triangle • right triangle • acute triangle • obtuse triangle • trapezoid • parallelogram • rectangle • rhombus • square
	Assessment Options:	**Topic 16 Assessment -** Geometric Measurement: Classify Two-Dimensional Figures *(print or online)* ***(pp.879-880)*** **Topic 16 Performance Assessment -** Geometric Measurement: Classify Two-Dimensional Figu *881-882)*	

District-Wide Standards-Based Benchmark #4 due by June 6		
Assessment Tasks – Topic 16		
	Procedural Check	**Application Task**
5.G.3	A parallelogram has four sides, with both sets of opposite sides parallel. What types of quadrilaterals are parallelograms? (DOK 2) Regular polygons have congruent sides and angles. Name and draw some regular polygons. (DOK 1)	Provide a series of "sometimes, never, or always" questions, and require that students provide a written or pictorial explanation for each answer. For example: • A parallelogram is a square- sometimes, never, or always? Explain how you know Explain why all squares are rectangles, but not all rectangles are squares. (DOK 3)
5.G.4	Have students identify all the polygons they can find in a piece of geometric art. (DOK 1) http://interiorcomplex.com/accessories/20-modern-geometric-art-prints/#	Have students make their own piece of art, making sure to include a variety of polygons. Have students use their understanding of hierarchy to make the piece (for example, the top of the picture can contain any polygons, but as it goes down it has to use more specific types of polygons). (DOK 2) Name at least two other quadrilaterals that a square can be classified as. Explain using what you know about the properties of quadrilaterals. Use pictures and labels to justify your answer. (DOK 3)

5th Grade Additional Assessment Tasks

Assessment Tasks		
	Procedural Check	**Application Task**
5.OA.2	Write an expression for calculations given in words such as the following: "Divide 144 by 12, and then subtract 7." (DOK 2) Answer: (144 ÷ 12) - 7	Mara bought 6 bags of Skittles at $0.85 each and 9 packs of gum at $1.20 each. Write an expression. Explain your reasoning. How do you know your expression fits the problem? (DOK 3) Answer: (6 X 0.85) + (9 X 1.20)
5.OA.2	**Write an expression with each.** A plane can fly 425 miles for each hour it flies. How far will it fly in h hours? A number of students divided into 4 teams. 4 times a number n increased by 3. Explain how you came up with each expression. (DOK 2)	Miles has n number of baseball cards. He keeps the same number of cards in each of three boxes. What expression represents the number of baseball cards Miles can put in each box? Use what you know about expressions to justify why your answer is correct. (DOK 3)
5 NBT.1	How many tenths are in 2.5? (DOK 1) How much larger is 200 than 20? How much smaller is 0.3 than 3? (DOK 2)	Using base-ten blocks, show at least two ways to represent the number 3.2 (e.g., use a flat as a unit, a stick as 1/10 and a single as 1/100) Give evidence to justify your answer. (DOK 3) Use words, numbers, and a model to explain why each of the digits in the number 4.44 has a different value. Be sure to use "ten times as large" and "one tenth as large" in your explanation. (DOK 3)
5NBT.1	One pound of apples costs $1.99. How much would ten pounds cost? (DOK 1)	Explain why 4 ÷ 10 = 0.4 Draw pictures to illustrate your explanations. (DOK 3)

		Jesse puts 10 jellybeans on a scale and the scale reads 12.0 grams. How much would you expect 1 jellybean to weigh? Explain your thinking. (DOK 3) How are .7, 7, 70 related? Draw pictures to explain your reasoning. (DOK 3)
5.NBT.2	Express the missing divisor using a power of 10. Explain your reasoning using a place value model. (DOK 2) a. 5.2 ÷ _____ = 0.052 b. 7,650 ÷ _____ = 7.65	Generate and solve another division problem with the same quotient and remainder as the two problems below. Explain your strategy for creating the new problem. $\begin{array}{r} 3 \\ 17\overline{)63} \\ 51 \\ \hline 12 \end{array}$ $\begin{array}{r} 3 \\ 42\overline{)138} \\ 126 \\ \hline 12 \end{array}$ (DOK 3)
5 NBT.2	Explain why the following multiplication and division problems with the powers of ten are true. (DOK 3) $432 \text{ X } 10^3 = 432000$ $4.32 \text{ X } 10^2 = 432$ $43.2 \text{ X } 10^1 = 432$	Martha earned $4.20 each day for ten days of babysitting. Over a year's time, she worked ten times ten days. Write an expression using exponents of 10 to show how much she earned in ten days. (DOK 2) Joe is multiplying 64.15 x 10 so he put a zero at the end of the number to get his answer. 64.15 x 10 = 64.150. Explain why you agree or disagree with the Jack's thinking. (DOK 3)
5.NBT.6	Divide the following and show your work 588 ÷ 14. (DOK 1) Samantha wants to split a collection of stickers into groups of 48. Samantha has 1,008 stickers. How many groups will be created? Show two ways to find the answer. (DOK 1)	Write a division problem using a 4-digit dividend and a 2-digit divisor that results in an even quotient. Explain your thinking. (DOK 3)

5 NBT.6	Divide 2789 ÷ 72 = Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (DOK 2)	• The quotient of a division problem is 20 when rounded to the nearest ten. • The divisor of the same problem is 50 when rounded to the nearest ten. • The dividend is between 1,000 and 1,200. What is a possible quotient and divisor in this problem? Explain why these numbers would work for this problem using calculations, rectangular arrays or area models. (DOK 3)
5.NBT.7	Connor is buying tickets to a play. The play he and his friends want to see costs $4.75 per ticket. Connor has $26.00 in his pocket. What is the greatest number of tickets Connor can buy? (DOK 1)	There are 167 students, 15 chaperones and 6 teachers going on a field trip. Each bus can hold 24 people. How many buses will be needed to transport everyone going on the trip? (DOK 2) In this calculation some numbers are missing. What might they be? How do you know? (DOK 3) 3 . _ _ + _ .7 _ = 1 _ .3
5.NF.1	 (DOK 1)	Two fractions with unlike denominators have a sum of 1. Write 3 pairs of addends that are fractions with unlike denominators and that equal 1. Draw pictures to justify your answer. (DOK 3) Kina wants to run a total of 7 5/8 miles every Tuesday and Thursday. If she runs 4 4/16 miles on Tuesday and 3 3/8 miles on Thursday, will he meet his goal for this week? Explain using pictures, numbers and/or words. (DOK 3)

5.NF.2	Solve the problem with a visual model and equation. Also, use benchmark fractions to check the reasonableness of your answer. Claire took 2 ¾ hours to read a book. Here brother, Dan, took 2/3 hour less to read his book. How much more time did Claire spend reading than Dan? Extension Question: How much time did they spend altogether reading their books? (DOK 1)	1. Create a word problem that could be solved by adding two specific fractions with unlike denominators. Example: 2 ½ + ¾ = 2. Represent the problem using both a diagram and an equation. 3. Solve your problem. Show all your work. 4. Use benchmark fractions to explain how you know that your answer is reasonable. (DOK 3)
5.G.2	Have students overlay a coordinate grid on a real map. They should use their knowledge of coordinate geometry and ordered pairs to find locations as they travel along the map. (DOK 1)	Sara has saved $20. She earns $8 for each hour she works. If Sara saves all of her money, how much will she have after working 3 hours? 5 hours? 10 hours? Create a graph that shows the relationship between the hours Sara worked and the amount of money she has saved. What other information do you know from analyzing the graph? (DOK 3)

North Carolina Department of Public Instruction
INSTRUCTIONAL SUPPORT TOOLS
FOR ACHIEVING NEW STANDARDS

5th Grade Mathematics ● Unpacked Content

For the new Common Core State Standards that will be effective in all North Carolina schools in the 2012-13 school year.

This document is designed to help North Carolina educators teach the Common Core (Standard Course of Study). NCDPI staff are continually updating and improving these tools to better serve teachers.

What is the purpose of this document?
To increase student achievement by ensuring educators understand specifically what the new standards mean a student must know, understand and be able to do. This document may also be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the sequence, pacing, and units of study for grade-level curricula. This document, along with on-going professional development, is one of many resources used to understand and teach the CCSS.

What is in the document?
Descriptions of what each standard means a student will know, understand and be able to do. The "unpacking" of the standards done in this document is an effort to answer a simple question "What does this standard mean that a student must know and be able to do?" and to ensure the description is helpful, specific and comprehensive for educators.

How do I send Feedback?
We intend the explanations and examples in this document to be helpful and specific. That said, we believe that as this document is used, teachers and educators will find ways in which the unpacking can be improved and made ever more useful. Please send feedback to us at feedback@dpi.state.nc.us and we will use your input to refine our unpacking of the standards. Thank You!

Just want the standards alone?
You can find the standards alone at http://corestandards.org/the-standards

Standards for Mathematical Practices

The Common Core State Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that students complete.

Mathematic Practices	Explanations and Examples
1. Make sense of problems and persevere in solving them.	Mathematically proficient students in grade 5should solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?".
2. Reason abstractly and quantitatively.	Mathematically proficient students in grade 5should recognize that a number represents a specific quantity. They connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others.	In fifth grade mathematical proficient students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
4. Model with mathematics.	Mathematically proficient students in grade 5 experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fifth graders should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems.
5. Use appropriate tools strategically.	Mathematically proficient fifth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems or make predictions from real world data.
6. Attend to precision.	Mathematically proficient students in grade 5 continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism they record their answers in cubic units.
7. Look for and make use of structure.	In fifth grade mathematically proficient students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation.
8. Look for and express regularity in repeated reasoning.	Mathematically proficient fifth graders use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit numbers and perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and begin to formulate generalizations.

Grade 5 Critical Areas

The Critical Areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction. The Critical Areas for fifth grade can be found on page 33 in the *Common Core State Standards for Mathematics*.

1. **Developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions).**
   Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

2. **Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations.**
   Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

3. **Developing understanding of volume.**
   Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

Operations and Algebraic Thinking 5.0A

Common Core Cluster

Write and interpret numerical expressions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **parentheses, brackets, braces, numerical expressions**

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	The order of operations is introduced in third grade and is continued in fourth. This standard calls for students to evaluate expressions with parentheses ( ), brackets [ ] and braces { }. In upper levels of mathematics, evaluate means to substitute for a variable and simplify the expression. However at this level students are to only simplify the expressions because there are no variables. Example: Evaluate the expression 2{ 5[12 + 5(500 - 100) + 399]} Students should have experiences working with the order of first evaluating terms in parentheses, then brackets, and then braces. The first step would be to subtract 500 – 100 = 400. Then multiply 400 by 5 = 2,000. Inside the bracket, there is now [12 + 2,000 + 399]. That equals 2,411. Next multiply by the 5 outside of the bracket. 2,411 x 5 = 12,055. Next multiply by the 2 outside of the braces. 12,055 x 2= 24,110. Mathematically, there cannot be brackets or braces in a problem that does not have parentheses. Likewise, there cannot be braces in a problem that does not have both parentheses and brackets. This standard builds on the expectations of third grade where students are expected to start learning the conventional order. Students need experiences with multiple expressions that use grouping symbols throughout the year to develop understanding of when and how to use parentheses, brackets, and braces. First, students use these symbols with whole numbers. Then the symbols can be used as students add, subtract, multiply and divide decimals and fractions. Example: • (26 + 18) ÷ 4 Solution: 11 • {[2 x (3+5)] – 9} + [5 x (23-18)] Solution: 32 • 12 – (0.4 x 2) Solution: 11.2 • (2 + 3) x (1.5 – 0.5) Solution: 5 • $6-\left(\frac{1}{2}+\frac{1}{3}\right)$ Solution: 5 1/6

	• { 80 ÷ [2 x (3 ½ + 1 ½)] }+ 100 Solution: 108 To further develop students' understanding of grouping symbols and facility with operations, students place grouping symbols in equations to make the equations true or they compare expressions that are grouped differently. Example: • 15 - 7 – 2 = 10 → 15 - (7 – 2) = 10 • 3 x 125 ÷ 25 + 7 = 22 → [3 x (125 ÷ 25)] + 7 = 22 • 24 ÷ 12 ÷ 6 ÷ 2 = 2 x 9 + 3 ÷ ½ → 24 ÷ [(12 ÷ 6) ÷ 2] = (2 x 9) + (3 ÷ ½) • Compare 3 x 2 + 5 and 3 x (2 + 5) • Compare 15 – 6 + 7 and 15 – (6 + 7) In fifth grade, students work with exponents only dealing with powers of ten (5.NBT.2). Students are expected to evaluate an expression that has a power of ten in it. Example: 3 {**2 + 5 [5 + 2 x 10^4]+ 3**} In fifth grade students begin working more formally with expressions. They write expressions to express a calculation, e.g., writing 2 x (8 + 7) to express the calculation "add 8 and 7, then multiply by 2." They also evaluate and interpret expressions, e.g., using their conceptual understanding of multiplication to interpret 3 x (18932 x 921) as being three times as large as 18932 + 921, without having to calculate the indicated sum or product. Thus, students in Grade 5 begin to think about numerical expressions in ways that prefigure their later work with variable expressions (e.g., three times an unknown length is 3 · *L*). In Grade 5, this work should be viewed as exploratory rather than for attaining mastery; for example, expressions should not contain nested grouping symbols, and they should be no more complex than the expressions one finds in an application of the associative or distributive property, e.g., (8 + 27) + 2 or (6 x 30) (6 x 7). Note however that the numbers in expressions need not always be whole numbers. (*Progressions for the CCSSM, Operations and Algebraic Thinking*, CCSS Writing Team, April 2011, page 32)

5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*	This standard refers to expressions. Expressions are a series of numbers and symbols (+, -, x, ÷) without an equals sign. Equations result when two expressions are set equal to each other (2 + 3 = 4 + 1). Example: 4(5 + 3) is an expression. When we compute 4(5 + 3) we are evaluating the expression. The expression equals 32. 4(5 + 3) = 32 is an equation. This standard calls for students to verbally describe the relationship between expressions without actually calculating them. This standard calls for students to apply their reasoning of the four operations as well as place value while describing the relationship between numbers. The standard does not include the use of variables, only numbers and signs for operations. Example: Write an expression for the steps "double five and then add 26." Student *(2 x 5) + 26* Describe how the expression 5(10 x 10) relates to 10 x 10. Student The expression 5(10 x 10) is 5 times larger than the expression 10 x 10 since I know that I that 5(10 x 10) means that I have 5 groups of (10 x 10).

Common Core Cluster

Analyze patterns and relationships.

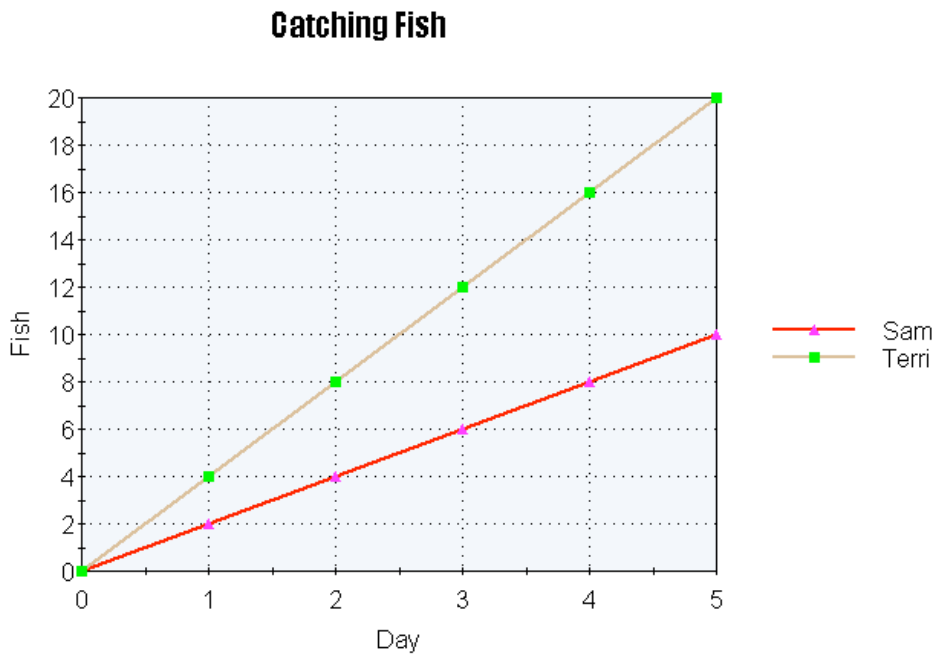
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language.
The terms students should learn to use with increasing precision with this cluster are: **numerical patterns, rules, ordered pairs, coordinate plane**

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*	This standard extends the work from Fourth Grade, where students generate numerical patterns when they are given one rule. In Fifth Grade, students are given two rules and generate two numerical patterns. The graphs that are created should be line graphs to represent the pattern. This is a linear function which is why we get the straight lines. The Days are the independent variable, Fish are the dependent variables, and the constant rate is what the rule identifies in the table. Make a chart (table) to represent the number of fish that Sam and Terri catch.

Days	Sam's Total Number of Fish	Terri's Total Number of Fish
0	0	0
1	2	4
2	4	8
3	6	12
4	8	16
5	10	20

Example:
Describe the pattern:
Since Terri catches 4 fish each day, and Sam catches 2 fish, the amount of Terri's fish is always greater. Terri's fish is also always twice as much as Sam's fish. Today, both Sam and Terri have no fish. They both go fishing each day. Sam catches 2 fish each day. Terri catches 4 fish each day. How many fish do they have after each of the five days? Make a graph of the number of fish.

Plot the points on a coordinate plane and make a line graph, and then interpret the graph.
Student:
My graph shows that Terri always has more fish than Sam. Terri's fish increases at a higher rate since she catches 4 fish every day. Sam only catches 2 fish every day, so his number of fish increases at a smaller rate than Terri. Important to note as well that the lines become increasingly further apart. Identify apparent relationships between corresponding terms. Additional relationships: The two lines will never intersect; there will not be a day in which boys have the same total of fish, explain the relationship between the number of days that has passed and the number of fish a boy has (2n or 4n, n being the number of days).



Example:

Use the rule “add 3” to write a sequence of numbers. Starting with a 0, students write 0, 3, 6, 9, 12, . . .

Use the rule “add 6” to write a sequence of numbers. Starting with 0, students write 0, 6, 12, 18, 24, . . .

After comparing these two sequences, the students notice that each term in the second sequence is twice the corresponding terms of the first sequence. One way they justify this is by describing the patterns of the terms. Their justification may include some mathematical notation (See example below). A student may explain that both sequences start with zero and to generate each term of the second sequence he/she added 6, which is twice as much as was added to produce the terms in the first sequence. Students may also use the distributive property to describe the relationship between the two numerical patterns by reasoning that 6 + 6 + 6 = 2 (3 + 3 + 3).

$0, \ ^{+3}3, \ ^{+3}6, \ ^{+3}9, \ ^{+3}12, \ldots$

$0, \ ^{+6}6, \ ^{+6}12, \ ^{+6}18, \ ^{+6}24, \ldots$

Once students can describe that the second sequence of numbers is twice the corresponding terms of the first sequence, the terms can be written in ordered pairs and then graphed on a coordinate grid. They should recognize that each point on the graph represents two quantities in which the second quantity is twice the first quantity.

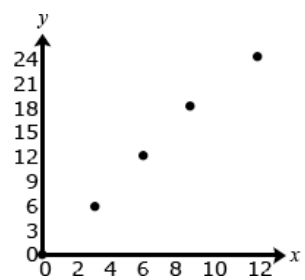
<u>Ordered pairs</u>

(0, 0)

(3, 6)

(6, 12)

(9, 18)



Number and Operations in Base Ten 5.NBT

Common Core Cluster

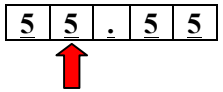
Understand the place value system.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **place value, decimal, decimal point, patterns, multiply, divide, tenths, thousands, greater than, less than, equal to, ‹, ›, =, compare/comparison, round**

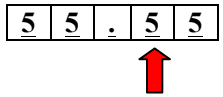
Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	Students extend their understanding of the base-ten system to the relationship between adjacent places, how numbers compare, and how numbers round for decimals to thousandths. This standard calls for students to reason about the magnitude of numbers. Students should work with the idea that the tens place is ten times as much as the ones place, and the ones place is $1/10^{th}$ the size of the tens place. In fourth grade, students examined the relationships of the digits in numbers for whole numbers only. This standard extends this understanding to the relationship of decimal fractions. Students use base ten blocks, pictures of base ten blocks, and interactive images of base ten blocks to manipulate and investigate the place value relationships. They use their understanding of unit fractions to compare decimal places and fractional language to describe those comparisons. Before considering the relationship of decimal fractions, students express their understanding that in multi-digit whole numbers, a digit in one place represents 10 times what it represents in the place to its right and 1/10 of what it represents in the place to its left. Example: The 2 in the number 542 is different from the value of the 2 in 324. The 2 in 542 represents 2 ones or 2, while the 2 in 324 represents 2 tens or 20. Since the 2 in 324 is one place to the left of the 2 in 542 the value of the 2 is 10 times greater. Meanwhile, the 4 in 542 represents 4 tens or 40 and the 4 in 324 represents 4 ones or 4. Since the 4 in 324 is one place to the right of the 4 in 542 the value of the 4 in the number 324 is $1/10^{th}$ of its value in the number 542. Example: A student thinks, "I know that in the number 5555, the 5 in the tens place (55<u>5</u>5) represents 50 and the 5 in the hundreds place (5<u>5</u>55) represents 500. So a 5 in the hundreds place is ten times as much as a 5 in the tens place or a 5 in the tens place is 1/10 of the value of a 5 in the hundreds place. Base on the base-10 number system digits to the left are times as great as digits to the right; likewise, digits to the right are 1/10th of digits to the left. For example, the 8 in 845 has a value of 800 which is ten times as much as the 8 in the number 782. In the same spirit, the 8 in 782 is 1/10th the value of the 8 in 845.

To extend this understanding of place value to their work with decimals, students use a model of one unit; they cut it into 10 equal pieces, shade in, or describe 1/10 of that model using fractional language ("This is 1 out of 10 equal parts. So it is 1/10". I can write this using 1/10 or 0.1"). They repeat the process by finding 1/10 of a 1/10 (e.g., dividing 1/10 into 10 equal parts to arrive at 1/100 or 0.01) and can explain their reasoning, "0.01 is 1/10 of 1/10 thus is 1/100 of the whole unit."

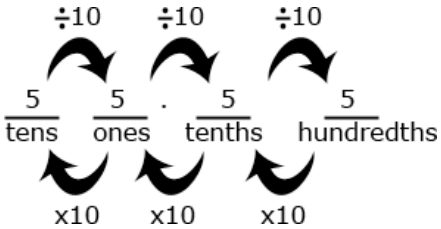
In the number 55.55, each digit is 5, but the value of the digits is different because of the placement.



The 5 that the arrow points to is 1/10 of the 5 to the left and 10 times the 5 to the right. The 5 in the ones place is 1/10 of 50 and 10 times five tenths.

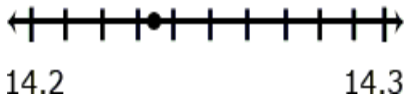


The 5 that the arrow points to is 1/10 of the 5 to the left and 10 times the 5 to the right. The 5 in the tenths place is 10 times five hundredths.

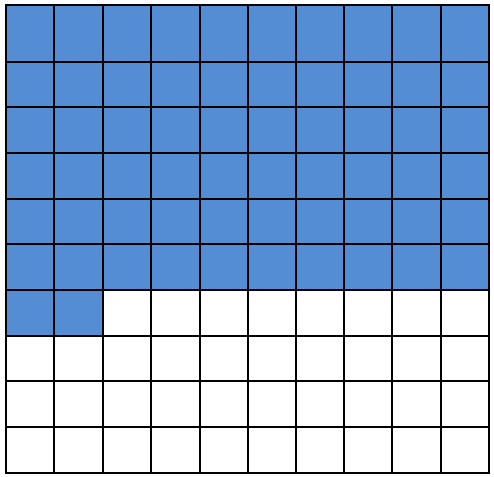


5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	New at Grade 5 is the use of whole number exponents to denote powers of 10. Students understand why multiplying by a power of 10 shifts the digits of a whole number or decimal that many places to the left. Example: Multiplying by 104 is multiplying by 10 four times. Multiplying by 10 once shifts every digit of the multiplicand one place to the left in the product (the product is ten times as large) because in the base-ten system the value of each place is 10 times the value of the place to its right. So multiplying by 10 four times shifts every digit 4 places to the left. Patterns in the number of 0s in products of a whole numbers and a power of 10 and the location of the decimal point in products of decimals with powers of 10 can be explained in terms of place value. Because students have developed their understandings of and computations with decimals in terms of multiples rather than powers, connecting the terminology of multiples with that of powers affords connections between understanding of multiplication and exponentiation. (*Progressions for the CCSSM, Number and Operation in Base Ten*, CCSS Writing Team, April 2011, page 16) This standard includes multiplying by multiples of 10 and powers of 10, including 10^2 which is 10 x 10=100, and 10^3 which is 10 x 10 x 10=1,000. Students should have experiences working with connecting the pattern of the number of zeros in the product when you multiply by powers of 10. Example: $2.5 \times 10^3 = 2.5 \times (10 \times 10 \times 10) = 2.5 \times 1{,}000 = 2{,}500$ Students should reason that the exponent above the 10 indicates how many places the decimal point is moving (not just that the decimal point is moving but that you are multiplying or making the number 10 times greater three times) when you multiply by a power of 10. Since we are multiplying by a power of 10 the decimal point moves to the right. $350 \div 10^3 = 350 \div 1{,}000 = 0.350 = 0.35$ 350/10 = 35, 35 /10 = 3.5 3.5 /10 =.0.35, or 350 x 1/10, 35 x 1/10, 3.5 x 1/10 this will relate well to subsequent work with operating with fractions. This example shows that when we divide by powers of 10, the exponent above the 10 indicates how many places the decimal point is moving (how many times we are dividing by 10 , the number becomes ten times smaller). Since we are dividing by powers of 10, the decimal point moves to the left.

	Students need to be provided with opportunities to explore this concept and come to this understanding; this should not just be taught procedurally. Example: Students might write: • $36 \times 10 = 36 \times 10^1 = 360$ • $36 \times 10 \times 10 = 36 \times 10^2 = 3600$ • $36 \times 10 \times 10 \times 10 = 36 \times 10^3 = 36{,}000$ • $36 \times 10 \times 10 \times 10 \times 10 = 36 \times 10^4 = 360{,}000$ Students might think and/or say: • I noticed that every time, I multiplied by 10 I added a zero to the end of the number. That makes sense because each digit's value became 10 times larger. To make a digit 10 times larger, I have to move it one place value to the left. • When I multiplied 36 by 10, the 30 became 300. The 6 became 60 or the 36 became 360. So I had to add a zero at the end to have the 3 represent 3 one-hundreds (instead of 3 tens) and the 6 represents 6 tens (instead of 6 ones). Students should be able to use the same type of reasoning as above to explain why the following multiplication and division problem by powers of 10 make sense. • $523 \times 10^3 = 523{,}000$ The place value of 523 is increased by 3 places. • $5.223 \times 10^2 = 522.3$ The place value of 5.223 is increased by 2 places. • $52.3 \div 10^1 = 5.23$ The place value of 52.3 is decreased by one place.
5.NBT.3 Read, write, and compare decimals to thousandths. **a.** Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$	This standard references expanded form of decimals with fractions included. Students should build on their work from Fourth Grade, where they worked with both decimals and fractions interchangeably. Expanded form is included to build upon work in 5.NBT.2 and deepen students' understanding of place value. Students build on the understanding they developed in fourth grade to read, write, and compare decimals to thousandths. They connect their prior experiences with using decimal notation for fractions and addition of fractions with denominators of 10 and 100. They use concrete models and number lines to extend this understanding to decimals to the thousandths. Models may include base ten blocks, place value charts, grids, pictures, drawings, manipulatives, technology-based, etc. They read decimals using fractional language and write decimals in fractional form, as well as in expanded notation. This investigation leads them to understanding equivalence of decimals (0.8 = 0.80 = 0.800).

b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	Comparing decimals builds on work from fourth grade. Example: Some equivalent forms of 0.72 are: 72/100 — 70/100 + 2/100 7/10 + 2/100 — 0.720 7 x (1/10) + 2 x (1/100) — 7 x (1/10) + 2 x (1/100) + 0 x (1/1000) 0.70 + 0.02 — 720/1000 Students need to understand the size of decimal numbers and relate them to common benchmarks such as 0, 0.5 (0.50 and 0.500), and 1. Comparing tenths to tenths, hundredths to hundredths, and thousandths to thousandths is simplified if students use their understanding of fractions to compare decimals. Example: Comparing 0.25 and 0.17, a student might think, "25 hundredths is more than 17 hundredths". They may also think that it is 8 hundredths more. They may write this comparison as 0.25 > 0.17 and recognize that 0.17 < 0.25 is another way to express this comparison. Comparing 0.207 to 0.26, a student might think, "Both numbers have 2 tenths, so I need to compare the hundredths. The second number has 6 hundredths and the first number has no hundredths so the second number must be larger. Another student might think while writing fractions, "I know that 0.207 is 207 thousandths (and may write 207/1000). 0.26 is 26 hundredths (and may write 26/100) but I can also think of it as 260 thousandths (260/1000). So, 260 thousandths is more than 207 thousandths.
5.NBT.4 Use place value understanding to round decimals to any place.	This standard refers to rounding. Students should go beyond simply applying an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line to support their work with rounding. Example: Round 14.235 to the nearest tenth. Students recognize that the possible answer must be in tenths thus, it is either 14.2 or 14.3. They then identify that 14.235 is closer to 14.2 (14.20) than to 14.3 (14.30).  14.2 14.3

Students should use benchmark numbers to support this work. Benchmarks are convenient numbers for comparing and rounding numbers. 0., 0.5, 1, 1.5 are examples of benchmark numbers.
Example:
Which benchmark number is the best estimate of the shaded amount in the model below? Explain your thinking.



Common Core Cluster

Perform operations with multi-digit whole numbers and with decimals to hundredths.

Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **multiplication/multiply, division/divide, decimal, decimal point, tenths, hundredths, products, quotients, dividends, rectangular arrays, area models, addition/add, subtraction/subtract, (properties)-rules about how numbers work, reasoning**

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm.	In fifth grade, students fluently compute products of whole numbers using the standard algorithm. Underlying this algorithm are the properties of operations and the base-ten system. Division strategies in fifth grade involve breaking the dividend apart into like base-ten units and applying the distributive property to find the quotient place by place, starting from the highest place. (Division can also be viewed as finding an unknown factor: the dividend is the product, the divisor is the known factor, and the quotient is the unknown factor.) Students continue their fourth grade work on division, extending it to computation of whole number quotients with dividends of up to four digits and two-digit divisors. Estimation becomes relevant when extending to two-digit divisors. Even if students round appropriately, the resulting estimate may need to be adjusted. **Recording division after an underestimate** 1655 ÷ 27 Rounding 27 to 30 produces the underestimate 50 at the first step but this method allows the division process to be continued 1 10 (30) 50 27) 1655 -1350 305 -270 35 -27 8 61 (*Progressions for the CCSSM, Number and Operation in Base Ten*, CCSS Writing Team, April 2011, page 16) **Computation algorithm.** A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. **Computation strategy.** Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. This standard refers to fluency which means accuracy (correct answer), efficiency (a reasonable amount of steps), and flexibility (using strategies such as the distributive property or breaking numbers apart also using strategies according to the numbers in the problem, 26 x 4 may lend itself to (25 x 4) + 4 where as another problem might lend itself to making an equivalent problem 32 x 4 = 64 x 2)). This standard builds upon students' work with multiplying numbers in third and fourth grade. In fourth grade, students developed understanding of multiplication through using various strategies. While the standard algorithm is mentioned, alternative strategies are also appropriate to help students develop

conceptual understanding. The size of the numbers should NOT exceed a three-digit factor by a two-digit factor.

Examples of alternative strategies:
There are 225 dozen cookies in the bakery. How many cookies are there?

Student 1	Student 2	Student 3
225 x 12 I broke 12 up into 10 and 2. 225 x 10 = 2,250 225 x 2 = 450 2,250 + 450 = 2,700	225x12 I broke up 225 into 200 and 25. 200 x 12 = 2,400 I broke 25 up into 5 x 5, so I had 5 x 5 x12 or 5 x 12 x 5. 5 x12= 60. 60 x 5 = 300 I then added 2,400 and 300 2,400 + 300 = 2,700.	I doubled 225 and cut 12 in half to get 450 x 6. I then doubled 450 again and cut 6 in half to get 900 x 3. 900 x 3 = 2,700.

Draw a array model for 225 x 12…. 200 x 10, 200 x 2, 20 x 10, 20 x 2, 5 x 10, 5 x 2

225 x 12

	200	20	5
10	2,000	200	50
2	400	40	10

2,000
400
200
40
50
+ 10
2,700

5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

This standard references various strategies for division. Division problems can include remainders. Even though this standard leads more towards computation, the connection to story contexts is critical. Make sure students are exposed to problems where the divisor is the number of groups and where the divisor is the size of the groups. In fourth grade, students' experiences with division were limited to dividing by one-digit divisors. This standard extends students' prior experiences with strategies, illustrations, and explanations. When the two-digit divisor is a "familiar" number, a student might decompose the dividend using place value.

Example:
There are 1,716 students participating in Field Day. They are put into teams of 16 for the competition. How many teams get created? If you have left over students, what do you do with them?

Student 1
1,716 divided by 16
There are 100 16's in 1,716.
1,716 – 1,600 = 116
I know there are at least 6 16's.
116 - 96 = 20
I can take out at least 1 more 16.
20 - 16 = 4
There were 107 teams with 4 students left over. If we put the extra students on different team, 4 teams will have 17 students.

Student 2
1,716 divided by 16.
There are 100 16's in 1,716.
Ten groups of 16 is 160. That's too big.
Half of that is 80, which is 5 groups.
I know that 2 groups of 16's is 32.
I have 4 students left over.

1716 -1600	100
116 -80	5
36 -32	2
4	

Student 3
1,716 ÷ 16 =
I want to get to 1,716
I know that 100 16's equals 1,600
I know that 5 16's equals 80
1,600 + 80 = 1,680
Two more groups of 16's equals 32, which gets us to 1,712
I am 4 away from 1,716
So we had 100 + 6 + 1 = 107 teams
Those other 4 students can just hang out

Student 4
How many 16's are in 1,716?
We have an area of 1,716. I know that one side of my array is 16 units long. I used 16 as the height. I am trying to answer the question what is the width of my rectangle if the area is 1,716 and the height is 16. 100 + 7 = 107 R 4

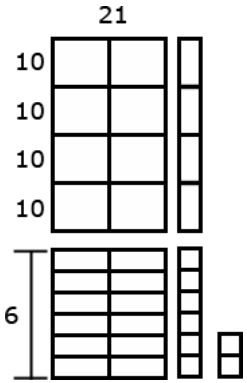
	100	**7**
16	100 x 16 = 1,600	7 x 16 =112
	1,716 - 1,600 = 116	116 - 112 = 4

Example:
Using expanded notation 2682 ÷ 25 = (2000 + 600 + 80 + 2) ÷ 25
Using understanding of the relationship between 100 and 25, a student might think ~

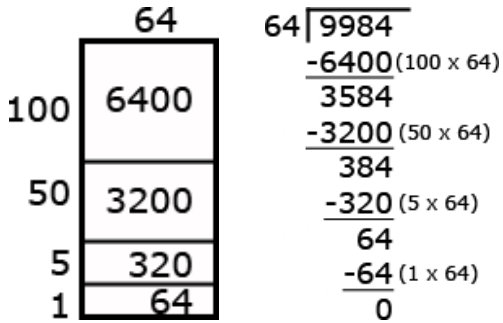
- I know that 100 divided by 25 is 4 so 200 divided by 25 is 8 and 2000 divided by 25 is 80.
- 600 divided by 25 has to be 24.
- Since 3 x 25 is 75, I know that 80 divided by 25 is 3 with a reminder of 5.
  (Note that a student might divide into 82 and not 80)
- I can't divide 2 by 25 so 2 plus the 5 leaves a remainder of 7.
- 80 + 24 + 3 = 107. So, the answer is 107 with a remainder of 7.

Using an equation that relates division to multiplication, 25 x *n* = 2682, a student might estimate the answer to be slightly larger than 100 because s/he recognizes that 25 x 100 = 2500.

Example: 968 ÷ 21
Using base ten models, a student can represent 962 and use the models to make an array with one dimension of 21. The student continues to make the array until no more groups of 21 can be made. Remainders are not part of the array.



Example: 9984 ÷ 64
An area model for division is shown below. As the student uses the area model, s/he keeps track of how much of the 9984 is left to divide.



5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	Because of the uniformity of the structure of the base-ten system, students use the same place value understanding for adding and subtracting decimals that they used for adding and subtracting whole numbers. Like base-ten units must be added and subtracted, so students need to attend to aligning the corresponding places correctly (this also aligns the decimal points). It can help to put 0s in places so that all numbers show the same number of places to the right of the decimal point. Although whole numbers are not usually written with a decimal point, but that a decimal point with 0s on its right can be inserted (e.g., 16 can also be written as 16.0 or 16.00). The process of composing and decomposing a base-ten unit is the same for decimals as for whole numbers and the same methods of recording numerical work can be used with decimals as with whole numbers. For example, students can write digits representing new units below on the addition or subtraction line, and they can decompose units wherever needed before subtracting. General methods used for computing products of whole numbers extend to products of decimals. Because the expectations for decimals are limited to thousandths and expectations for factors are limited to hundredths at this grade level, students will multiply tenths with tenths and tenths with hundredths, but they need not multiply hundredths with hundredths. Before students consider decimal multiplication more generally, they can study the effect of multiplying by 0.1 and by 0.01 to explain why the product is ten or a hundred times as small as the multiplicand (moves one or two places to the right). They can then extend their reasoning to multipliers that are single-digit multiples of 0.1 and 0.01 (e.g., 0.2 and 0.02, etc.). There are several lines of reasoning that students can use to explain the placement of the decimal point in other products of decimals. Students can think about the product of the smallest base-ten units of each factor. For example, a tenth times a tenth is a hundredth, so 3.2 x 7.1 will have an entry in the hundredth place. Note, however, that students might place the decimal point incorrectly for 3.2 x 8.5 unless they take into account the 0 in the ones place of 32 x 85. (Or they can think of 0.2 x 0.5 as 10 hundredths.) Students can also think of the decimals as fractions or as whole numbers divided by 10 or 100.[5.NF.3] When they place the decimal point in the product, they have to divide by a 10 from each factor or 100 from one factor. For example, to see that 0.6 x 0.8 = 0.48, students can use fractions: 6/10 x 8/10 = 48/100.[5.NF.4] Students can also reason that when they carry out the multiplication without the decimal point, they have multiplied each decimal factor by 10 or 100, so they will need to divide by those numbers in the end to get the correct answer. Also, students can use reasoning about the sizes of numbers to determine the placement of the decimal point. For example, 3.2 x 8.5 should be close to 3 x 9, so 27.2 is a more reasonable product for 3.2 x 8.5 than 2.72 or 272. This estimation-based method is not reliable in all cases, however, especially in cases students will encounter in later grades. For example, it is not easy to decide where to place the decimal point in 0.023 x 0.0045 based on estimation. Students can summarize the results of their reasoning such as those above as specific numerical patterns and then as one general overall pattern such as "the number of decimal places in the product is the sum of the number of decimal places in each factor." General methods used for computing quotients of whole numbers extend to decimals with the additional issue of placing the decimal point in the quotient. As with decimal multiplication, students can first examine the cases of dividing by 0.1 and 0.01 to see that the quotient becomes 10 times or 100 times as large as the dividend. For example, students can view 7 ÷ 0.1 =

as asking how many tenths are in 7.[5.NF.7b] Because it takes 10 tenths make 1, it takes 7 times as many tenths to make 7, so 7 ÷ 0.1 = 7 x 10 = 70. Or students could note that 7 is 70 tenths, so asking how many tenths are in 7 is the same as asking how many tenths are in 70 tenths, which is 70. In other words, 7 ÷ 0.1 is the same as 70 ÷1. So dividing by 0.1 moves the number 7 one place to the left, the quotient is ten times as big as the dividend. As with decimal multiplication, students can then proceed to more general cases. For example, to calculate 7 ÷ 0.2, students can reason that 0.2 is 2 tenths and 7 is 70 tenths, so asking how many 2 tenths are in 7 is the same as asking how many 2 tenths are in 70 tenths. In other words, 7 ÷ 0.2 is the same as 70 ÷ 2; multiplying both the 7 and the 0.2 by 10 results in the same quotient. Or students could calculate 7 ÷ 0.2 by viewing 0.2 as 2 x 0.1, so they can first divide 7 by 2, which is 3.5, and then divide that result by 0.1, which makes 3.5 ten times as large, namely 35. Dividing by a decimal less than 1 results in a quotient larger than the dividend[5.NF.5] and moves the digits of the dividend one place to the left. Students can summarize the results of their reasoning as specific numerical patterns then as one general overall pattern such as "when the decimal point in the divisor is moved to make a whole number, the decimal point in the dividend should be moved the same number of places."(*Progressions for the CCSSM, Number and Operation in Base Ten*, CCSS Writing Team, April 2011, page 17-18)

This standard builds on the work from fourth grade where students are introduced to decimals and compare them. In fifth grade, students begin adding, subtracting, multiplying and dividing decimals. This work should focus on concrete models and pictorial representations, rather than relying solely on the algorithm. The use of symbolic notations involves having students record the answers to computations (2.25 x 3= 6.75), but this work should not be done without models or pictures. This standard includes students' reasoning and explanations of how they use models, pictures, and strategies.

This standard requires students to extend the models and strategies they developed for whole numbers in grades 1-4 to decimal values. Before students are asked to give exact answers, they should estimate answers based on their understanding of operations and the value of the numbers.

Examples:

- 3.6 + 1.7

A student might estimate the sum to be larger than 5 because 3.6 is more than 3 ½ and 1.7 is more than 1 ½.

- 5.4 – 0.8

A student might estimate the answer to be a little more than 4.4 because a number less than 1 is being subtracted.

- 6 x 2.4

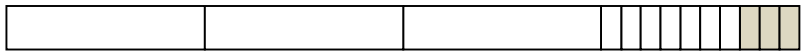
A student might estimate an answer between 12 and 18 since 6 x 2 is 12 and 6 x 3 is 18. Another student might give an estimate of a little less than 15 because s/he figures the answer to be very close, but smaller than 6 x 2 ½ and think of 2 ½ groups of 6 as 12 (2 groups of 6) + 3 (½ of a group of 6).

Students should be able to express that when they add decimals they add tenths to tenths and hundredths to hundredths. So, when they are adding in a vertical format (numbers beneath each other), it is important that they write numbers with the same place value beneath each other. This understanding can be reinforced by connecting

addition of decimals to their understanding of addition of fractions. Adding fractions with denominators of 10 and 100 is a standard in fourth grade.

Example: 4 - 0.3

3 tenths subtracted from 4 wholes. The wholes must be divided into tenths. (solution is 3 and 7/10 or 3.7)

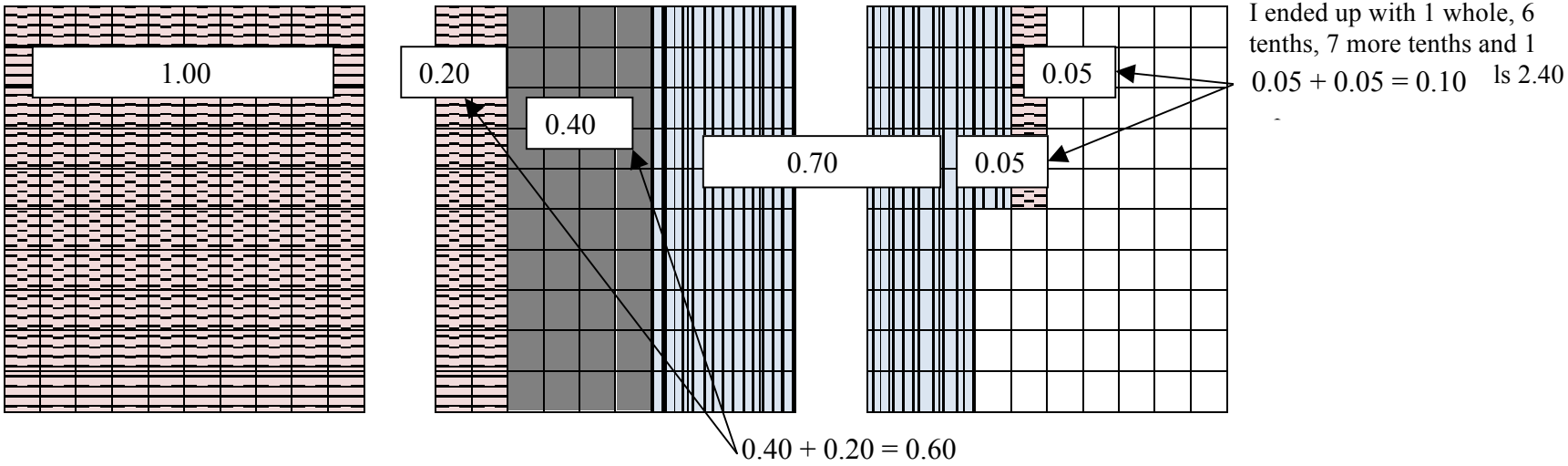


Additional examples on next page.

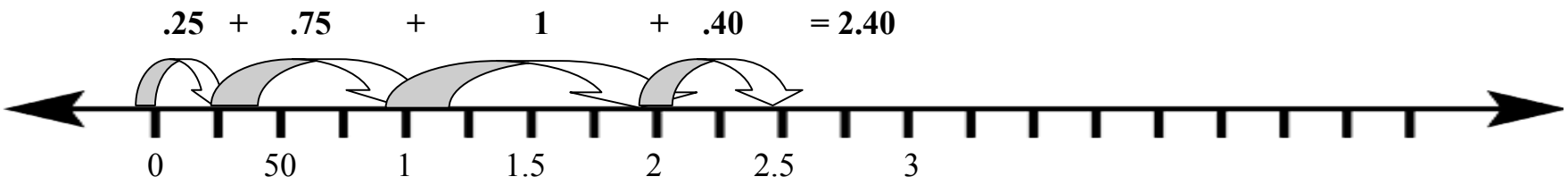
Example:
A recipe for a cake requires 1.25 cups of milk, 0.40 cups of oil, and 0.75 cups of water. How much liquid is in the mixing bowl?

Student 1
1.25 + 0.40 + 0.75

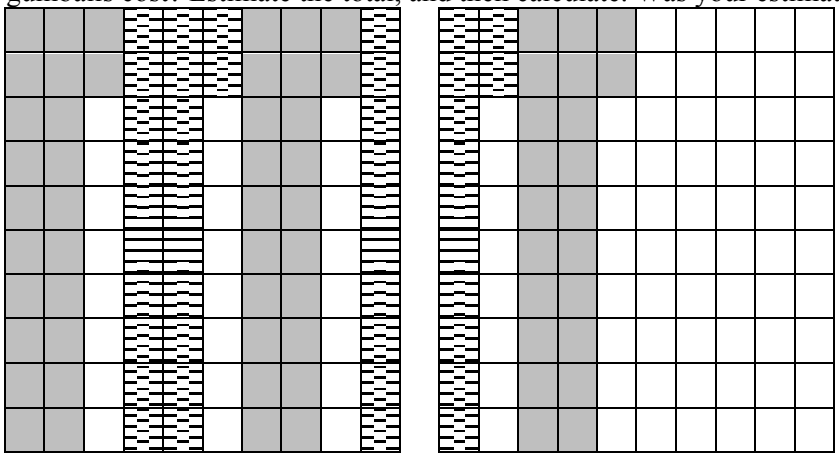
First, I broke the numbers apart:
I broke 1.25 into 1.00 + 0.20 + 0.05
I left 0.40 like it was.
I broke 0.75 into 0.70 + 0.05
I combined my two 0.05s to get 0.10.
I combined 0.40 and 0.20 to get 0.60.
I added the 1 whole from 1.25.



Student 2
I saw that the 0.25 in 1.25 and the 0.75 for water would combine to equal 1 whole.
I then added the 2 wholes and the 0.40 to get 2.40.

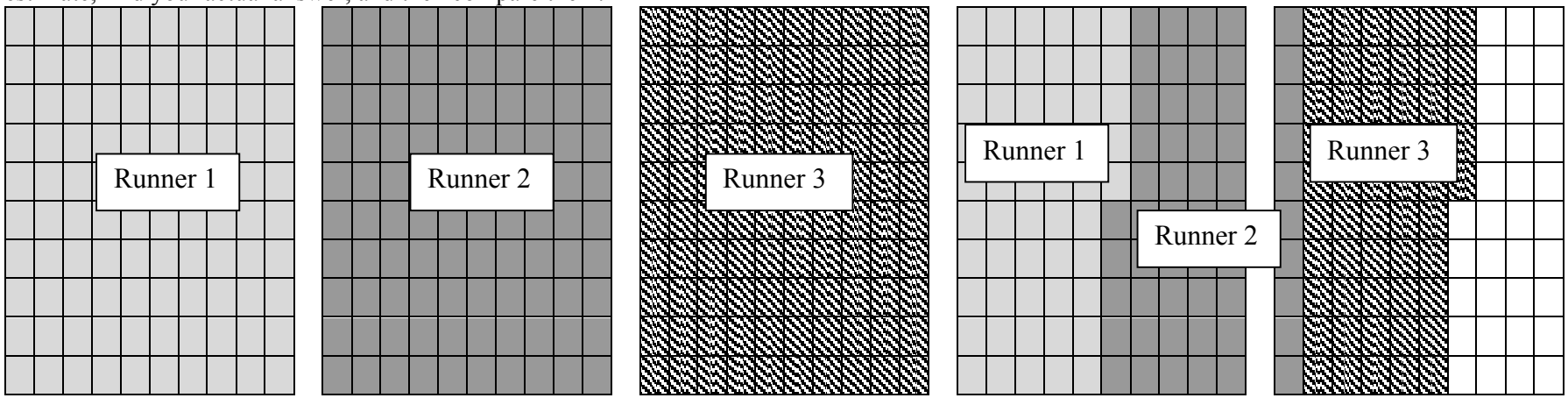


Example of Multiplication:
A gumball costs $0.22. How much do 5 gumballs cost? Estimate the total, and then calculate. Was your estimate close?



I estimate that the total cost will be a little more than a dollar. I know that 5 20's equal 100 and we have 5 22's.
I have 10 whole columns shaded and 10 individual boxes shaded. The 10 columns equal 1 whole. The 10 individual boxes equal 10 hundredths or 1 tenth. My answer is $1.10.
My estimate was a little more than a dollar, and my answer was $1.10. I was really close.

Example of Division:
A relay race lasts 4.65 miles. The relay team has 3 runners. If each runner goes the same distance, how far does each team member run? Make an estimate, find your actual answer, and then compare them.

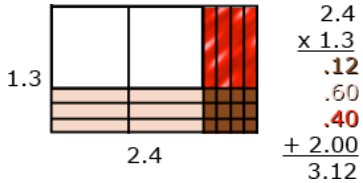
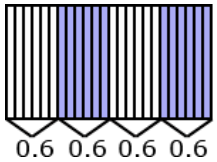
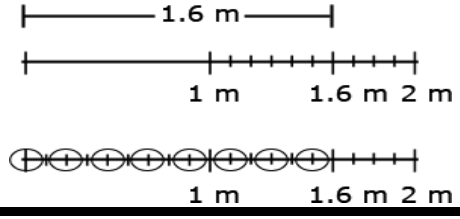


My estimate is that each runner runs between 1 and 2 miles. If each runner went 2 miles, that would be a total of 6 miles which is too high. If each runner ran 1 mile, that would be 3 miles, which is too low.

I used the 5 grids above to represent the 4.65 miles. I am going to use all of the first 4 grids and 65 of the squares in the 5th grid. I have to divide the 4 whole grids and the 65 squares into 3 equal groups. I labeled each of the first 3 grids for each runner, so I know that each team member ran at least 1 mile. I then have 1 whole grid and 65 squares to divide up. Each column represents one-tenth. If I give 5 columns to each runner, that means that each runner has run 1 whole mile and 5 tenths of a mile. Now, I have 15 squares left to divide up. Each runner gets 5 of those squares. So each runner ran 1 mile, 5 tenths and 5 hundredths of a mile. I can write that as 1.55 miles.

My answer is 1.55 and my estimate was between 1 and 2 miles. I was pretty close.

Additional multiplication and division examples:

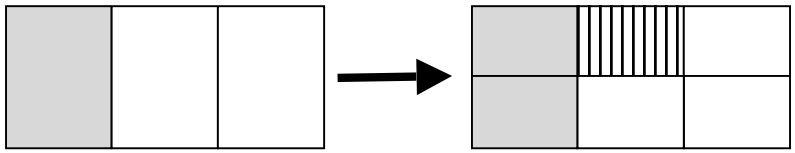
An area model can be useful for illustrating products.  	Students should be able to describe the partial products displayed by the area model. For example, "3/10 times 4/10 is 12/100. 3/10 times 2 is 6/10 or 60/100. 1 group of 4/10 is 4/10 or 40/100. 1 group of 2 is 2."
Example of division: finding the number in each group or share. Students should be encouraged to apply a fair sharing model separating decimal values into equal parts such as $2.4 \div 4 = 0.6$  	Example of division: finding the number of groups. Joe has 1.6 meters of rope. He has to cut pieces of rope that are 0.2 meters long. How many can he cut?
Example of division: finding the number of groups. Students could draw a segment to represent 1.6 meters. In doing so, s/he would count in tenths to identify the 6 tenths, and be able identify the number of 2 tenths within the 6 tenths. The student can then extend the idea of counting by tenths to divide the one meter into tenths and determine that there are 5 more groups of 2 tenths.  	Students might count groups of 2 tenths without the use of models or diagrams. Knowing that 1 can be thought of as 10/10, a student might think of 1.6 as 16 tenths. Counting 2 tenths, 4 tenths, 6 tenths, . . .16 tenths, a student can count 8 groups of 2 tenths. Use their understanding of multiplication and think, "8 groups of 2 is 16, so 8 groups of 2/10 is 16/10 or 1 6/10."

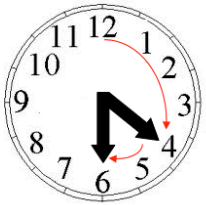
Number and Operation – Fractions 5.NF

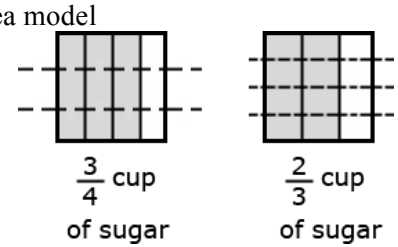
Common Core Cluster

Use equivalent fractions as a strategy to add and subtract fractions.
Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them.

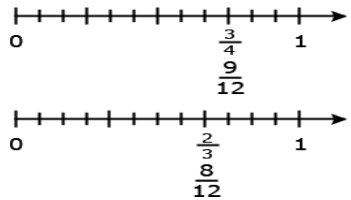
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **fraction, equivalent, addition/ add, sum, subtraction/subtract, difference, unlike denominator, numerator, benchmark fraction, estimate, reasonableness, mixed numbers**

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*	**5.NF.1** builds on the work in fourth grade where students add fractions with like denominators. In fifth grade, the example provided in the standard 2/3 + ¾ has students find a common denominator by finding the product of both denominators. This process should come after students have used visual fraction models (area models, number lines, etc.) to build understanding before moving into the standard algorithm describes in the standard The use of these visual fraction models allows students to use reasonableness to find a common denominator prior to using the algorithm. For example, when adding 1/3 + 1/6, Grade 5 students should apply their understanding of equivalent fractions and their ability to rewrite fractions in an equivalent form to find common denominators. Example: 1/3 + 1/6  1/3 is the same as 2/6 I drew a rectangle and shaded 1/3. I knew that if I cut every third in half then I would have sixths. Based on my picture, 1/3 equals 2/6. Then I shaded in another 1/6 with stripes. I ended up with an answer of 3/6, which is equal to 1/2. On the contrary, based on the algorithm that is in the example of the Standard, when solving 1/3 + 1/6, multiplying 3 and 6 gives a common denominator of 18. Students would make equivalent fractions 6/18 + 3/18 = 9/18 which is also equal to one-half. Please note that while multiplying the denominators will always give a common denominator, this may not result in the smallest denominator.

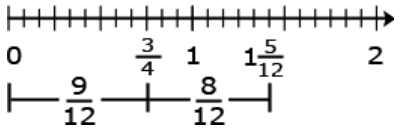
	Students should apply their understanding of equivalent fractions and their ability to rewrite fractions in an equivalent form to find common denominators. They should know that multiplying the denominators will always give a common denominator but may not result in the smallest denominator. Examples: $\frac{2}{5}+\frac{7}{8}=\frac{16}{40}+\frac{35}{40}=\frac{51}{40}$ $3\frac{1}{4}-\frac{1}{6}=3\frac{3}{12}-\frac{2}{12}=3\frac{1}{12}$ Fifth grade students will need to express both fractions in terms of a new denominator with adding unlike denominators. For example, in calculating 2/3 + 5/4 they reason that if each third in 2/3 is subdivided into fourths and each fourth in 5/4 is subdivided into thirds, then each fraction will be a sum of unit fractions with denominator 3 x 4 = 4 x 3 + 12: $\frac{2}{3}+\frac{5}{4}=\frac{2\times 4}{3\times 4}+\frac{5\times 3}{4\times 3}=\frac{8}{12}+\frac{15}{12}=\frac{23}{12}.$ It is **not** necessary to find a least common denominator to calculate sums of fractions, and in fact the effort of finding a least common denominator is a distraction from understanding adding fractions. (*Progressions for the CCSSM, Number and Operation – Fractions*, CCSS Writing Team, August 2011, page 10) Example: Present students with the problem 1/3 + 1/6. Encourage students to use the clock face as a model for solving the problem. Have students share their approaches with the class and demonstrate their thinking using the clock model. 
5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations	This standard refers to number sense, which means students' understanding of fractions as numbers that lie between whole numbers on a number line. Number sense in fractions also includes moving between decimals and fractions to find equivalents, also being able to use reasoning such as 7/8 is greater than ¾ because 7/8 is missing only 1/8 and ¾ is missing ¼ so 7/8 is closer to a whole Also, students should use benchmark fractions to estimate and examine the reasonableness of their answers. Example here such as 5/8 is greater than 6/10 because

to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*	5/8 is 1/8 larger than ½(4/8) and 6/10 is only 1/10 larger than ½ (5/10) Example: Your teacher gave you 1/7 of the bag of candy. She also gave your friend 1/3 of the bag of candy. If you and your friend combined your candy, what fraction of the bag would you have? Estimate your answer and then calculate. How reasonable was your estimate? Student 1 1/7 is really close to 0. 1/3 is larger than 1/7, but still less than 1/2. If we put them together we might get close to 1/2. 1/7 + 1/3= 3/21 + 7/21 = 10/21. The fraction does not simplify. I know that 10 is half of 20, so 10/21 is a little less than ½. Another example: 1/7 is close to 1/6 but less than 1/6, and 1/3 is equivalent to 2/6, so I have a little less than 3/6 or ½. Example: Jerry was making two different types of cookies. One recipe needed 3/4 cup of sugar and the other needed 2/3 cup of sugar. How much sugar did he need to make both recipes? • Mental estimation: A student may say that Jerry needs more than 1 cup of sugar but less than 2 cups. An explanation may compare both fractions to ½ and state that both are larger than ½ so the total must be more than 1. In addition, both fractions are slightly less than 1 so the sum cannot be more than 2. • Area model  $\frac{3}{4}=\frac{9}{12}$ $\frac{2}{3}=\frac{8}{12}$ $\frac{3}{4}+\frac{2}{3}=\frac{17}{12}=\frac{12}{12}+\frac{5}{12}=1\frac{5}{12}$

- Linear model



Solution:

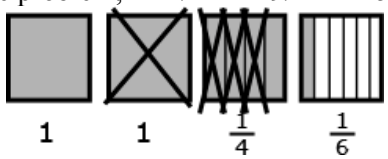


Example: Using a bar diagram

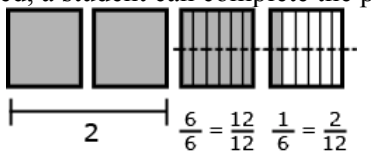
- Sonia had 2 1/3 candy bars. She promised her brother that she would give him ½ of a candy bar. How much will she have left after she gives her brother the amount she promised?
- If Mary ran 3 miles every week for 4 weeks, she would reach her goal for the month. The first day of the first week she ran 1 ¾ miles. How many miles does she still need to run the first week?
  - Using addition to find the answer:1 ¾ + n = 3
  - A student might add 1 ¼ to 1 ¾ to get to 3 miles. Then he or she would add 1/6 more. Thus 1 ¼ miles + 1/6 of a mile is what Mary needs to run during that week.

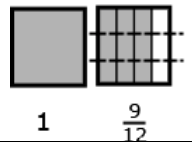
Example: Using an area model to subtract

- This model shows 1 ¾ subtracted from 3 1/6 leaving 1 + ¼ = 1/6 which a student can then change to 1 + 3/12 + 2/12 = 1 5/12. 3 1/6 can be expressed with a denominator of 12. Once this is done a student can complete the problem, 2 14/12 – 1 9/12 = 1 5/12.



- This diagram models a way to show how 3 1/6 and 1 ¾ can be expressed with a denominator of 12. Once this is accomplished, a student can complete the problem, 2 14/12 – 1 9/12 = 1 5/12.





Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies for calculations with fractions extend from students' work with whole number operations and can be supported through the use of physical models.

Example:
Elli drank 3/5 quart of milk and Javier drank 1/10 of a quart less than Ellie.
How much milk did they drink all together?

$$\frac{3}{5} - \frac{1}{10} = \frac{6}{10} - \frac{1}{10} = \frac{5}{10}$$

Solution:

$$\frac{3}{5} - \frac{1}{10} = \frac{6}{10} - \frac{1}{10} = \frac{5}{10}$$ This is how much milk Javier drank.

$$\frac{3}{5} + \frac{5}{10} = \frac{6}{10} + \frac{5}{10} = \frac{11}{10}$$ Together they drank 1 1/10 quarts of milk.

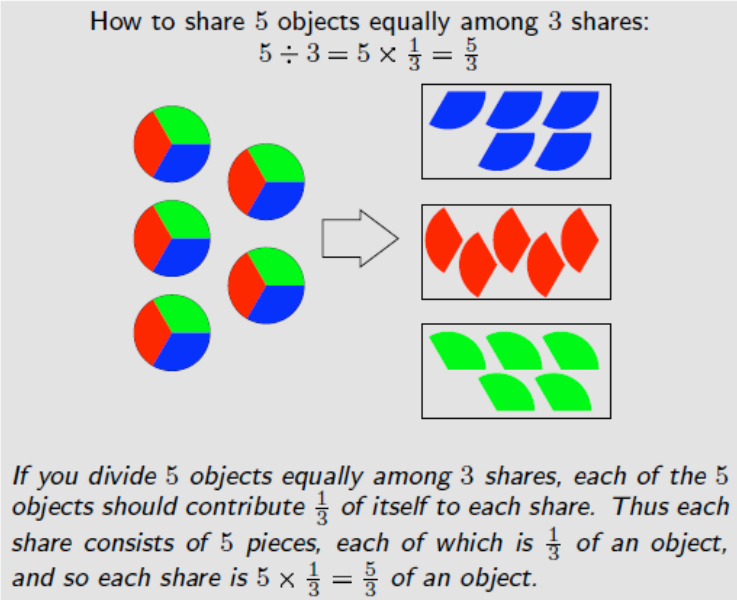
This solution is reasonable because Ellie drank more than ½ quart and Javier drank ½ quart so together they drank slightly more than one quart.

Students make sense of fractional quantities when solving word problems, estimating answers mentally to see if they make sense.
Example:
Ludmilla and Lazarus each have a lemon. They need a cup of lemon juice to make hummus for a party. Ludmilla squeezes 1/2 a cup from hers and Lazarus squeezes 2/5 of a cup from his. How much lemon juice do they have? Is it enough?
Students estimate that there is almost but not quite one cup of lemon juice, because 2/5 < 1/2. They calculate 1/2 + 2/5 = 9/10, and see this as 1/10 less than 1, which is probably a small enough shortfall that it will not ruin the recipe. They detect an incorrect result such as 2/5 + 2/5 = 3/7 by noticing that 3/7 < 1/2.
(*Progressions for the CCSSM, Number and Operation – Fractions*, CCSS Writing Team, August 2011, page 11)

Common Core Cluster

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.
Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **fraction, numerator, denominator, operations, multiplication/multiply, division/divide, mixed numbers, product, quotient, partition, equal parts, equivalent, factor, unit fraction, area, side lengths, fractional sides lengths, scaling, comparing**

Common Core Standard	Unpacking What does this standards mean a child will know and be able to do?
5.NF.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*	Fifth grade student should connect fractions with division, understanding that 5 ÷ 3 = 5/3 Students should explain this by working with their understanding of division as equal sharing.  (*Progressions for the CCSSM, Number and Operation – Fractions*, CCSS Writing Team, August 2011, page 11) Students should also create story contexts to represent problems involving division of whole numbers.

Example:
If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? This can be solved in two ways. First, they might partition each pound among the 9 people, so that each person gets 50 x 1/9 = 50/9 pounds.
Second, they might use the equation 9 x 5= 45 to see that each person can be given 5 pounds, with 5 pounds remaining. Partitioning the remainder gives 5 5/9 pounds for each person.
(*Progressions for the CCSSM, Number and Operation – Fractions*, CCSS Writing Team, August 2011, page 11)

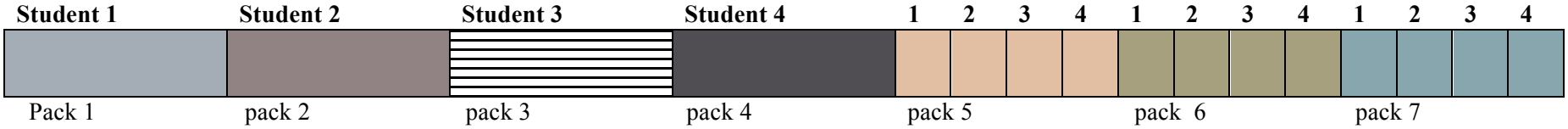
This standard calls for students to extend their work of partitioning a number line from third and fourth grade. Students need ample experiences to explore the concept that a fraction is a way to represent the division of two quantities.

Students are expected to demonstrate their understanding using concrete materials, drawing models, and explaining their thinking when working with fractions in multiple contexts. They read 3/5 as "three fifths" and after many experiences with sharing problems, learn that 3/5 can also be interpreted as "3 divided by 5."
Examples:
Ten team members are sharing 3 boxes of cookies. How much of a box will each student get?
When working this problem a student should recognize that the 3 boxes are being divided into 10 groups, so s/he is seeing the solution to the following equation, 10 x n = 3 (10 groups of some amount is 3 boxes) which can also be written as n = 3 ÷ 10. Using models or diagram, they divide each box into 10 groups, resulting in each team member getting 3/10 of a box.

Two afterschool clubs are having pizza parties. For the Math Club, the teacher will order 3 pizzas for every 5 students. For the student council, the teacher will order 5 pizzas for every 8 students. Since you are in both groups, you need to decide which party to attend. How much pizza would you get at each party? If you want to have the most pizza, which party should you attend?

The six fifth grade classrooms have a total of 27 boxes of pencils. How many boxes will each classroom receive? Students may recognize this as a whole number division problem but should also express this equal sharing problem as $^{27}/_{6}$. They explain that each classroom gets $^{27}/_{6}$ boxes of pencils and can further determine that each classroom get 4 $^{3}/_{6}$ or 4 $^{1}/_{2}$ boxes of pencils.

Example:
Your teacher gives 7 packs of paper to your group of 4 students. If you share the paper equally, how much paper does each student get?



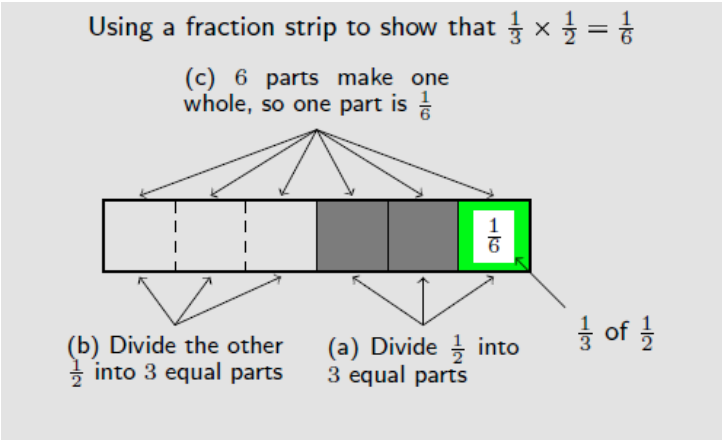
Each student receives 1 whole pack of paper and ¼ of the each of the 3 packs of paper. So each student gets 1 ¾ packs of paper.

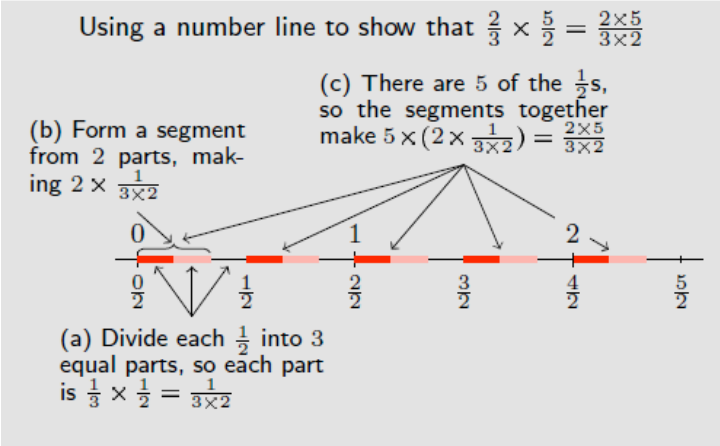
5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*

Students need to develop a fundamental understanding that the multiplication of a fraction by a whole number could be represented as repeated addition of a unit fraction (e.g., 2 x (1/4) = 1/4 + ¼

This standard extends student's work of multiplication from earlier grades. In fourth grade, students worked with recognizing that a fraction such as 3/5 actually could be represented as 3 pieces that are each one-fifth (3 x (1/5)). This standard references both the multiplication of a fraction by a whole number and the multiplication of two fractions. Visual fraction models (area models, tape diagrams, number lines) should be used and created by students during their work with this standard.





(*Progressions for the CCSSM, Number and Operation – Fractions*, CCSS Writing Team, August 2011, page 11)

As they multiply fractions such as 3/5 x 6, they can think of the operation in more than one way.

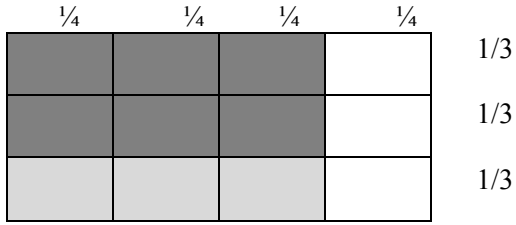
- 3 x (6 ÷ 5) or (3 x 6/5)
- (3 x 6) ÷ 5 or 18 ÷ 5 (18/5)

Students create a story problem for 3/5 x 6 such as,

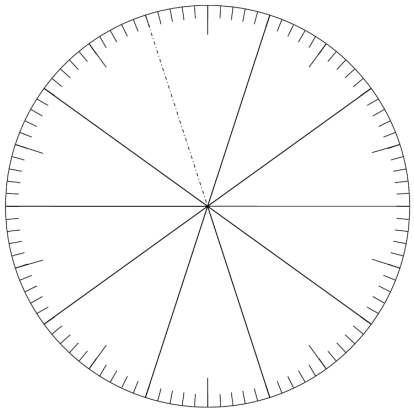
- Isabel had 6 feet of wrapping paper. She used 3/5 of the paper to wrap some presents. How much does she have left?
- Every day Tim ran 3/5 of mile. How far did he run after 6 days? (Interpreting this as 6 x 3/5)

Example:
Three-fourths of the class is boys. Two-thirds of the boys are wearing tennis shoes. What fraction of the class are boys with tennis shoes?
This question is asking what 2/3 of ¾ is, or what is 2/3 x ¾. What is 2/3 x ¾, in this case you have 2/3 groups of size ¾ (a way to think about it in terms of the language for whole numbers is 4 x 5 you have 4 groups of size 5.
The array model is very transferable from whole number work and then to binomials.

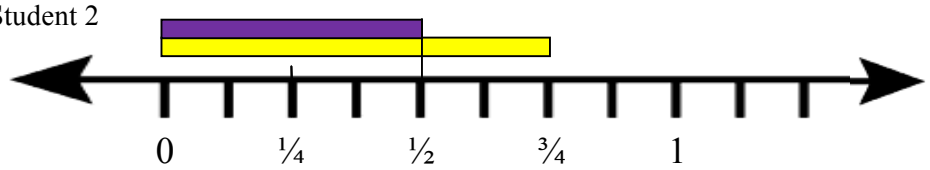
Student 1
I drew a rectangle to represent the whole class. The four columns represent the fourths of a class. I shaded 3 columns to represent the fraction that are boys. I then split the rectangle with horizontal lines into thirds. The dark area represents the fraction of the boys in the class wearing tennis shoes, which is 6 out of 12. That is 6/12, which equals 1/2.



Student 3
Fraction circle could be used to model student thinking. First I shade the fraction circle to show the ¾ and then overlay with 2/3 of that?



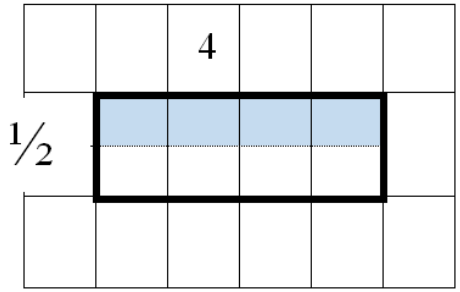
Student 2



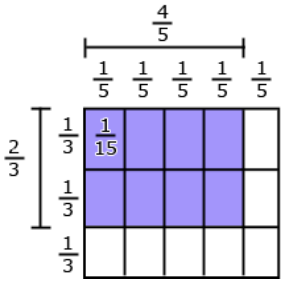
b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

This standard extends students' work with area. In third grade students determine the area of rectangles and composite rectangles. In fourth grade students continue this work. The fifth grade standard calls students to continue the process of covering (with tiles). Grids (see picture) below can be used to support this work.

Example:
The home builder needs to cover a small storage room floor with carpet. The storage room is 4 meters long and half of a meter wide. How much carpet do you need to cover the floor of the storage room? Use a grid to show your work and explain your answer. In the grid below I shaded the top half of 4 boxes. When I added them together, I added ½ four times, which equals 2. I could also think about this with multiplication ½ x 4 is equal to 4/2 which is equal to 2.



Example:
In solving the problem $\frac{2}{3} \times \frac{4}{5}$, students use an area model to visualize it as a 2 by 4 array of small rectangles each of which has side lengths 1/3 and 1/5. They reason that 1/3 x 1/5 = 1/(3 x 5) by counting squares in the entire rectangle, so the area of the shaded area is (2 x 4) x 1/(3 x 5) = $\frac{2 \times 4}{3 \times 5}$. They can explain that the product is less than $\frac{4}{5}$ because they are finding $\frac{2}{3}$ of $\frac{4}{5}$. They can further estimate that the answer must be between $\frac{2}{5}$ and $\frac{4}{5}$ because $\frac{2}{3}$ of $\frac{4}{5}$ is more than $\frac{1}{2}$ of $\frac{4}{5}$ and less than one group of $\frac{4}{5}$.



The area model and the line segments show that the area is the same quantity as the product of the side lengths.

5.NF.5 Interpret multiplication as scaling (resizing), by:

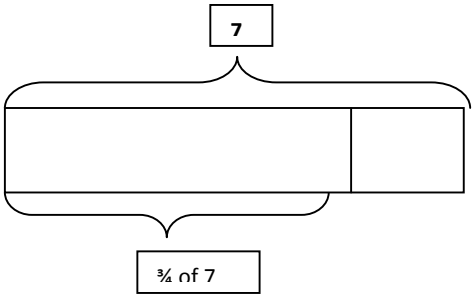
a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

This standard calls for students to examine the magnitude of products in terms of the relationship between two types of problems. This extends the work with 5.OA.1.

Example 1:
Mrs. Jones teaches in a room that is 60 feet wide and 40 feet long. Mr. Thomas teaches in a room that is half as wide, but has the same length. How do the dimensions and area of Mr. Thomas' classroom compare to Mrs. Jones' room? Draw a picture to prove your answer.

Example 2:
How does the product of 225 x 60 compare to the product of 225 x 30? How do you know?
Since 30 is half of 60, the product of 22 5x 60 will be double or twice as large as the product of 225 x 30.

Example:
$\frac{3}{4} \times 7$ is less than 7 because 7 is multiplied by a factor less than 1 so the product must be less than 7.



b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.	This standard asks students to examine how numbers change when we multiply by fractions. Students should have ample opportunities to examine both cases in the standard: a) when multiplying by a fraction greater than 1, the number increases and b) when multiplying by a fraction less the one, the number decreases. This standard should be explored and discussed while students are working with 5.NF.4, and should not be taught in isolation. Example: Mrs. Bennett is planting two flower beds. The first flower bed is 5 meters long and 6/5 meters wide. The second flower bed is 5 meters long and 5/6 meters wide. How do the areas of these two flower beds compare? Is the value of the area larger or smaller than 5 square meters? Draw pictures to prove your answer. Example: $2\frac{2}{3}$ x 8 must be more than 8 because 2 groups of 8 is 16 and $2\frac{2}{3}$ is almost 3 groups of 8. So the answer must be close to, but less than 24. $\frac{3}{4} = \frac{5 \times 3}{5 \times 4}$ because multiplying $\frac{3}{4}$ by $\frac{5}{5}$ is the same as multiplying by 1.

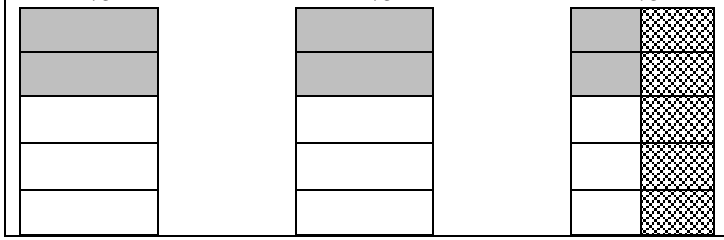
5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

This standard builds on all of the work done in this cluster. Students should be given ample opportunities to use various strategies to solve word problems involving the multiplication of a fraction by a mixed number. This standard could include fraction by a fraction, fraction by a mixed number or mixed number by a mixed number.

Example:
There are 2 ½ bus loads of students standing in the parking lot. The students are getting ready to go on a field trip. 2/5 of the students on each bus are girls. How many busses would it take to carry ***only*** the girls?

Student 1
I drew 3 grids and 1 grid represents 1 bus. I cut the third grid in half and I marked out the right half of the third grid, leaving 2 ½ grids. I then cut each grid into fifths, and shaded two-fifths of each grid to represent the number of girls. When I added up the shaded pieces, 2/5 of the 1st and 2nd bus were both shaded, and 1/5 of the last bus was shaded.

2/5 + 2/5 + 1/5 = 5/5 = 1 whole bus.



Student 2
2 ½ x 2/5=
I split the 2 ½ into 2 and ½
2 x 2/5 = 4/5
½ x 2/5 = 2/10
I then added 4/5 and 2/10. That equals 1 whole bus load.

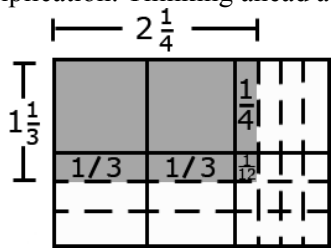
Example:
Evan bought 6 roses for his mother. $\frac{2}{3}$ of them were red. How many red roses were there?
Using a visual, a student divides the 6 roses into 3 groups and counts how many are in 2 of the 3 groups.

A student can use an equation to solve.
$\frac{2}{3} \times 6 = \frac{12}{3} = 4$ red roses

Example:

Mary and Joe determined that the dimensions of their school flag needed to be $1\frac{1}{3}$ ft. by $2\frac{1}{4}$ ft. What will be the area of the school flag?

A student can draw an array to find this product and can also use his or her understanding of decomposing numbers to explain the multiplication. Thinking ahead a student may decide to multiply by $1\frac{1}{3}$ instead of $2\frac{1}{4}$.



The explanation may include the following:

- First, I am going to multiply $2\frac{1}{4}$ by 1 and then by $\frac{1}{3}$.
- When I multiply $2\frac{1}{4}$ by 1, it equals $2\frac{1}{4}$.
- Now I have to multiply $2\frac{1}{4}$ by $\frac{1}{3}$.
- $\frac{1}{3}$ times 2 is $\frac{2}{3}$.
- $\frac{1}{3}$ times $\frac{1}{4}$ is $\frac{1}{12}$.
- So the answer is $2\frac{1}{4} + \frac{2}{3} + \frac{1}{12}$ or $2\frac{3}{12} + \frac{8}{12} + \frac{1}{12} = 2\frac{12}{12} = 3$

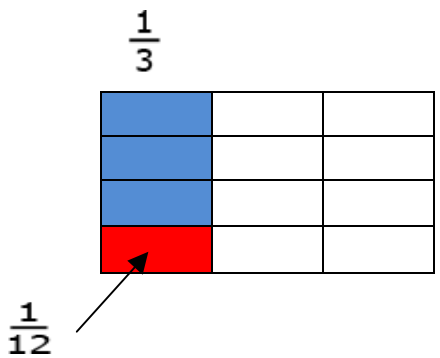
5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]

a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*

[1] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

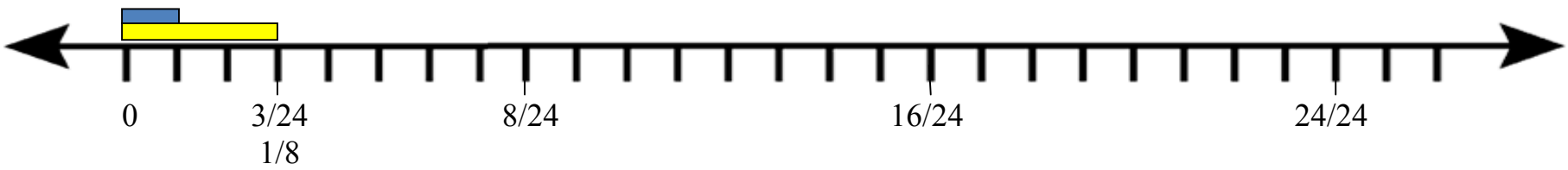
5.NF.7 is the first time that students are dividing with fractions. In fourth grade students divided whole numbers, and multiplied a whole number by a fraction. The concept *unit fraction* is a fraction that has a one in the denominator. For example, the fraction 3/5 is 3 copies of the unit fraction 1/5. 1/5 + 1/5 + 1/5 = 3/5 = 1/5 x 3 or 3 x 1/5

Example:
Knowing the number of groups/shares and finding how many/much in each group/share
Four students sitting at a table were given 1/3 of a pan of brownies to share. How much of a pan will each student get if they share the pan of brownies equally?
The diagram shows the 1/3 pan divided into 4 equal shares with each share equaling 1/12 of the pan.

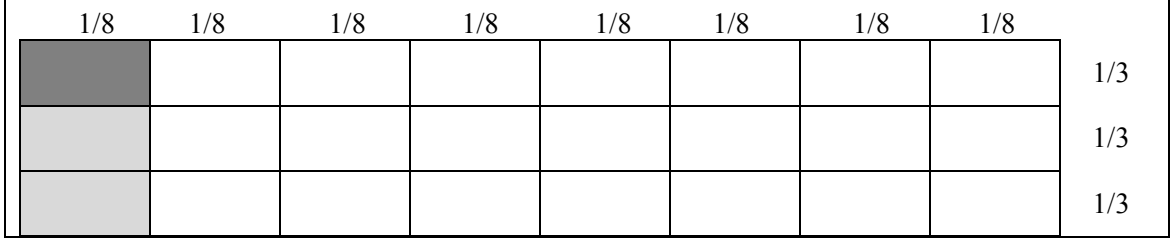


5.NF.7a This standard asks students to work with story contexts where a unit fraction is divided by a non-zero whole number. Students should use various fraction models and reasoning about fractions.
Example:
You have 1/8 of a bag of pens and you need to share them among 3 people. How much of the bag does each person get?

Student 1
Expression 1/ 8 ÷ 3



Student 2
I drew a rectangle and divided it into 8 columns to represent my 1/8. I shaded the first column. I then needed to divide the shaded region into 3 parts to represent sharing among 3 people. I shaded one-third of the first column even darker. The dark shade is 1/24 of the grid or 1/24 of the bag of pens.



Student 3
1/8 of a bag of pens divided by 3 people. I know that my answer will be less than 1/8 since I'm sharing 1/8 into 3 groups. I multiplied 8 by 3 and got 24, so my answer is 1/24 of the bag of pens. I know that my answer is correct because (1/24) x 3 = 3/24 which equals 1/8.

b. Interpret division of a whole number by a unit fraction, and compute such quotients**.** *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*

5.NF.7b This standard calls for students to create story contexts and visual fraction models for division situations where a whole number is being divided by a unit fraction.

Example:
Create a story context for 5 ÷ 1/6. Find your answer and then draw a picture to prove your answer and use multiplication to reason about whether your answer makes sense. How many 1/6 are there in 5?

> Student
> The bowl holds 5 Liters of water. If we use a scoop that holds 1/6 of a Liter, how many scoops will we need in order to fill the entire bowl?
>
> I created 5 boxes. Each box represents 1 Liter of water. I then divided each box into sixths to represent the size of the scoop. My answer is the number of small boxes, which is 30. That makes sense since 6 x 5 = 30.
>
> 
>
> 1 = 1/6 + 1/6 + 1/6 + 1/6 + 1/6 a whole has 6/6 so five wholes would be 6/6 + 6/6 + 6/6 + 6/6 + 6/6 =30/6

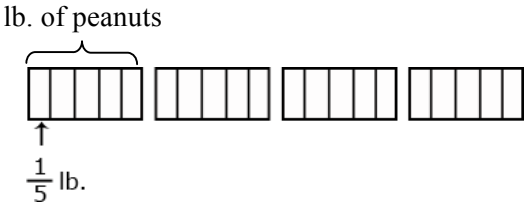
c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share ½ lb of chocolate equally? How many 1/3-cup servings are 2 cups of raisins?*

5.NF.7c extends students' work from other standards in 5.NF.7. Student should continue to use visual fraction models and reasoning to solve these real-world problems.

Example:
How many 1/3-cup servings are in 2 cups of raisins?

> Student
> I know that there are three 1/3 cup servings in 1 cup of raisins. Therefore, there are 6 servings in 2 cups of raisins. I can also show this since 2 divided by 1/3 = 2 x 3 = 6 servings of raisins.

Examples:
Knowing how many in each group/share and finding how many groups/shares
Angelo has 4 lbs of peanuts. He wants to give each of his friends 1/5 lb. How many friends can receive 1/5 lb of peanuts?
A diagram for 4 ÷ 1/5 is shown below. Students explain that since there are five fifths in one whole, there must be 20 fifths in 4 lbs.



	Example: How much rice will each person get if 3 people share 1/2 lb of rice equally? $\frac{1}{2} \div 3 = \frac{3}{6} \div 3 = \frac{1}{6}$ A student may think or draw ½ and cut it into 3 equal groups then determine that each of those part is 1/6. A student may think of ½ as equivalent to 3/6. 3/6 divided by 3 is 1/6.

Measurement and Data 5.MD

Common Core Cluster

Convert like measurement units within a given measurement system.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **conversion/convert, metric and customary measurement**
From previous grades: relative size, liquid volume, mass, length, kilometer (km), meter (m), centimeter (cm), kilogram (kg), gram (g), liter (L), milliliter (mL), inch (in), foot (ft), yard (yd), mile (mi), ounce (oz), pound (lb), cup (c), pint (pt), quart (qt), gallon (gal), hour, minute, second

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.	**5.MD.1** calls for students to convert measurements within the same system of measurement in the context of multi-step, real-world problems. Both customary and standard measurement systems are included; students worked with both metric and customary units of length in second grade. In third grade, students work with metric units of mass and liquid volume. In fourth grade, students work with both systems and begin conversions within systems in length, mass and volume. Students should explore how the base-ten system supports conversions within the metric system. Example: 100 cm = 1 meter.

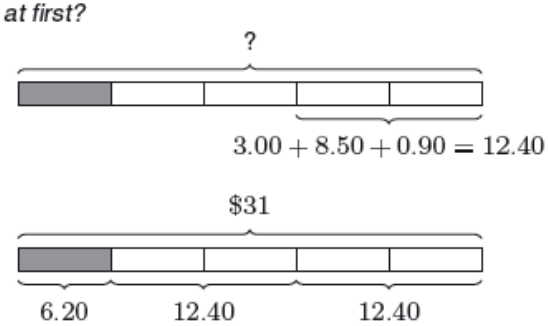
In Grade 5, students extend their abilities from Grade 4 to express measurements in larger or smaller units within a measurement system. This is an excellent opportunity to reinforce notions of place value for whole numbers and decimals, and connection between fractions and decimals (e.g., 2 ½ meters can be expressed as 2.5 meters or 250 centimeters). For example, building on the table from Grade 4, Grade 5 students might complete a table of equivalent measurements in feet and inches. Grade 5 students also learn and use such conversions in solving multi-step, real world problems (see example below).

Feet	Inches
0	0
	1
	2
	3

In Grade 6, this table can be discussed in terms of ratios and proportional relationships (see the Ratio and Proportion Progression). In Grade 5, however, the main focus is on arriving at the measurements that generate the table.

Multi-step problem with unit conversion

Kumi spent a fifth of her money on lunch. She then spent half of what remained. She bought a card game for $3, a book for $8.50, and candy for 90 cents. How much money did she have at first?



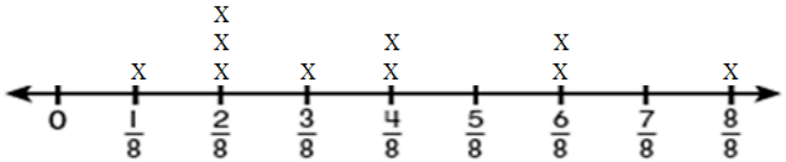
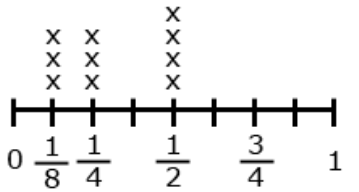
Students can use tape diagrams to represent problems that involve conversion of units, drawing diagrams of important features and relationships (MP1).

(*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, August 2011, page 26)

Common Core Cluster

Represent and interpret data.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **line plot, length, mass, liquid volume**

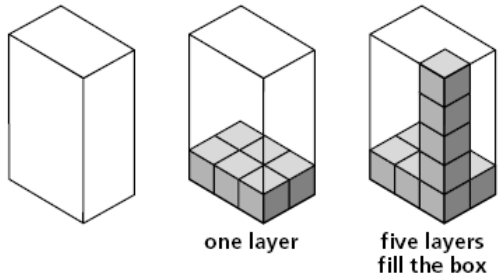
Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
5. MD.2 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*	**5.MD.2** This standard provides a context for students to work with fractions by measuring objects to one-eighth of a unit. This includes length, mass, and liquid volume. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot. Example: Students measured objects in their desk to the nearest ½, ¼, or 1/8 of an inch then displayed data collected on a line plot. How many object measured ¼? ½? If you put all the objects together end to end what would be the total length of **all** the objects?  Example: Ten beakers, measured in liters, are filled with a liquid. Liquid in Beakers  Amount of Liquid (in Liters) The line plot above shows the amount of liquid in liters in 10 beakers. If the liquid is redistributed equally, how much liquid would each beaker have? (This amount is the mean.) Students apply their understanding of operations with fractions. They use either addition and/or multiplication to determine the total number of liters in the beakers. Then the sum of the liters is shared evenly among the ten beakers.

Common Core Cluster

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **measurement, attribute, volume, solid figure, right rectangular prism, unit, unit cube, gap, overlap, cubic units (cubic cm, cubic in., cubic ft., nonstandard cubic units), multiplication, addition, edge lengths, height, area of base**

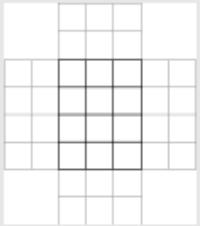
Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
5. MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.	**5. MD.3, 5.MD.4,** and **5. MD.5** These standards represent the first time that students begin exploring the concept of volume. In third grade, students begin working with area and covering spaces. The concept of volume should be extended from area with the idea that students are covering an area (the bottom of cube) with a layer of unit cubes and then adding layers of unit cubes on top of bottom layer (see picture below). Students should have ample experiences with concrete manipulatives before moving to pictorial representations. Students' prior experiences with volume were restricted to liquid volume. As students develop their understanding volume they understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. This cube has a length of 1 unit, a width of 1 unit and a height of 1 unit and is called a cubic unit. This cubic unit is written with an exponent of 3 (e.g., in^3, m^3). Students connect this notation to their understanding of powers of 10 in our place value system. Models of cubic inches, centimeters, cubic feet, etc are helpful in developing an image of a cubic unit. Students' estimate how many cubic yards would be needed to fill the classroom or how many cubic centimeters would be needed to fill a pencil box.
5. MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	 (3 x 2) represented by first layer (3 x 2) x 5 represented by number of 3 x 2 layers (3 x 2) + (3 x 2) + (3 x 2) + (3 x 2)+ (3 x 2) = 6 + 6 + 6 + 6 + 6 + 6 = 30 6 representing the size/area of one layer
5. MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the	

edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.	The major emphasis for measurement in Grade 5 is volume. Volume not only introduces a third dimension and thus a significant challenge to students' spatial structuring, but also complexity in the nature of the materials measured. That is, solid units are "packed," such as cubes in a three-dimensional array, whereas a liquid "fills" three-dimensional space, taking the shape of the container. The unit structure for liquid measurement may be psychologically one dimensional for some students. "Packing" volume is more difficult than iterating a unit to measure length and measuring area by tiling. Students learn about a unit of volume, such as a cube with a side length of 1 unit, called a unit cube.5.MD.3 They pack cubes (without gaps) into right rectangular prisms and count the cubes to determine the volume or build right rectangular prisms from cubes and see the layers as they build.5.MD.4 They can use the results to compare the volume of right rectangular prisms that have different dimensions. Such experiences enable students to extend their spatial structuring from two to three dimensions. That is, they learn to both mentally decompose and recompose a right rectangular prism built from cubes into layers, each of which is composed of rows and columns. That is, given the prism, they have to be able to decompose it, understanding that it can be partitioned into layers, and each layer partitioned into rows, and each row into cubes. They also have to be able to compose such as structure, multiplicatively, back into higher units. That is, they eventually learn to conceptualize a layer as a unit that itself is composed of units of units—rows, each row composed of individual cubes—and they iterate that structure. Thus, they might predict the number of cubes that will be needed to fill a box given the net of the box. Another complexity of volume is the connection between "packing" and "filling." Often, for example, students will respond that a box can be filled with 24 centimeter cubes, or build a structure of 24 cubes, and still think of the 24 as individual, often discrete, not necessarily *units of volume*. They may, for example, not respond confidently and correctly when asked to fill a graduated cylinder marked in cubic centimeters with the amount of liquid that would fill the box. That is, they have not yet connected their ideas about filling volume with those concerning packing volume. Students learn to move between these conceptions, e.g., using the same container, both filling (from a graduated cylinder marked in ml or cc) and packing (with cubes that are each 1 cm^3). Comparing and discussing the volume-units and what they represent can help students learn a general, complete, and interconnected conceptualization of volume as filling three-dimensional space. Students then learn to determine the volumes of several right rectangular prisms, using cubic centimeters, cubic inches, and cubic feet. With guidance, they learn to increasingly apply multiplicative reasoning to determine volumes, looking for and making use of structure. That is, they understand that multiplying the length times the width of a right rectangular prism can be viewed as determining how many cubes would be in each layer if the prism were packed with or built up from unit cubes.5.MD.5a They also learn that the height of the prism tells how many layers would fit in the prism. That is, they understand that volume is a derived attribute that, once a length unit is specified, can be computed as the product of three length measurements or as the product of one area and one length measurement.

Then, students can learn the formulas $V = l \times w \times h$ and $V = B \times h$ for right rectangular prisms as efficient methods for computing volume, maintaining the connection between these methods and their previous work with computing the number of unit cubes that pack a right rectangular prism.5.MD.5b They use these competencies to find the volumes of right rectangular prisms with edges whose lengths are whole numbers and solve real-world and mathematical problems involving such prisms.

Students also recognize that volume is additive and they find the total volume of solid figures composed of two right rectangular prisms.5.MD.5c For example, students might design a science station for the ocean floor that is composed of several rooms that are right rectangular prisms and that meet a set criterion specifying the total volume of the station. They draw their station and justify how their design meets the criterion.

Net for five faces of a right rectangular prism



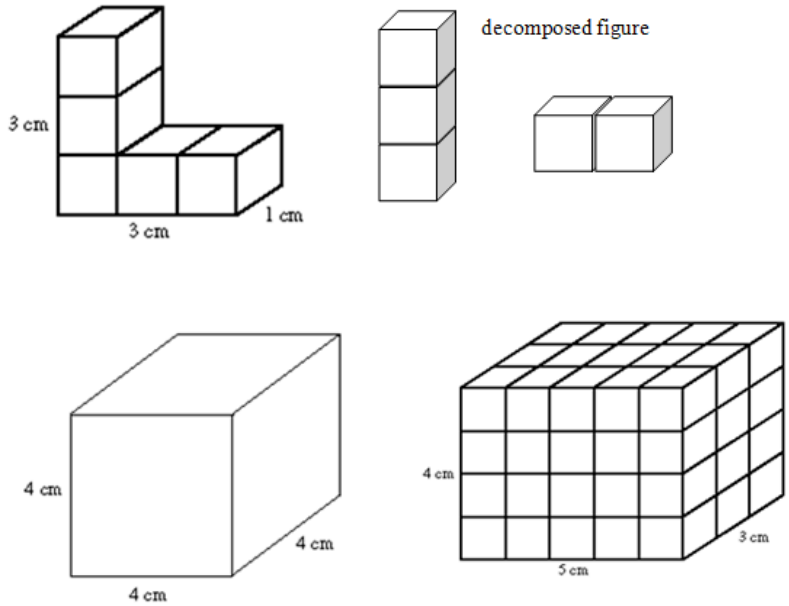
Students are given a net and asked to predict the number of cubes required to fill the container formed by the net. In such tasks, students may initially count single cubes or repeatedly add the number of cubes in a row to determine the number in each layer, and repeatedly add the number in each layer to find the total number of unit cubes. In folding the net to make the shape, students can see how the side rectangles fit together and determine the number of layers.

(*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, August 2011, page 26)

5. MD.5a & b These standards involve finding the volume of right rectangular prisms (see picture above).Students should have experiences to describe and reason about why the formula is true. Specifically, that they are covering the bottom of a right rectangular prism (length x width) with multiple layers (height). Therefore, the formula (length x width x height) is an extension of the formula for the area of a rectangle.

5.MD.5c This standard calls for students to extend their work with the area of composite figures into the context of volume. Students should be given concrete experiences of breaking apart (decomposing) 3-dimensional figures into right rectangular prisms in order to find the volume of the entire 3-dimensional figure.

Examples:

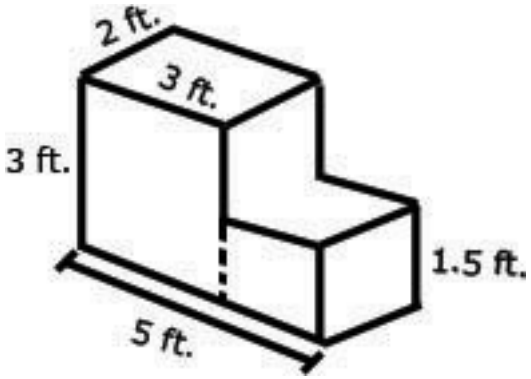


students need multiple opportunities to measure volume by filling rectangular prisms with cubes and looking at the relationship between the total volume and the area of the base. They derive the volume formula (volume equals the area of the base times the height) and explore how this idea would apply to other prisms. Students use the associative property of multiplication and decomposition of numbers using factors to investigate rectangular prisms with a given number of cubic units.

Example:
When given 24 cubes, students make as many rectangular prisms as possible with a volume of 24 cubic units. Students build the prisms and record possible dimensions.

Length	Width	Height
1	2	12
2	2	6
4	2	3
8	3	1

Example:
Students determine the volume of concrete needed to build the steps in the diagram below.

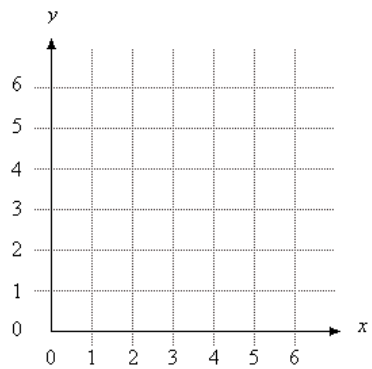


Geometry 5.G

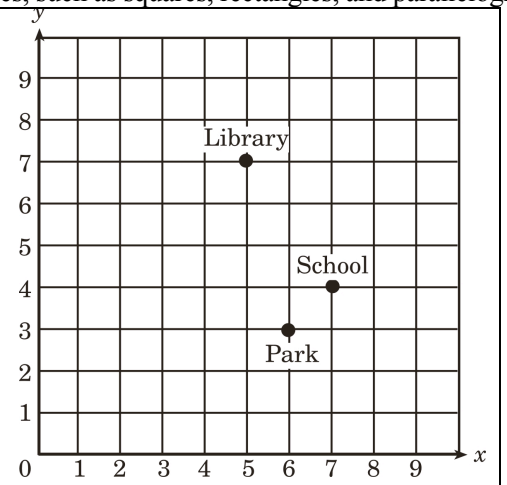
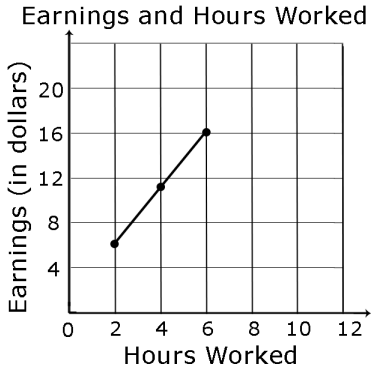
Common Core Cluster

Graph points on the coordinate plane to solve real-world and mathematical problems.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **coordinate system, coordinate plane, first quadrant, points, lines, axis/axes, x-axis, y-axis, horizontal, vertical, intersection of lines, origin, ordered pairs, coordinates, x-coordinate, y-coordinate**

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).	**5.G.1** and **5.G.2** These standards deal with only the first quadrant (positive numbers) in the coordinate plane. Although students can often "locate a point," these understandings are beyond simple skills. For example, initially, students often fail to distinguish between two different ways of viewing the point (2, 3), say, as instructions: "right 2, up 3"; and as the point defined by being a distance 2 from the *y*-axis and a distance 3 from the *x*-axis. In these two descriptions the 2 is first associated with the *x*-axis, then with the *y*-axis. Example: Connect these points in order on the coordinate grid below: (2, 2) (2, 4) (2, 6) (2, 8) (4, 5) (6, 8) (6, 6) (6, 4) and (6, 2). What letter is formed on the grid? *Solution: "M" is formed.* **Coordinate Grid** 

		Example: Plot these points on a coordinate grid. Point A: (2,6) Point B: (4,6) Point C: (6,3) Point D: (2,3) Connect the points in order. Make sure to connect Point D back to Point A. 1. What geometric figure is formed? What attributes did you use to identify it? 2. What line segments in this figure are parallel? 3. What line segments in this figure are perpendicular? *solutions: trapezoid, line segments AB and DC are parallel, segments AD and DC are perpendicular* Example: Emanuel draws a line segment from (1, 3) to (8, 10). He then draws a line segment from (0, 2) to (7, 9). If he wants to draw another line segment that is parallel to those two segments what points will he use?

5.G.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	This standard references real-world and mathematical problems, including the traveling from one point to another and identifying the coordinates of missing points in geometric figures, such as squares, rectangles, and parallelograms. Example: Using the coordinate grid, which ordered pair represents the location of the School? Explain a possible path from the school to the library.  Example: Sara has saved $20. She earns $8 for each hour she works. If Sara saves all of her money, how much will she have after working 3 hours? 5 hours? 10 hours? Create a graph that shows the relationship between the hours Sara worked and the amount of money she has saved. What other information do you know from analyzing the graph? Example: Use the graph below to determine how much money Jack makes after working exactly 9 hours. 

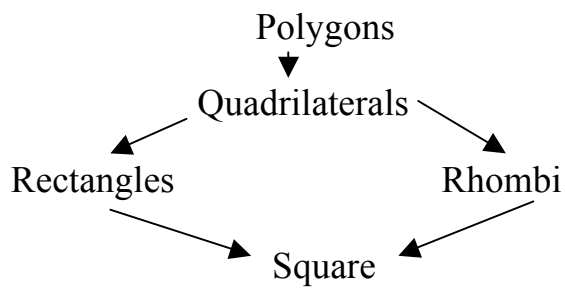
Common Core Cluster

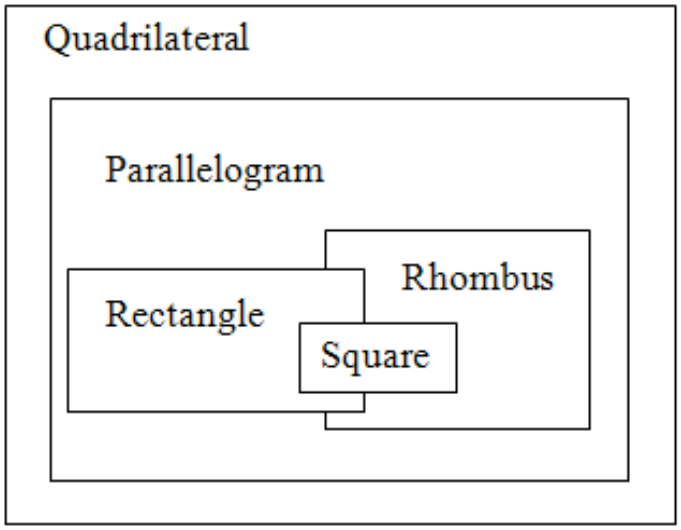
Classify two-dimensional figures into categories based on their properties.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **attribute, category, subcategory, hierarchy, (properties)-rules about how numbers work, two dimensional**
From previous grades: **polygon, rhombus/rhombi, rectangle, square, triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle, kite**

[1]The term "**property**" in these standards is reserved for those attributes that indicate a relationship between components of shapes. Thus, "having parallel sides" or "having all sides of equal lengths" are properties. "**Attributes**" and "**features**" are used interchangeably to indicate any characteristic of a shape, including properties, and other defining characteristics (e.g., straight sides) and nondefining characteristics (e.g., "right-side up").
(Progressions for the CCSSM, Geometry, CCSS Writing Team, June 2012, page 3 footnote)

Common Core Standard	Unpacking What do these standards mean a child will know and be able to do?
5.G.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*	This standard calls for students to reason about the attributes (properties) of shapes. Student should have experiences discussing the property of shapes and reasoning. Example: Examine whether all quadrilaterals have right angles. Give examples and non-examples. Example: If the opposite sides on a parallelogram are parallel and congruent, then rectangles are parallelograms A sample of questions that might be posed to students include: A parallelogram has 4 sides with both sets of opposite sides parallel. What types of quadrilaterals are parallelograms? Regular polygons have all of their sides and angles congruent. Name or draw some regular polygons. All rectangles have 4 right angles. Squares have 4 right angles so they are also rectangles. True or False? A trapezoid has 2 sides parallel so it must be a parallelogram. True or False? The notion of congruence ("same size and same shape") may be part of classroom conversation but the concepts of congruence and similarity do **not** appear until middle school. **TEACHER NOTE:** In the U.S., the term "trapezoid" may have two different meanings. Research identifies these as inclusive and exclusive definitions. The inclusive definition states: A trapezoid is a quadrilateral with *at least* one pair of parallel sides. The exclusive definition states: **A trapezoid is a quadrilateral with *exactly* one pair of parallel sides.** With this definition, a parallelogram is not a trapezoid. North Carolina has adopted the exclusive definition. (*Progressions for the CCSSM: Geometry*, The Common Core Standards Writing Team, June 2012.)

	http://illuminations.nctm.org/ActivityDetail.aspx?ID=70
5.G.4 Classify two-dimensional figures in a hierarchy based on properties.	This standard builds on what was done in 4th grade. Figures from previous grades: **polygon, rhombus/rhombi, rectangle, square, triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle, kite** A **kite** is a quadrilateral whose four sides can be grouped into two pairs of equal-length sides that are beside (adjacent to) each other. Example: Create a Hierarchy Diagram using the following terms: polygons – a closed plane figure formed from line segments that meet only at their endpoints. quadrilaterals - a four-sided polygon. rectangles - a quadrilateral with two pairs of congruent parallel sides and four right angles. rhombi – a parallelogram with all four sides equal in length. square – a parallelogram with four congruent sides and four right angles. Possible student solution: 

quadrilateral – a four-sided polygon. parallelogram – a quadrilateral with two pairs of parallel and congruent sides. rectangle – a quadrilateral with two pairs of congruent, parallel sides and four right angles. rhombus – a parallelogram with all four sides equal in length. square – a parallelogram with four congruent sides and four right angles.	Possible student solution: 

Student should be able to reason about the attributes of shapes by examining: What are ways to classify triangles? Why can't trapezoids and kites be classified as parallelograms? Which quadrilaterals have opposite angles congruent and why is this true of certain quadrilaterals?, and How many lines of symmetry does a regular polygon have?

TEACHER NOTE: In the U.S., the term "trapezoid" may have two different meanings. Research identifies these as inclusive and exclusive definitions. The inclusive definition states: A trapezoid is a quadrilateral with *at least* one pair of parallel sides. The exclusive definition states: **A trapezoid is a quadrilateral with *exactly* one pair of parallel sides.** With this definition, a parallelogram is not a trapezoid. North Carolina has adopted the exclusive definition. (*Progressions for the CCSSM: Geometry*, The Common Core Standards Writing Team, June 2012.)

Some examples used in this document are from the Arizona Mathematics Education Department

Glossary

Table 1 Common addition and subtraction situations[1]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3

	Total Unknown	Addend Unknown	Both Addends Unknown[2]
Put Together/ Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare[4]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).
[2]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.
[3]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.
[4]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2 Common multiplication and division situations[1]

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$, and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[2] Area[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[1]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.
[2]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3 The properties of operations

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

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